Stem and merchantable volume equations for hybrid aspen growing on farmland in Sweden

Tord Johansson
ABSTRACT

In this study, stem volume models for hybrid aspen were developed. The study was based on 29 stands located in middle and southern Sweden (latitudes 55 – 60° N.). The mean total age of the hybrid aspen was 22 years (range 15 – 50 years) with a mean stand density of 1006 stems ha\(^{-1}\) (90 – 2402) and a mean diameter at breast height (over bark) of 19.7 cm (8.5 – 40.8 cm). A number of equations were assessed for modeling stem volume. Standing volume was examined in relation to soil type. Multiple samples were collected for three types of soils: sediments of light clay and medium clay and till soils. A power equation, \(V = \beta_0 D^{\beta_1} H^{\beta_2}\), fits the data best. This model explained 99% of the observed variation in prediction of observed data and exhibited no apparent bias across the range of predicted stem volumes. Merchantable volume equations for the estimation of commercial volume for any top diameters and bole length were developed. They explained 99% of the observed variation in prediction of observed data. Soil types in the stands were light and medium clays and tills. The standing volume was calculated, the mean was 301±167 (range 111-701) m\(^3\) ha\(^{-1}\). Mean annual increment was also calculated, the mean was 14.6±8.0 (range 2.8-36.9 m\(^3\) ha\(^{-1}\) year\(^{-1}\). There were no significant differences between the soil types and MAI.

Key words: farmland, hybrid aspen, MAI, merchantable volume, soil type, stands, stem volume, total age
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SAMMANFATTNING

Hybridasp (Populus tremula x Populus tremuloides) är en hybrid mellan den europeiska aspen (Populus tremula L.) och den amerikanska aspen (Populus tremuloides Michx.). Den europeiska aspen är ett av de träd som har den största naturliga geografiska utbredningen. Den finns på den norra hemisfären från Storbritannien i väster till Kina i öster och förekommer i Norge med svag växt upp till latitud 70° N.). Den amerikanska aspen är den mest utbredda trädarten i Nordamerika och är spridd från östra till västra delen av landet.


INTRODUCTION

Hybridization between the European aspen (*Populus tremula* L.) and the trembling aspen (*Populus tremuloides* Michx.) was first described at the beginning of the 1920s in Germany (Wettstein 1933). The first artificial crosses between European and trembling aspens in Sweden were undertaken in 1939 (Rytter & Stener 2005) with the earliest cultivation of this hybrid aspen being achieved at the end of the 1930s (Johnsson 1953, 1976; Langhammer 1973). The Swedish Match Company started a breeding program in the late 1940s, focusing mainly on developing fast-growing individuals with a high yield (Christersson 1996). Later on, in the 1960s, interest in match production decreased in Sweden and this breeding work lessened. In the 1980s, interest for hybrid aspen as a fast-growing species with a high growth capacity increased. In a crossbreeding study on species in the poplar section Leuce, European aspen, trembling aspen and hybrid aspen, with their low demand for nutrients and water, were favored over most other poplar species (Leibundgut 1967; Mohrdiek 1977). In an Estonian study, the concentration of gravel and clay correlated, respectively, negatively and positively with sand on the fast growth of hybrid aspen (Tullus et al. 2007). Fast growth occurred on nutrient-rich, well aerated and moderately drained Albeluvisols, Luvisols and Planosols (Tullus et al. 2010).

The yield of planted hybrid aspen trees have been studied in Sweden, Finland, Norway, Denmark, the Baltic States and the Great Lakes Region of the USA (Møller 1965; Benson & Einspahr 1967; Christersson 1996; Li et al., 1998; Dickmann 2001; Yu et al. 2001; Karačić et al. 2003; Rytter & Stener 2003; Tullus et al. 2011; Johansson 2013a, 2013b). Until recently, hybrid aspen trees in Sweden were only grown on small plantations, most of which were established between 1980 and 1990 on farmland set aside for the assessment of their productivity. In Sweden, hybrid aspen is used mainly as pulpwood, biofuel and wood for the production of matches, with a small amount being used for the production of plywood.

Results from Nordic trials on hybrid aspen (*Populus tremula* L. x *Populus tremuloides* Michx.) stands established between 1940 and 1960 showed mean annual increments (MAI) between 8 and 26 m³ ha⁻¹ year⁻¹ (among others: Johnsson 1953, 1976; Langhammer 1973; Persson 1973; Jacobsen 1976; Eriksson 1984; Elfving 1986 a, b; Ilstedt and Gullberg 1993). Results from stands established at the end of 1980s confirm the figures to be above 12.7 (2-41) m³ ha⁻¹ year⁻¹ (Johansson 2010). The second generation of hybrid aspen, similar to the parent species, exhibits rapid establishment and fast growth. Hybrid aspen stands typically
produce 50,000 to 100,000 suckers per hectare after cutting (Rytter 2006). In a German study (Liesebach et al. 1999), the number of suckers per hectare established after harvesting the first plantation (10 years) was 165,000; after the second rotation (10 years), the number had increased to 215,000.

Volume equations for individual trees are an important topic of research in forest management (Fang & Bailey 1999). The height and diameter are easy to measure and estimate, but the stem form is a complex trait (Assmann 1970) that varies substantially within and between species, soil type, hydrology, altitude, climate, forest management practices (Steven & Benee 1988; Karlsson 2005) and interactions between all of these factors. This range of factors complicates the development of a universal model.

There are several methods for estimating stem volume (Clutter et al. 1983). Sections (1 – 3 m) of the stem can be used to calculate stem volume using several different formulas (Young and Wilson 1967). Summation of the section volumes and the top volume (using the cone formula) gives the (observed) tree volume. The volume of an individual tree depends on its height, diameter and stem form. Different equations for predicting stem volume have been devised. Equations based on basal area at breast height, tree height and a form factor are common volume estimation tools c.f. Näslund (1947). Equations for stem and commercial volume for specified commercial minimum diameters (i.e. one equation for each specification) are the most often used by Scandinavian forest management. Some equations are functions of DBH alone (Case & Hall 2008; Gautam & Thapa 2009). Other researchers have proposed equations that use a broader range of independent variables, such as the diameter at a specified upper height, the height at the crown base, the bark thickness, along with site indicator variables such as altitude, latitude, soil type and vegetation type (Burk et al. 1989; Brandel 1990). Most of the stem volume equations used in Swedish forestry have been developed for the three species Scots pine (*Pinus sylvestris* L.), Norway spruce (*Picea abies* (L.) Karst.) and birch (*Betula pendula* Roth and *Betula pubescens* Ehrh.), although equations have also been developed for less common species, including conifers and domestic broadleaf species.

Prior to this study, just one stem volume equation based on hybrid aspen stems in breeding plots in Sweden has been published (Johnsson 1953). An equation based on data from hybrid aspen stands growing on farmland could be used in surveys of the stem volume of individual
hybrid aspens, for estimating the volumes of hybrid aspen stands and for constructing volume tables.

There is a need for an equation to estimate the commercial part of the hybrid aspen stem. Merchantable volume equations for prediction of commercial volumes have been reported for some tree species (Burkhart 1977; Cao et al. 1980; Clutter 1980, Terwari et al. 2013). There are two types of merchantable volume equations: prediction of commercial stem volume based on top diameter or the bole length. The estimation could then be made by a merchantable volume equation or a taper equation.

OBJECTIVES

The objective was to evaluate different volume models for constructing a stem volume equation for hybrid aspen growing on farmland sites. The equation should be used for stem volume estimates of hybrids aspen. A second objective was to construct a variable-top and a variable-length merchantable volume equation. A minor aim was to study the influence of soil type on the standing volume. Data from the study was used for predicting volumes by a published equation on hybrid aspen and published stem equations developed for trembling and European aspens. Then the predicted volumes were compared with the volume predicted by the developed equation and the observed volumes. The final recommended stem and merchantable volume equations are intended to be tools for assisting in the management of hybrid aspen stands. The standing volume of the tested hybrid aspen stands was calculated.

MATERIALS AND METHODS

Data were collected from studies of hybrid aspen growing on farmland in central and southern Sweden (latitudes 55 – 61° N.) published by Johansson (2013a), Figure 1. After checking the field data (diameter and height) for outliers those data were declared outliers and excluded from further analysis. Data from 29 stands from the previous study were estimated, see Table 1. The stands were growing on farmland, with most of them planted between 1988 and 1992. They were established either as experimental stands, for commercial production or as
demonstration sites, and encompassed a variety of site and stand characteristics (Table 1). Hybrid aspens were the only species growing on the site. The method for determination of soil type was reported in a study of hybrid aspen (Johansson, 2013a). The soils at all but eight of the sites were clay sediments, with textures ranging from light (12) to medium clay (7) and till soils (8). Soils at the sites were grouped into three main soil type categories: sediments (light clay and medium clay) and tills. The stands’ ages ranged from 15 to 50 years. The mean number of stems per hectare was calculated based on the number of stems counted in whole stands. The area of the studied plantations varied between 0.02 and 2.4 ha, Table 1.

![Map of Sweden showing the locations of the sampling areas in this study](image)

The number of stems varied between 90 and 2402 per hectare; this range covers most of the common stand densities (Table 1). The initial spacing and number of plants were known for most of the stands. Some of the stands had not been thinned at all, while the others had been moderately to heavily thinned. The diameter at breast height (DBH) was measured for all aspens in the stand by cross-callipering (Table I). The weighted mean basal area diameter was calculated for each stand. The Site Index (H20 m) was estimated using a dominant hybrid aspen on each plot (Johansson 2013b). The height-diameter (DBH) relationship of registered trees in the stands is shown in Figure 2.
Table 1. Summary statistics for 29 stands and sample trees of hybrid aspen

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>SD</th>
<th>Min.</th>
<th>Max.</th>
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<td>years</td>
<td>22</td>
<td>8</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>Stem number</td>
<td>stems ha(^{-1})</td>
<td>1006</td>
<td>617</td>
<td>90</td>
<td>2402</td>
</tr>
<tr>
<td>Diameter (DBH)</td>
<td>cm</td>
<td>19.7</td>
<td>6.5</td>
<td>8.5</td>
<td>40.8</td>
</tr>
<tr>
<td>Height</td>
<td>m</td>
<td>20.7</td>
<td>3.4</td>
<td>14.0</td>
<td>27.7</td>
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<tr>
<td>Basal area</td>
<td>m(^2) ha(^{-1})</td>
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<td>12.3</td>
<td>7.6</td>
<td>66.1</td>
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<td>H(_{30}), m</td>
<td>20</td>
<td>5</td>
<td>15</td>
<td>27</td>
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<td>0.02</td>
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<tr>
<td>Diameter (DBH)</td>
<td>cm</td>
<td>22.2</td>
<td>5.5</td>
<td>14.5</td>
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<tr>
<td>Height</td>
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<td>20.7</td>
<td>3.5</td>
<td>14.0</td>
<td>29.4</td>
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<tr>
<td>Stem volume</td>
<td>m(^3)</td>
<td>0.403</td>
<td>0.279</td>
<td>0.129</td>
<td>1.566</td>
</tr>
<tr>
<td>Plot area</td>
<td>m(^2)</td>
<td>749</td>
<td>831</td>
<td>180</td>
<td>4000</td>
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<tr>
<td>No of sampled trees</td>
<td></td>
<td>29</td>
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</table>

Figure 2. Correlation between DBH and Height

Construction of a stem volume model for hybrid aspen

The plot area ranged from 180 to 4000 m\(^2\). The studied area was placed in the middle of the plantation area to avoid edge effects caused by factors such as wind, open areas, ditches and shading by adjacent stands. On each plot, one tree with a diameter close to the estimated arithmetic mean diameter was selected. The sampled trees were subjectively selected because the forest owners imposed certain conditions so as to avoid the study affecting their plans for future management of the stands. Border trees and suppressed trees were avoided. The sample tree was examined for damage, stem straightness and leader type (single vs. double or
The tree had to be free from rot. If any tree was damaged or rotten, another tree from the stand was selected instead. In total, 29 trees were sampled to obtain data for the developing of the volume equations. For each sample tree, the DBH and the height (m) were measured, see Table 1. The age was determined by counting annual rings from a disc cut at stump height (0.2 m). Then 1 – 2 years to the measured age at stump height was added to get the total age. In some cases planting date and seedling age was given by the owner.

The tree was cut as near to the ground as possible to make the measurement of tree height easier. The volume of each sample tree was calculated by summing up the volume (m$^3$) of 1 m sections and the top part. The diameter over bark (in cm) at breast height (1.3 m) and the 1 m sections using Smalian’s formula ($v = ((A_1 + A_2)/2) \times 1$) m$^3$ where $A$ (m$^2$) is the cross sectional area at the ends of the section. The volume of the top section was calculated using the formula of a cone ($1/3 \times (A \times \text{top length})$) m$^3$. The mean observed stem volume for hybrid aspen was $0.403 \pm 0.279$ (0.129 – 1.566) m$^3$, see Table 1.

The standing volume of the tested hybrid aspen stands was calculated. The volume of all hybrid aspens in a plantation was estimated and divided by the plantation area and then converted to volume per hectare.

Several equations for estimating the stem volume that used diameter or diameter and height were tested. In the initial test, fifteen stem and merchantable equations were tested. For developing merchantable volume equations data for total height, section volume and section diameter from the measured 1 m sections and DBH were used. Preliminary analysis consisted of fitting the equations to the data and testing their goodness of fit. The coefficient of determination ($R^2$) and root mean square error (RMSE) was used in the test. Then the following three volume and five merchantable volume equations were examined further:

\begin{align*}
v &= \beta_0 D^{\beta_1} \\
v &= \beta_0 D^{\beta_1} H \\
v &= \beta_0 D^{\beta_1} H^{\beta_2} \\
v_m &= v(1 - \beta_0(d_m/D_1)^{\beta_1}) \quad \text{(Alemdag 1988)} \\
v_m &= v(1 - \beta_0(d_m^{\beta_1}/H^{\beta_2})) \quad \text{(Burkhart, 1977)} \\
v_m &= v(1 - \beta_0(H - h_m)^{\beta_1}/H^{\beta_2}) \quad \text{(Cao et al. 1980)} \\
v_m &= v(1 - \beta_0(1 - h_m/H)^{\beta_1}) \quad \text{(Alemdag 1988)}
\end{align*}
where: \( v \) = stem volume, \( m^3 \); \( D \) = diameter at breast height (ob), cm; \( H \) = stem height, m;
\( \beta_0, \beta_1 \) and \( \beta_2 \) are parameters

**Statistical Procedure**

Regression analysis was carried out using the SAS statistical package (SAS 2006). The nlin procedure was used when fitting and developing the models to estimate parameters, and when evaluating the equations considered in the study. In order to determine how well the equations fitted the data, the following statistics were used for the equations and for the data on different stem levels:

\[
\text{Coefficient of determination} \quad R^2 = 1 - \frac{1}{n} \sum_{i=1}^{n} (v_i - \hat{v}_i)^2 / \sum_{i=1}^{n} (v_i - \bar{v})^2 \quad (Zar 1999) \quad (7)
\]

Average bias \( AB \)
\[
1 \frac{1}{n} \sum_{i=1}^{n} (v_i - \hat{v}_i) \quad (8)
\]

Average Absolute Bias \( AAB \)
\[
\frac{1}{n} \sum_{i=1}^{n} |v_i - \hat{v}_i| / n \quad (9)
\]

Root Mean Square Error \( RMSE \)
\[
\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{v_i - \bar{v}}{v_i} \right)^2} \quad (10)
\]

Residual standard error \( RSE \)
\[
\frac{1}{n} \sum_{i=1}^{n} (v_i - \hat{v}_i)^2 / \sqrt{n} \quad (11)
\]

where: \( v \) = stem volume; \( v_i, \bar{v} \) and \( \hat{v}_i \) are the observed, mean and predicted volumes (v).

According to Parresol et al. (1987), Average Absolute Bias (AAB) reveals a clear distinction between the equations examined.

The models were validated using the “leave-one-out cross-validation” procedure as the number of available stands was low (see Figure I, Table 1). Data points for the entire experiment were omitted one at a time and the parameters were estimated using the reduced dataset (Zhang 1993; Kozak & Kozak 2003; Nord-Larsen et al. 2009; Webb and Copsey 2011).

A rank analysis was used with (1) as the best for each criterion (Cao et al. 1980 and Figueiredo-Filho et al. 1996). Thus for \( R^2 \), an equation that had the highest value closest to
1.0 was given the highest rank. An equation which had the lowest value of AB, ABB, RMSE and RSE was given rank 1 respectively. The overall rankings of the criteria for the equation were summarized.

Comparison of mean MAIs among soil types for the sampled stands was carried out using the Bonferroni test. The level of significance accepted was $\alpha \leq 0.05$.
Throughout the results, means in statistical tests are presented along with their standard errors (SE) and predicted stem and merchantable volumes with their standard deviations (SD).

**RESULTS**

*General characteristics*

The correlation between DBH and tree height for the sample trees and DBH and observed stem volumes are shown in Figure 2. The low correlation between DBH and height for the sample trees indicates a rapid height growth for trees in the diameter range 15-20 cm.

*Stem volume equations*

The observed stem volume was 0.403±0.279 (0.129-1.566) m³, Table 1. A comparison of the tested equations of stem volume for 29 hybrid aspens was carried out. The predicted stem volumes were 0.405±0.273 (0.146-0.1568), 0.405±0.282 (0.123-0.1560) and 0.403±0.273 (0.127-1.554) for the equations (1-3) tested. All parameter estimates and evaluation statistics of the equations studied are summarized in Table 2. The coefficient of determination, $R^2$, was high, 0.99, for equations (1) – (3). Lowest RMSE value was found for equation (3) 0.002 m³. A good prediction of stem volume was indicated by low AAB values, 0.001 m³, for equation (3), see Table 2. Equation (1) had the highest RMSE value (0.047 m³) and the highest AAB value (0.035 m³). The residual distribution for the equations is shown in Figure 3. The “leave-one-out” cross-validation procedure yielded the same mean RMSE values compared with the estimated RMSE for equation (3) (Table 2) and across diameter classes (Table 3).

Due to the requirements for low levels of bias, absolute bias and RMSE, equation (3) is the most suitable for forest surveys and inventories.
Table 2. Estimated parameters for equations 1 – 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>R²</th>
<th>RSE</th>
<th>RMSE (m³)</th>
<th>AB (m³)</th>
<th>AAB (m³)</th>
<th>RMSE¹ (m³)</th>
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<td>0.047</td>
<td>-0.0020</td>
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<td>0.047</td>
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**Equation (1)**

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<tr>
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<th>R²</th>
<th>RSE</th>
<th>RMSE (m³)</th>
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**Equation (2)**

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<td>( b_0 )</td>
<td>0.892500</td>
<td>0.059500</td>
<td>0.99</td>
<td>0.002</td>
<td>0.015</td>
<td>-0.0010</td>
<td>0.002</td>
<td>0.015</td>
<td>2</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>2.630500</td>
<td>0.019700</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_2 )</td>
<td>2.600200</td>
<td>0.028800</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Equation (6)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>R²</th>
<th>RSE</th>
<th>RMSE (m³)</th>
<th>AB (m³)</th>
<th>AAB (m³)</th>
<th>RMSE¹ (m³)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_0 )</td>
<td>0.984200</td>
<td>0.004667</td>
<td>0.99</td>
<td>0.002</td>
<td>0.015</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.015</td>
<td>1</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>2.630900</td>
<td>0.019700</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. "Leave-one-out cross validation" across diameter classes.

<table>
<thead>
<tr>
<th>Diameter class cm, No of obs.</th>
<th>Equation no. (1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>0.048</td>
<td>0.008</td>
<td>0.002</td>
<td>0.032</td>
<td>0.023</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>20-24</td>
<td>0.047</td>
<td>0.008</td>
<td>0.002</td>
<td>0.033</td>
<td>0.023</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>25-</td>
<td>0.045</td>
<td>0.008</td>
<td>0.002</td>
<td>0.032</td>
<td>0.024</td>
<td>0.015</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Figure 3. Residuals for stem and merchantable volume equations for hybrid aspen

**Merchantable volume equations**

The parameter estimates and evaluation statistics of the equations are summarized in Table 2. The coefficient of determination, $R^2$, was high for all equations (0.99). The variable-top equation (5) had the lowest RMSE, AB and AAB values and the equation was ranked highest among equations (4) and (5). The variable-length equation 6) had the lowest RMSE and was ranked highest. The “leave-one-out cross validation” procedure yielded the same mean RMSE values compared with the estimated RMSE for equations (4) to (7), Table 2.
Standing volume for hybrid aspen stands

The mean standing volume was 301 ± 167 (range 111 – 701) m³ ha⁻¹. The standing mean annual increment (MAI) was 14.6 ± 8.0 (range 2.8 – 36.9) m³ ha⁻¹ year⁻¹. The mean MAI for light clay soils was 13.9±8.2 (range 5.8–28.9), 20.5±9.3 (range 9.7–36.9) for medium clay soils and 11.4±4.4 (range 2.8–18.3) for till soils. The t-test (Boniferroni) at 5% level indicated no significant differences in MAIs between soil types.

DISCUSSION

Systematic selection of trees for sampling should generally be avoided, but was necessary in this study because of restrictions imposed by the site owners. According to Kozak (1997), a systematic selection of sample trees may introduce bias into the regression coefficients and may result in the apparent variation being smaller than would be observed in a truly random sample. This was not a major issue in this study because conditions within the studied stands were relatively homogeneous and the trees did not differ greatly in size. Volume equations that are dependent on a measurement of the stem diameter at some upper height, as well as DBH and H, generate more accurate and precise predictions (Brandel 1990; Kozak 2004). However, in practice, measurements of the stem diameter at some upper heights are impractical and time-consuming and are not preferable here.

Equation (3) produced the most valid parameter values and is recommended. Equation (3) had the lowest value of AB and ABB. RMSE for the “leave-one-out” cross-validation was close to the calculated RMSE, which indicates that the evaluated equations fitted the data well. There were small differences of RMSE across diameter classes. The recommended volume equation can be used for accurate estimations of volume at stand level. The equation can also be used in future studies on sampled stems from representative plots of a selection of sites/stands with different heights, diameters, diameter distributions, basal area and stock levels. Volume calculations for individual stems and the sum of these volumes from plots can then be recalculated to give volume per hectare. This enables development of volume tables and matrixes of volume per hectare (m³ ha⁻¹), which are one of the most important and frequently used tools in forest planning and management operations in Sweden. It is also possible to use the volume equations in calculations and predictions of mean annual increment.
Hybrid aspen has a rapid growth with high MAI. In the present study the mean MAI was 15.1±7.9 (range 5.9-36.9) m³ ha⁻¹ year⁻¹. In a Danish study of 28-year-old hybrid aspen stands (140-2300 stems ha⁻¹) the MAI was 17.1 m³ ha⁻¹ year⁻¹ (Jacobsen, 1976). Studies of 11-32-year-old hybrid aspen stands growing in Sweden showed MAIs between 2-41 m³ ha⁻¹ year⁻¹ (Johnsson 1953, 1976; Eriksson 1984; Elfving 1986a b; Karačić et al. 2003; Rytter & Stener 2003; Johansson 2010). In a Norwegian study of 16-year-old stands the MAI was 21.3 m³ ha⁻¹ year⁻¹ (Langhammer 1973).

Estimates of stem volume (observed) using equation (3) and one equation (12) for hybrid aspen and three equations (13-15) developed for trembling and European aspens were compared with the stem analysis volume estimates (observed).

The following equations were tested:

An equation for single hybrid aspen stems growing on farmland in Sweden (Johnsson 1953):  
\[ V = 0.03186D^2H + 0.4300H + 0.0551D^2 - 0.4148D \]  
\[ (12) \]

An equation for trembling aspen (Schlagel 1975):  
\[ V = -10.366 + 1.004\ln(D^2H) \]  
\[ (13) \]

Two equations for European aspen:  
\[ V = -0.0475 + 0.00699D - 0.00023D^2 + 0.00004D^2 \times H \ (\text{Opdahl 1992}) \]  
\[ (14) \]
\[ V = -0.01548D^2 + 0.03255D^2H + 0.000047D^2H^2 - 0.01333DH + 0.004859DH^2 \ (\text{Eriksson 1973}) \]  
\[ (15) \]

As shown in Table 4, the estimated volumes for equation (3) were closest to the observed volumes. Equations 12 and 13 underestimated stem volumes (98% and 99% by observed) and equations 14 and 15 overestimated (111% and 105% by observed).

Table IV. Comparison of observed stem volumes for hybrid aspen between the constructed equation 3 and equations 12–15.

<table>
<thead>
<tr>
<th>Volume, m³</th>
<th>Observed</th>
<th>Equation 3</th>
<th>Equation 12</th>
<th>Equation 13</th>
<th>Equation 14</th>
<th>Equation 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.403</td>
<td>0.402</td>
<td>0.394</td>
<td>0.399</td>
<td>0.447</td>
<td>0.422</td>
</tr>
<tr>
<td>SD</td>
<td>0.279</td>
<td>0.279</td>
<td>0.309</td>
<td>0.322</td>
<td>0.336</td>
<td>0.335</td>
</tr>
<tr>
<td>Range</td>
<td>0.129–1.566</td>
<td>0.128–1.562</td>
<td>0.110–1.700</td>
<td>0.106–1.759</td>
<td>0.128–1.845</td>
<td>0.110–1.818</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Prior to this study, one stem volume equation (Johnsson, 1953) had been developed for use with hybrid aspen growth under Nordic conditions. An increasing interest in the use of hybrid aspen for biofuel, pulp wood and timber in Sweden makes it important to have a tool for prediction of hybrid aspen yields. Currently, there are mature hybrid aspen stands, which must be thinned or clear felled soon. An indication of present yield is valuable for the owner to help with future management.

The estimation of stem volume best fit the data when DBH and stem height were included in the equation. The preferred equation was valid for trees having a DBH larger than 5 cm and less than 45 cm. Calculation with a DBH > 45 cm should be carried out with care. Measuring stem height is more time-consuming and laborious than DBH. A commonly-used method is to sample tree heights per diameter class. Based on the diameter to height correlation, the stem height for all diameters in the stand or plot can be calculated. However, the equation (no. 1) based solely on DBH could be used when rapid estimations need to be carried out and precision is less important. The evaluated stem volume equation (3) for hybrid aspens is recommended for the prediction of stem volume of hybrid aspens growing on farmland but could be practiced on forest land. When using a published equation for European aspen or trembling aspen for prediction of stem volume for hybrid aspen the value will be slightly underestimated and equations for European aspen overestimated stem volumes.

The merchantable volume equations fitted the data well. The flexible top diameter and bole length merchantable volume equations represent useful tools for the estimation of commercial parts of a stem. When the timber volume has been calculated, the top volume could be calculated by subtracting the timber and pulp wood volume from the total bole volume. Then the top volume, used as biomass, could be calculated by using the basic density (m³ kg⁻¹) for hybrid aspen.

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