Multidimensional auctions for long-term procurement contracts under the threat of early exit: the case of conservation auctions

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MULTIDIMENSIONAL AUCTIONS FOR LONG-TERM PROCUREMENT CONTRACTS 
UNDER THE THREAT OF EARLY EXIT: THE CASE OF CONSERVATION CONTRACTS

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Abstract

In this paper we study how early-exit options, embedded in long-term procurement contracts which do not provide for sufficiently strong incentives against contract breach, can affect bidding behaviors in multidimensional procurement auctions and the parties' expected payoffs. We show first that bidders' payoff is lower when competing for contracts with unenforceable contract terms. Secondly, that neglecting the risk of opportunistic behavior by sellers can lead to contract awards that do not maximize the buyer's potential payoff. Finally, we make suggestions about how to mitigate potential misallocations, by pointing out the role of eligibility rules and competition among bidders.

Keywords: Public procurement; Scoring auctions; Contract breach; Real options; Conservation contracts.  
JEL Classification: C61, D44, D86, Q24, Q28.
1 Introduction

Premature termination of public procurement contracts, and other types of public-private partnerships, is not uncommon. An illustrative example is the rail franchise between Edinburgh and London awarded in 2007 to National Express on the basis that it would pay the Department of Transport £1.4 billion over seven and a half years. However, since passenger revenues proved to be lower than expected because of the economic downturn, just two years later National Express announced that it wanted to opt out the contract. In November 2009 the Department accepted to terminate the franchise and received £120 million from the company. In evaluating the case, the UK Parliament’s Public Accounts Committee noted that "[...] the Department did not undertake sufficient due diligence on the bid by National Express". In addition, by telling that the termination would not be held against the company if it bid for future franchises, "the Department has potentially incentivised other holding companies with loss-making franchises to terminate [their contract] as they know doing so [...] will not affect their ability to compete for other contracts" (UK Parliament - Public Accounts Committee, 2011).

Along with the increasing use of competitive procedures for public sector contracts, the auction literature has expanded in recent years. Theoretical models, however, have largely focused on the one-shot provision of a single item, in so doing overlooking the possibility that contractors could breach contracts which instead require the supply of goods or services over an extended period of time.

One interesting example of public-private agreements involving long-term obligations are conservation contracts, which have become quite popular in recent years as a means of procuring environmental services, such as biodiversity maintenance, carbon sequestration, soil erosion control, flood water storage or visual amenities. Broadly speaking, these contracts commit landowners to keep natural areas in their pristine state, or to remove cropland from production. In general, contractors are also required to act proactively, in order to enhance environmental quality, by implementing for example wildlife protection measures on enrolled forestlands and wetlands, or by establishing permanent native grasses on set-aside cropland. In exchange, landowners receive a payment flow for the direct and opportunity costs of conservation practices.

Traditionally, governments have offered fixed subsidies for compliance with a predetermined
set of conservation activities. However, along with the expansion of environmental contracting, interest in bidding mechanisms has grown, in order to increase the cost-effectiveness, transparency and political acceptance of environmental payments (Latacz-Lohmann and Schilizzi, 2005).

Competitive tenders for conservation contracts usually come in the form of multi-dimensional auctions in which agents are required to make offers on both price and environmental activities, and bids are evaluated according to predefined scoring criteria. Examples include the Conservation Reserve Program (CRP) auctions employed in the USA after 1990, the BushTender Trial in Australia and the Challenge Fund Scheme in UK.

As in other procurement literatures, conservation auction models are generally built on the (implicit) assumption that service time commitments will be met (see for example Kirwan et al., 2005; Claassen et al. 2008; Espinosa-Arredondo, 2008; Vukina et al., 2008; Wu and Lin, 2010). Landowners, however, could find it profitable to terminate the contract when the opportunity cost of compliance with conservation requirements proves to be higher-than-expected, because, for example, of sharp rises of crop prices, increases in timber prices, or the increased demand of land for housing or industrial uses.

Generally speaking, opportunistic behaviors by sellers may be discouraged by informal sanctions, such as the threat of losing reputation and future business. However, the relatively limited role of reputational incentives in exchanges between the government and the private sector (Kelman, 1990; Spagnolo, 2012) makes more compelling the need for formal remedies, which typically take on the form of financial "penalties". Even these contractual claims, however, can prove to be ineffective in enforcing compliance, because of technical, legal, or other factors.

First, since contract provisions are typically established prior to launch the tender, contracting agencies can find it difficult to tailor contractual penalties owing to the lack of precise information on the value of the bidders’ outside options.  

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1 The term "penalty" is used here in a broad sense, to encompass both contractual provisions aimed at enforcing compliance with contractual obligations (penalties \textit{stricto sensu}) or at protecting the promisee from the expected costs of breach ("liquidated damages" in the legal jargon).

2 For example, in the USA, besides returning all the cost-share funds already paid, with interest, owners willing to take their land out of the CRP program face a penalty of 25 per cent of rental payments received. The Secretary of Agriculture, however, is allowed to release land from CRP without penalty, an option which has been exercised twice, in 1995 and 1996 (Stubbs, 2013).
Second, general legal principles can limit the freedom to stipulate payments for non-compliance with contractual obligations. For example, in Common Law jurisdictions, payments, even though mutually agreed, can be subsequently voided (in part or in their entirety) by courts if it appears that they were designed to be a deterrent rather a reasonable pre-estimate of the loss that would be suffered in the event of breach (DiMatteo, 2001).³

Third, governments can face political pressure to soft early termination fees. In the USA, for example, agricultural associations have frequently lobbied for reducing payments for early release of CRP acres, and in 2011 some Members of the Congress asked President Obama to release CRP land without penalty for the purpose of grain production (Stubbs, 2013).

Finally, the effectiveness of contractual claims can be threatened by costly litigation and inefficient dispute settlement processes. For instance, institutional failures, leading to incomplete contract enforcement, have been emphasized in recent works on programs aimed at reducing deforestation and degradation of tropical forests in developing countries (Palmer, 2011; Cordero Salas and Roe, 2012; Cordero Salas, 2013).

When sellers do not face sufficiently strong incentives against breach of contracts, either because of the lack of reputational incentives, or because of the intrinsic weakness of penalties or because of the lack of credibility and the weak enforcement of contractual claims, bidders will compete for contracts which, *de facto*, include an "early-exit option". The question addressed in this paper is how this can affect bidding behaviors in multidimensional auctions and the parties’ expected payoffs.

Our main contributions are the following. First, we show that bidders’ payoff is lower when competing for contracts which, either explicitly or implicitly, do not provide for enforceable time commitments. Secondly, that neglecting the risk of opportunistic behavior by sellers can lead to contract awards that do not maximize the buyer’s potential payoff. Finally, we make suggestions about how to mitigate potential misallocations, by pointing out the role of eligibility rules and competition.

The remainder of the paper is organized as follows. The next section provides a brief overview of the related literatures. In section 3 we set up the model. Section 4 illustrates the benchmark

³This explains why, for example, the CRP contract specifies that the fee due in the event of early release "shall be due as liquidated damages [...] and not as a penalty" (USDA, 2013, § 10).
case in which the contractual duration is enforceable. In section 5 we derive the equilibrium of the auction game when bidders do not face sufficiently strong incentives against early-exit and in section 6 we discuss the impacts of ignoring the risk of a premature termination of contracts and possible remedies. We conclude in section 7. The Appendix contains the proofs omitted from the text.

2 Related literature

This article is related to various literatures which have developed in a largely independent fashion.

The first strand of literature is that on scoring auctions in which bidders compete on both price and non-price (quality) dimensions. Though the guarantee of supply over the stipulated contract period is generally considered to be one of the most important aspects in procurement, to the best of our knowledge, the risk of premature interruption of supply has not been addressed in the literature on multi-dimensional auctions. In his seminal paper, Che (1993) showed that, under specific conditions on the cross partial derivatives of the cost function, an auction in which price enters linearly into the scoring rule implements the optimal scheme by distorting quality downward with respect to the efficient level. A contribution of our paper is that it proves that Che’s result still holds even when bidders, facing on-going changes in outside conditions, incorporate into their bidding strategies potential terminations of supply.

The second literature is the one on real options analysis. Rights, but not obligations, to invest capital in productive assets have traditionally been traced back to collective opportunities, such as the possibility to penetrate a new geographical market without barriers to competitive entry, or to exclusive rights of exercise ("proprietary options") resulting from copyrights, patents, or from a company’s managerial resources or unique knowledge of a technological process which competitors cannot replicate (Kester, 1984; Dixit and Pindyck, 1994; Trigeorgis, 1996). Proprietary options can also be embedded in public contracts, such as concessions for public utilities, oil and gas leases or radio spectrum licenses. This for instance occurs when allottees have contractual discretion as to when, if at all, to develop the lease or to supply the market by using the assigned spectrum (Dosi and Moretto, 2010). However, even when contracts do not explicitly provide such discretion,

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4See, for instance, Che (1993), Bushnell and Oren (1994) and Asker and Cantillon (2008; 2010).
real options may emerge as a result of the contracting authority’s inability to enforce compliance with contractual obligations (Dosi and Moretto, 2015). For instance, in a procurement context, this can occur when contractors do not face sufficiently strong penalties against delays in project implementation or, as in the case addressed in this paper, against early termination of supply.

A number of papers, using real option theory, have analyzed the value of managerial flexibility from the promisee’s perspective (see for example Ford et al., 2002; Ho and Liu, 2002; Garvin and Cheah, 2004; You and Tam, 2006; Lo et al., 2007; D’Alpaos et al., 2013). These works, however, tend to overlook the feedback effects of flexibility on bidding behavior as well as the impacts in terms of contract allocation which can be substantial (Kogan and Morgan, 2010). One of the few exceptions is the paper by Dosi and Moretto (2015) that, viewing the imperfect enforcement of contract terms as a source of real options, studies the effects of the inability to enforce compliance with delivery schedules on competitive bids for public works. The authors, however, limit their analysis to homogeneous projects, where, unlike what we do here, bidding is restricted to the price dimension.

A third strand of literature, broadly related to the present work, is that on auctions with contingent payments.\(^5\) In particular, DeMarzo et al. (2005), by comparing the parties’ expected payoffs when bids are independent on future events ("cash auctions") with those achievable when bids are securities whose value derives from the value of an underlying asset, showed that cash auctions yield higher bidders’ payoffs than security-bid auctions when the underlying asset is a call-like ("growth") option. Even though our framework is different, in that the contract considered here does not provide for subsequent price adjustments, our findings are similar. In particular, we show that when the contract duration is not enforceable, the impact on the bidders’ payoff is similar to the effect of auctioning a contract with contingent payments, as it might occur if, for example, subsidies for setting aside cropland were indexed to changes in agricultural prices.\(^6\)

Finally, it is worth mentioning that the continuous-time setting that we use here is rather common in the literature studying the principal-agent relationship under moral hazard and/or adverse selection, when some asset (a natural resource, an investment project, etc.) owned by a risk-neutral principal is contracted out to a risk-neutral agent. DeMarzo and Sannikov (2006),

\(^5\) See for example Hendricks et al. (2003), Board (2007), Haile et al. (2010), and for an excellent survey Skrzypacz (2013).

\(^6\) On payments for environmental services with an indexed component, see Engel et al. (2015).
for instance, consider a continuous-time financial contracting model where the agent may divert cash for personal gain, while Sung (2005) and Sannikov (2007) present a continuous-time model of dynamic agency with both moral hazard and adverse selection.

3 Model set up

Consider \( n \geq 2 \) risk-neutral agents, each one holding one unit of a natural asset \( L \) that is currently kept "idle", meaning that it does not provide the owners with any (relevant) private benefit.

All parcels are suitable for producing one unit of a marketable good (\textit{good 1}). This, however, requires developing the asset (e.g. draining a wetland for agricultural use), by affording a sunk investment cost \( K(\theta_i) \), where \( \theta_i \in [\theta^l, \theta^u] \) denotes the agent’s innate managerial skills, with \( K_\theta < 0 \).

Normalizing at zero operational costs, the investment’s returns depend on the exogenous output price, which is assumed to evolve as follows:

\[
dp(t) = \sigma p(t)d\chi(t), \text{ with } p(0) = p_0
\]

where \( \sigma \) is the instantaneous volatility and \( d\chi(t) \) is the increment of the standard Wiener process satisfying \( E_0 [d\chi(t)] = 0 \) and \( E_0 [d\chi(t)^2] = dt \). The process (1), which is independent of \( \theta_i \), is common knowledge, and its realizations \( \{p(t), t > 0\} \) are publicly observed information.

Consider now a public agency (henceforth, "the buyer") that wishes to procure a public environmental service (\textit{good 2}) over a period of time \( (\tilde{T}) \) pre-specified by the buyer.

In accordance with the additionality principle, we assume that the buyer is allowed to pay only for the provision of services that would not have been supplied without public payments.\(^{11}\)

\(^{7}\)We normalize the quantity of \textit{good 1} at no cost in terms of robustness of our results.

\(^{8}\)\(K(\theta_i)\) can also be thought as the present value of a flow of periodic fixed costs \( k(\theta_i) = r K(\theta_i) \) to which the agent commits once investment is undertaken.

\(^{9}\)Note that if no arbitrage opportunities exist and the market is complete, the assumption of risk neutrality can be relaxed. In this case, in order to provide an appropriate adjustment for risk, it suffices to take expectations with respect to a distribution of \( p(t) \) adjusted for risk neutrality. See Cox and Ross (1976) for further details.

\(^{10}\)In Eq. (1) we abstract from the drift in order to focus on the impact that uncertainty has on outcome of the bidding process. Note, however, that by the Markov property of Eq. (1), our results would not be qualitatively altered by using a non-zero trend for \( p(t) \). It can be also easily shown that Eq. (1) is consistent with the case of a firm maximizing instantaneous operating profits under a Cobb-Douglas production technology (see Dixit and Pindyck, 1994, pp. 195-199).

\(^{11}\)On the additionality principle, see for example Ferraro (2008).
Provision of good 2 (e.g., soil erosion control) requires keeping $L$ in its current state and undertaking on-site conservation-oriented activities (e.g., native plant restoration, placement of buffer strips, etc.) affecting the quality and/or quantity of the environmental service.\textsuperscript{12} Denoting with $g$ the final environmental outcome, the per-unit-of-time direct cost of service provision is defined as $c(g, \theta_i)$, with the following properties: $c(0, \theta_i) = 0$, $c_g > 0$, $c_{gg} > 0$, $c_\theta \leq 0$ and $c_g \theta < 0$.\textsuperscript{13} \textsuperscript{14}

Prior to bidding agents have private information about their own type $\theta_i$. Bidder $i$ only knows that $\theta_j$, $j \neq i$ is drawn from a common prior cumulative distribution $F(\theta_i)$, with continuously differentiable density $f(\theta_i)$ defined on a positive support $[\theta^l, \theta^u] \subseteq R_+$. The contract is awarded by a first-score-sealed-bid auction. Specifically, agents are solicited at time $t = 0$ to bid on the environmental service ($g > 0$) and on the payment ($b > 0$) required for supplying $g$ at each time period $t \in (0, T]$. Offers are then evaluated according to a scoring rule $S(b, g)$, announced prior to bid opening, and the winner is the bidder with the highest score.\textsuperscript{15}

\textsuperscript{12}Programs such as Payments for ecosystem services (PES) fit within this category. Our model, however, can be easily extended to situations, as the one addressed by Gulati and Vercammen (2006), where agents currently using a resource depleting technology (A) are encouraged to switch to a resource conserving one (B). Basically, in this case, the seller accepts to use B until date $T$, and then may switch back to A once the contract expires. Comparing this case with ours, the main difference is given by the initial state of the input asset used to provide the environmental service, since the seller is required to suspend operations under technology A. In contrast, in our case, the seller is asked not to exercise the option of adopting A until the contract expires.

\textsuperscript{13}The cost function $c(g, \theta_i)$ can be defined as:

$$c(g, \theta_i) = \min_{x \geq 0} \text{w} x \quad \text{s.t. } \varphi(x, \theta_i) \geq g$$

where $\text{w}$ is the vector of input prices and $\varphi(x, \theta_i)$ is a strictly concave production function indicating the efficient level of environmental output for any given input vector $x$, with $\varphi_\theta \geq 0$.

\textsuperscript{14}For a discussion of outcome vs. action-based conservation contracts, see Latacz-Lohmann and Schilizzi (2005), Whitten et al. (2007), Gibbons et al. (2011) and White and Sadler (2011).

\textsuperscript{15}Note that the simplifying assumption that each bidder holds only one eligible asset can be relaxed. The model can, in fact, be easily extended to a multi-item procurement auction where each bidder holds multiple assets, as long as each agent is allowed to bid only for one contract. In addition, most of our general results hold also in the case of auctions with a discriminatory format, as long as the number of bidders is sufficiently high (see Krishna, 2012, Ch. 12, for a discussion on this point).
In accordance to the literature on conservation auctions, we assume the following scoring rule:\(^{16}\)

\[
S(b, g) = \int_0^T s(b, g) e^{-rz} \, dz = (v(g) - b) \frac{1 - e^{-rT}}{r},
\]

where \(r\) is the discount rate and \(v(g)\) is a function mapping the social utility attached to \(g\), with \(v_g(g) > 0\) and \(v_{gg}(g) \leq 0\).

Finally, we add the following assumptions.

- **Assumption 1:** The inverse hazard rate \(F(\theta_i)/f(\theta_i)\) is increasing in \(\theta\).
- **Assumption 2:** \(\Delta(g, \theta_i) = c(g, \theta_i) - rK(\theta_i)\) is positive and non-increasing in \(\theta_i\) for every \(\theta_i \in [\theta^l, \theta^u]\) and its derivative is bounded above.
- **Assumption 3:** At each time period \(g\) is verifiable by all parties.
- **Assumption 4:** The buyer is able to commit to carry out the terms of the contract for its entire duration.

Assumption 1 is standard in the auction literature (see for example Krishna, 2002). Assumption 2 is made in order to guarantee strict monotonicity of the scoring rule (Che, 1993). It implies that, as \(\theta\) increases, the cost-efficiency in the provision of good 2 dominates the cost-efficiency in the production of good 1. The underlying assumption is that, being good 1 a rather conventional product, individual managerial skills play a relatively less important role in explaining cost differences across agents. Assumption 3, which is made to focus purely on the effects of opportunistic behavior leading to early exit, indicates that the buyer is able to monitor compliance with the promised service and to stop rental payments should \(g\) fall below the contractual specifications.\(^{17}\) Finally, Assumption 4 rules out the possibility that the buyer could use, right after the auction \((t > 0)\), the information extracted through the bidding process to ask the contractor to increase efforts in service provision or to lower the bid price.

\(^{16}\)This functional form is consistent with the scoring rules used by Kirwan et al. (2005), Vukina et al. (2008) and Wu and Lin (2010) to analyze the effects of the Conservation Reserve Program. Notice that our framework can be easily extended to the case where \(g(t)\) evolves deterministically at a given exogenous rate.

\(^{17}\)Problems related to imperfect monitoring of conservation activities (or final environmental outputs) have been discussed in a series of papers dealing with agri-environmental contracts. See, among others, Giannakas and Kaplan (2005) and Hart and Latacz-Lohmann (2005).
4  Perfect enforcement

To get a benchmark case, we first analyze the outcome of the bidding process when the contractual duration is enforceable. Implicitly, we assume that the buyer does not face any constraints to stipulate and to costlessly enforce arbitrarily large penalties against breach or, equivalently, that the level of liquidated damages is such that agents bid knowing that they will never find it convenient to prematurely terminate the contract. This might, for instance, be the case when contractors were required to refund all the payments already received, with interest, plus an exit fee (see footnote 25 for a formal proof).\footnote{This is actually the approach adopted in the US Conservation Reserve Program for handling potential early outs. The observed low number of CRP acres withdrawn earlier seems to confirm the deterrent effect of this incentive scheme.}

4.1  Preferences

Prior to bidding each agent contemplates the opportunity of developing $L$ for producing good 1. Denoting by $\hat{p}_i$ the market price triggering investment in such venture for the $i$th agent, the value of the development project is given by:

$$\tilde{\Pi}(\theta_i) = E_0[e^{-r\tilde{T}_i} \int_{\tilde{T}_i}^{\infty} p(z)e^{-r(z-\tilde{T}_i)}dz - K(\theta_i)] = E_0[e^{-r\tilde{T}_i}](\frac{\hat{p}_i}{r} - K(\theta_i)), \quad (3)$$

where the optimal time of investment, $\tilde{T}_i = \inf\{t \geq 0 \mid p(t) = \hat{p}_i\}$, is a random variable, and $E_0$ is the expectation taken at the starting period $t = 0$ over the process $\{p(t), t \geq 0\}$. Notice that Eq. (3) represents both the winning bidder’s opportunity cost of keeping $L$ in its pristine state and the value of the asset for the bidders who will not be awarded the contract.

The optimal trigger $\hat{p}_i$ is the solution of the following problem:

$$\hat{V}(\theta_i) = \max_{\tilde{T}_i} E_0[e^{-r\tilde{T}_i}](\frac{\hat{p}_i}{r} - K(\theta_i)) = \max_{\hat{p}_i} (\frac{p_0}{\hat{p}_i})^\beta (\frac{\hat{p}_i}{r} - K(\theta_i)), \quad (4)$$

where $E_0[e^{-r\tilde{T}_i}] = (p_0/\hat{p}_i)^\beta$ is the expected discount factor, and $\beta > 1$ is the positive root of the characteristic equation $\phi(\beta) = (\sigma^2/2)\beta(\beta - 1) - r = 0$.\footnote{The expected present value $E_0[e^{-r\tilde{T}_i}]$ can be determined by using dynamic programming (see for example Dixit and Pindyck (1994, pp. 315-316).}
Solving problem (4), we get:

\[
\hat{p}_i \equiv \hat{p}(\theta_i) = \left[\beta/(\beta - 1)\right]rK(\theta_i),
\]

(4.1)
\[
\hat{V}(\theta_i) = \left[\Gamma(p_0)/(\beta - 1)\right]K(\theta_i)^{1-\beta},
\]

(4.2)

where \(\Gamma(p_0) = \left[\frac{p_0}{p_0/r-(\beta-1)}\right]^{\beta}\).

Thus, as standard in the real-option literature, the optimal trigger is given by the user cost of capital, \(rK(\theta_i)\), corrected by the option multiple \(\beta/(\beta - 1)\) which accounts for the irreversibility and the uncertainty characterizing the decision to develop \(L\) for commercial use.

Notice that \(d\hat{V}(\theta_i)/d\theta_i > 0\) and \(d\hat{p}(\theta_i)/d\theta_i < 0\). That is, the more efficient is the agent, the higher are the value of the asset and the opportunity cost of keeping \(L\) idle, and the lower is the output price making profitable to develop \(L\) for commercial use.

According to the additionality principle, public payments are allowed only for supporting private decisions that would have not been made anyway. In our frame, additionality passes through the actual threat of having \(L\) developed for commercial use. Hence, to ensure that all \(n\) bidders are eligible for conservation funding, we add the following assumption, which says that, at the time of bidding, market revenues are such that none of the bidders would continue to keep \(L\) idle without conservation subsidies.\(^{20}\)

- **Assumption 5**: \(p_0 = \hat{p}(\theta^l)\).

By Assumption 5, given the properties of \(\hat{p}(\theta_i)\), our frame can be normalized by setting \(\hat{\Pi}(\theta^l) = 0\). This operation, which does not affect the underlying ranking of agents with respect to the profitability of developing \(L\), implies that bidders can be ranked according to the following reservation value:

\[
\hat{\Pi}(\theta_i) = \frac{p_0}{r} - K(\theta_i), \text{ for } \theta_i \in [\theta^l, \theta^n],
\]

(3.1)

which is increasing in the cost parameter \(\theta\).

Now consider the winning bidder. The *ex-post* value of the contract is given by:\(^{21}\)

\[
\Pi(b_i, g_i; \theta_i) = E_0 \left\{ \int_0^T \left( b_i - c(g_i, \theta_i) \right) e^{-r z} dz + e^{-r T} E_T \left[ e^{-r (\hat{\pi} - T)(\hat{p}/r - K(\theta_i))} \right] \right\}
\]
\[ \Pi(b_i, g_i; \theta_i) = (b_i - c(g_i, \theta_i)) \frac{1 - e^{-rT}}{r} + (p_0 - rK(\theta_i)) \frac{e^{-rT}}{r}, \]  

(5)

where \( E_0[p_{T}] = p_0 \) is a straightforward implication of the Markov property of the diffusion process (1).\(^{22}\)

By subtracting Eq. (3.1) from Eq. (5), we get:

\[ \Pi(b_i, g_i; \theta_i) - \widehat{\Pi}(\theta_i) = \begin{cases} 
[(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))] \frac{1-e^{-rT}}{r}, & \text{for } \theta_i \in (\theta^l, \theta^u], \\
(b_i - c(g_i, \theta_i)) \frac{1-e^{-rT}}{r}, & \text{for } \theta_i = \theta^l.
\end{cases} \]

(6)

Hence, the seller’s pay-off (accounting for the reservation value) is given by the difference between the present value of rental payments (net of the direct cost of performing \( g_i \)) and the opportunity cost of not developing the asset until the expiration of the contract. Notice that, by Assumption 5, the latter is null for the agent having the highest development cost (\( \theta^l \)).

### 4.2 Equilibrium strategy

Given the information available at time \( t = 0 \), each agent will choose the optimal bidding strategy by maximizing the following functional:

\[ \Pi(\overline{S}_i) = \Pi(b_i, g_i; \theta_i) \cdot \Pr(\text{of win}/\overline{S}_i) + \Pi(0, 0; \theta_i) \cdot (1 - \Pr(\text{of win}/\overline{S}_i)), \]

(7)

where \( \Pr(\text{of win}/\overline{S}_i) \) is the probability of winning the auction, conditional on the reported score \( \overline{S}_i(b_i, g_i) \), and \( \Pi(0, 0; \theta_i) = \widehat{\Pi}(\theta_i) \) is the reservation value.

Hence, with probability \( \Pr(\text{of win}/\overline{S}_i) \), agent \( i \) will be entitled to receive a flow of net payments worth \( (b_i - c(g_i, \theta_i))(1 - e^{-rT})/r \), plus the value of developing the asset at the expiration of the contract, \( e^{-rT}[(p_0/r) - K(\theta_i)] \). Instead, with probability \( (1 - \Pr(\text{of win}/\overline{S}_i)) \), the agent will simply get the reservation value \( \Pi(0, 0; \theta_i) = \widehat{\Pi}(\theta_i) \geq 0 \).

that is, the present value of the flow of rental payments, \( b_i \), minus direct costs, \( c(g_i, \theta_i) \), accruing up to \( T \) (first term), plus the value attached at \( T \) to the option to invest in the production of good 1 (second term). Note that by using the stochastic discount factor \( E_\tau[e^{-r(T - \tau)}] \) we are also accounting for the likelihood of having \( p_T \leq \widehat{p}_i \). However, given the information available at \( t = 0 \), by the law of iterated expectations and Assumption 5 such option is always in the money and the above expression reduces to:

\[ \Pi(b_i, g_i; \theta_i) = (b_i - c(g_i, \theta_i)) \frac{1 - e^{-rT}}{r} + (E_0[p_T] - rK(\theta_i)) \frac{e^{-rT}}{r}. \]

\(^{22}\)See for example Dixit and Pindyck (1994, pp. 71-74).
Agents participate in the auction only if the following individual rationality constraint holds:

$$\Pi(\mathcal{S}_i) \geq \Pi(\theta_i) \geq 0. \quad (8)$$

Since agents bid knowing that $T$ is enforceable, the probability of winning, $\Pr(\text{of win}/\mathcal{S}_i)$, is equivalent to $\Pr(\text{of win}/s_i)$, where $s_i$ is the instantaneous score. Thus, using (6), we can rearrange (8) and define agent $i$’s objective function as follows:

$$\tilde{W}(b_i, g_i; \theta_i) = \{(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))\frac{1 - e^{-rT}}{r}\} \Pr(\text{of win}/s_i). \quad (9)$$

The solution of the bidding game is as follows:

**Proposition 1** When the contract duration is enforceable, for any finite number of bidders $n$ it will always exist an equilibrium in symmetric and strictly increasing strategies $\bar{s}(\theta_i)$ characterized by:

i) the environmental output:

$$g(\theta_i) = \arg\max [v(g_i) - c(g_i, \theta_i)]$$

with $dg(\theta_i)/d\theta_i = \frac{c_{g\theta}(g(\theta_i), \theta_i)}{v_{gg}(g(\theta_i)) - c_{gg}(g(\theta_i), \theta_i)} > 0,$

for all $\theta_i \in [\theta^l, \theta^u]$, 

$$\Delta g(\theta_i) = \frac{c_{g\theta}(g(\theta_i), \theta_i)}{v_{gg}(g(\theta_i)) - c_{gg}(g(\theta_i), \theta_i)} > 0,$$

ii) the bidding function:

$$\bar{b}(\theta_i) = c(g(\theta_i), \theta_i) + (p_0 - rK(\theta_i)) - \int_{\theta^l}^{\theta_i} \Delta g(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i)} dx,$$

iii) the expected profits:

$$\tilde{W}(\theta_i) = -\int_{\theta^l}^{\theta_i} \Delta g(x) \frac{1 - e^{-rT}}{r} F^{(n)}(x) dx,$$

where $\Delta g(x) = c_g(g(x), x) - rK_g(x) < 0$.

**Proof.** See Appendix A.1. ■

The bid service in (10) can be easily determined by using Che’s Lemma 1.23 This result allows us to illustrate the agent’s bidding strategy by determining the score $s_i$ and the output (quantity

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23 Note in fact that, once secured the enforcement of the contract duration $T$, our quasi-linear scoring rule is consistent with the frame developed in Che (1993, pp. 671-672).
or quality) level \( g_i \). Note also that since \( T \) is enforceable, the buyer’s total payoff simply comes from the instantaneous score, \( s(\theta_i) = v(g(\theta_i)) - b(\theta_i) \). Therefore, the auction efficiency is ensured by showing that \( d\bar{s}(\theta_i)/d\theta_i > 0 \) (see Appendix A.1).

Eq. (11) shows that the bid price covers both the direct cost of performing the promised environmental service, \( c(g(\theta_i), \theta_i) \), and the opportunity cost of not developing \( L \) for commercial use, \((p_0 - rK(\theta_i))\). Moreover, agents must be compensated by information rents, \(-\int_{\theta_i}^{\theta_i^*} \Delta g(x) \frac{F^{(\nu)}(x)}{F^{(\nu)}(\theta_i)} dx\). As standard in the auction literature, no rents will be paid to the least cost-efficient agent, i.e. \( b(\theta_i^*) = c(g(\theta_i^*), \theta_i^*) + (p_0 - rK(\theta_i^*)) \).

5 Non-enforceable contract duration

Now suppose that the seller does not face sufficiently strong penalties against contract breach. As already argued, this can be attributed either to the lack of reputational incentives, or to the weakness of contractual penalties, or to the weak enforcement of contractual claims. For the sake of simplicity, here we simply assume that the seller does not face any (credibly-enforceable) penalty for early exit.\(^{24}\) In other words, bidders bid knowing that, should an attractive outside option arise, they can terminate the contract at the only cost of losing from that time onward conservation payments. As shown henceforth, this implies that, unlike the previous case, bidding strategies will be affected by endogenous timing considerations.

5.1 Preferences

As above, prior to bidding, each agent contemplates the opportunity of developing \( L \), which is worth \( \hat{\Pi}(\theta_i) \) as defined by Eq. (3.1).

Now consider the winner. Denoting by \( p_i^* \) the optimal threshold for developing \( L \), the ex-post value of the project is given by:

\[
\Pi(b_i, g_i; \theta_i) = E_0[\int_0^{T_i} (b_i - c(g_i, \theta_i)) e^{-rT} dz + e^{-rT_i} (\int_{T_i}^{\infty} p(z) e^{-r(z-T_i)} dz - K(\theta_i))] = (b_i - c(g_i, \theta_i)) \frac{1 - E_0[e^{-rT_i}]}{r} + (p_i^* - rK(\theta_i)) \frac{E_0[e^{-rT_i}]}{r}, \tag{14}
\]

\(^{24}\)Note, however, that the model can be easily extended, to include a probability-based penalty for early exit, provided the expected penalty for breach does not exceed the seller’s exit-option value (see Dosi and Moretto, 2015).
where \( T_i = \inf\{t \geq 0 \mid p(t) = p_i^*\} \) is the optimal time for breaching the contract. The first term in Eq. (14) is the expected net present value of conservation payments, while the second term is the expected net present value of switching to the production of good 1.

By rearranging Eq. (14), the seller’s optimal trigger is given by the solution of the following problem:

\[
V(b_i, g_i, \theta_i) = \max_{T_i} E_0[e^{-rT_i}] \frac{p_i^*}{r} - \frac{[(b_i - c(g_i, \theta_i)) + rK(\theta_i)]}{r}
\]

\[
= \max_{p_i^* \in \ddot{P}_i} \frac{p_0}{p_i^*} \beta p_i^* - \frac{[(b_i - c(g_i, \theta_i)) + rK(\theta_i)]}{r}.
\]

(15)

where \( E_0[e^{-rT_i}] = (p_0/p_i^*)^\beta < 1 \). In (15) the term in squared brackets represents the cost of switching to the production of good 1, which, besides the direct cost, \( rK(\theta_i) \), must also account for the forgone net rental payments, \( b_i - c(g_i, \theta_i) \).

Solving problem (15), we get:

\[
p_i^* = p^*(b_i, g_i, \theta_i) = \frac{\beta}{\beta - 1} [(b_i - c(g_i, \theta_i)) + rK(\theta_i)],
\]

(15.1)

\[
V(b_i, g_i, \theta_i) = \frac{\Gamma(p_0)/(\beta - 1)}{r} \frac{b_i - c(g_i, \theta_i)}{r} + K(\theta_i)^{1-\beta}.
\]

(15.2)

Subtracting \( \widehat{\Pi}(\theta_i) \) from Eq. (14) yields the value attached to having the contract awarded:

\[
\Pi(b_i, g_i; \theta_i) - \widehat{\Pi}(\theta_i) = \begin{cases}
[(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))] \frac{1-E_0[e^{-rT_i}]}{r} + \\
+(p_i^* - p_0) E_0[e^{-rT_i}] + (b_i - c(g_i, \theta_i)) \frac{1-E_0[e^{-rT_i}]}{r} + (p_i^* - p_0) E_0[e^{-rT_i}], & \text{for } \theta_i \in (\theta^l, \theta^u] \\
(b_i - c(g_i, \theta_i)) \frac{1-E_0[e^{-rT_i}]}{r} + (p_i^* - p_0) E_0[e^{-rT_i}], & \text{for } \theta_i = \theta^l.
\end{cases}
\]

(16)

Comparison between Eq. (6) and Eq. (16) points out the value of the managerial flexibility embedded in the opportunity of early-exit. It also shows that the lack of enforcement of contract terms alters the seller’s expected payoff, namely, by lowering the opportunity cost of participating to the conservation program.

### 5.2 Equilibrium strategy

Suppose the buyer ignored the risk of opportunistic behavior by the seller and, as above, ranked bids on the basis of the instantaneous score \( s(b_l, g_i) \), by assuming that \( T \) will be obeyed.

\[ \Pi(b_i, g_i; \theta_i) = E_0[\int_0^{T_i} r(t) c'(g_i, \theta_i) e^{-r z} dz + e^{-r T_i} \int_0^\infty p(z)e^{-r(s-T_i)} dz - K(\theta_i)] < \widehat{\Pi}(\theta_i) \]

In this case, none of the bidders would find it profitable to prematurely terminate the contract.

---

25 Notice that if, in the event of early exit (\( T_i < \ddot{T} \)), the buyer imposed the repayment of the whole funds already paid:

\[ \Pi(b_i, g_i; \theta_i) = E_0[\int_0^{T_i} r(t) c'(g_i, \theta_i) e^{-r z} dz + e^{-r T_i} \int_0^\infty p(z)e^{-r(s-T_i)} dz - K(\theta_i)] < \widehat{\Pi}(\theta_i) \]
Therefore, bidders will make their bids by maximizing the following functional:

$$\Pi(s_i) = \Pi(b_i, g_i; \theta_i) \cdot \Pr(\text{of win}/s_i) + \Pi(0, 0; \theta_i) \cdot (1 - \Pr(\text{of win}/s_i)), \quad (17)$$

where $$\Pi(0, 0; \theta_i) = \tilde{\Pi}(\theta_i)$$ is the reservation value, and agents will participate in the auction only if the individual rationality constraint holds, i.e. $$\Pi(s_i) \geq \tilde{\Pi}(\theta_i)$$.

Using Eq. (16) and Eq. (17), bidder i’s objective can be defined as follows:

$$W(b_i, g_i; \theta_i) = \{[(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - E_0[e^{-rT_i}]}{r} + (p^*_i - p_0) \frac{E_0[e^{-rT_i}]}{r} \} \Pr(\text{of win}/s_i). \quad (18)$$

The solution of the bidding game is as follows.

**Proposition 2** When the contract duration is not enforceable, for any finite $$n$$ it will always exist an equilibrium in symmetric, strictly increasing strategies $$s(\theta_i)$$, characterized by:

i) the environmental output $$g(\theta_i)$$ (defined by Eq. (10)),

ii) the bidding function:

$$b(\theta_i) = c(g(\theta_i), \theta_i) + (p_0 - rK(\theta_i)) + \int_{\theta^i}^{\theta^*} \Delta_{\theta}(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) dx + (p^*(\theta_i) - p_0) \frac{E_0[e^{-rT_i(\theta_i)}]}{r}, \quad (19)$$

iii) the expected profits:

$$W(\theta_i) = - \int_{\theta^i}^{\theta^*} \Delta_{\theta}(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) dx, \quad (20)$$

where, by (8), $$W(\theta^i) = 0$$.

**Proof.** See Appendix A.2. ■

Note, first, that the bid service is equal to that offered under enforceable contract terms. This is an important result which allows extending the use of Lemma 1 by Che (1993) also to the case of contracts with non-enforceable duration (see Appendix A.2). Hence, once secured its monotonicity, i.e. $$ds(\theta_i)/d\theta_i > 0$$, the use of the instantaneous score still allows to select the most efficient (least-cost) agent.

The second remark concerns the bid price which, as above, covers both the direct and opportunity costs of conservation practices. However, comparison of Eq. (11) with Eq. (19)
shows that when the contract duration is not enforceable, the bid price is lowered by the term \((p^*(\theta_i) - p_0)\frac{E_0[e^{-rT_i(\theta_i)}]}{r}\), which accounts for the potential gains associated with early-exit.\(^\text{26}\) In other words, bidders will bid more "aggressively" when they do not face enforceable project deadlines.

Clearly, the magnitude of the bid reduction will depend on the uncertainty about the profits resulting from putting \(L\) into commercial use. In fact, it is easy to show that: \(\sigma \to \infty, E_0[e^{-rT_i(\theta_i)}] \to 0\) for all \(\theta_i\). That is, the higher is the uncertainty about market profits the lower is the difference between the bid price with and without contract enforcement, because the today’s value to develop \(L\) tends to vanish as uncertainty increases.

Since information rents are null for agent \(\theta^l\), we get:

\[
\tilde{b}(\theta^l) = c(g(\theta^l), \theta^l) + (p_0 - rK(\theta^l)) - p_0 \frac{E_0[e^{-rT_i(\theta^l)}]}{\beta - (1 - E_0[e^{-rT_i(\theta^l)}])} < \tilde{b}(\theta^l),
\]

from which it is easy to show that \(\beta/(\beta - 1)(\tilde{b}(\theta^l) - c(g(\theta^l), \theta^l) + rK(\theta^l)) = p^*(\theta^l) > p_0\). In other words, the managerial flexibility, spurred by the non-enforcement of contract terms, tends to intensify the competition among the bidders.

In light of these results, let us analyze the effect of managerial flexibility upon the parties’ individual payoffs.

Let us first consider the seller.

**Proposition 3** Whatever is \(\overline{T}\): (i) the rental payment is lower when the contract duration is not enforceable, \(b(\theta_i) < \tilde{b}(\theta_i)\); (ii) the seller’s expected payoff is lower when the contract duration is not enforceable, \(W(\theta_i) < \tilde{W}(\theta_i)\), unless \(T_i > \overline{T}\), in which case \(W(\theta_i) = \tilde{W}(\theta_i)\).

**Proof.** See Appendix A.3. ■

The first result is consistent with other findings, such as those of Spulber (1990), who pointed out that, in the absence of enforcement, the most efficient (low-cost) bidders will be forced to bid low in order to preserve their chances of winning. This, in turn, raises the probability of breach of contracts. Unlike Spulber, however, we find that the possibility of adjusting the service period allows preserving the efficiency of the bidding process. In other words, the auction does not fail to allocate the contract to the bidder having the lowest cost of undertaking conservation activities.

\(^{26}\)Notice that both the information rents and the gains associated with contract breach are annualized by the term \((1 - E_0[e^{-rT_i(\theta_i)}])/r\).
The second result states that bidders’ expected payoff is higher when facing an enforceable contract deadline, since the potential benefits, stemming from the early-exit option, are outweighed by the stronger bid competition spurred by the non-enforceability of contract terms.

This result conforms with the findings of the literature on security-bid auctions. In fact, when the contract duration is enforceable, bidders bid knowing that, upon winning, they will give up the opportunity to develop $L$ for $T$ time periods. By taking the buyer’s perspective, this is equivalent to purchasing a call-like ("growth") option by a cash auction, by paying a price which in our framework takes on the form of a flow of rental payments (covering also the direct costs of environmental quality enhancements).

On the other hand, when $T$ is not enforceable, the price received for selling the growth option also includes the option to prematurely terminate the contract. The inclusion of this last element in the bid price makes, de facto, state-contingent the value attached to the contract. Thus, not surprisingly, the value accruing to the seller is lower than that achievable under an only-cash auction.

6 The buyer’s payoff and the risk of opportunistic behavior

6.1 The buyer’s payoff

When the contract duration is not enforceable, the buyer’s total expected payoff is given by:

$$S(\theta_i) = s(\theta_i) \left(1 - E_0/[e^{-rT}(\theta_i)]\right),$$

where $T$ is the seller’s optimal time for breaching the contract.

Since $T \leq \bar{T}$, allocating the contract on the basis of the highest instantaneous score $s(\theta_i)$ might not be the best choice for the buyer, because the winner (least-cost agent) could be more prone than others to early-exit.

This becomes clear if we take a closer look at the derivative of Eq. (15.1) with respect to the bidders’ types:

$$\frac{dp^*(\theta_i)}{d\theta_i} = -\beta \left( \frac{\Delta \theta}{\beta - 1} + \frac{ds(\theta_i)}{d\theta_i} \right).$$

Since $\Delta \theta(\theta_i) < 0$ and $ds(\theta_i)/d\theta_i > 0$, the sign of $dp^*(\theta_i)/d\theta_i$ is ambiguous. In other words, higher values of $\theta$ can either translate into an increase, or a reduction of the optimal trigger for
breaching the contract. In the latter case, the combination of high instantaneous net benefits and a short service period can turn out not being the one giving the buyer the highest total payoff.

Notice that, in our framework, by Assumption 5 ("additionality"), all losing bidders will develop \( L \) for commercial use by affording a sunk capital cost. Hence, in case of early exit by the winner, the possibility of procuring the service contractualized in the first place by running a new auction with the same agents is ruled out.

### 6.2 Accounting for the risk of opportunistic behavior

The risk of not selecting the agent providing the highest potential payoff could be avoided if the buyer: (i) exploited the information gathered through the bidding process and (ii) used the total expected payoff, \( S(\theta_i) \), rather than the instantaneous score, \( s(\theta_i) \), to allocate the contract.

This would come at no cost in terms of auction efficiency, provided we introduce the following assumption on information rents which strengthens Assumption 1.

**Assumption 6:** \(-\Delta_\phi(\theta_i)(F(\theta_i)/f(\theta_i))\) is increasing in \( \theta_i \) and is bounded above by \((n - 1)/[(1 - E_0[e^{-rT_i(\theta_i)}])]/r \) for each \( \theta_i \in [\theta^l, \theta^u] \)

Notice that if \( \Delta(\theta_i) \equiv [c(g, \theta_i) - rK(\theta_i)] \) is concave in \( \theta_i \), Assumption 1 would suffice for having \(-\Delta_\phi(\theta_i)(F(\theta_i)/f(\theta_i))\) increasing in \( \theta_i \). This, for instance, corresponds to the well-known sufficient conditions (i.e., Spence-Mirrlees single-crossing and monotonicity conditions) for implementing the optimal contract in a static framework (see, for example, Guesnerie and Laffont, 1984 and Fudenberg and Tirole, 1991).

In our dynamic frame, however, the standard conditions do not ensure the monotonicity of \( T(\theta_i) \) and, therefore, the monotonicity of \( S(\theta_i) \). In fact, the information rents for the most efficient agents might be so high that they can competitively bid on the instantaneous score and win the auction, even though there might be other bidders able to ensure a longer service provision and, as a whole, higher total benefits to the buyer.

Hence, the risk of opportunistic behavior calls for a restriction not just on \( \Delta_\phi(\theta_i)(F(\theta_i)/f(\theta_i)) \), but on both \( T(\theta_i) \) and \( \Delta_\phi(\theta_i)(F(\theta_i)/f(\theta_i)) \). In fact, by rearranging equation Eq. (22) as follows (see Appendix A.4):

\[
\frac{dp^*(\theta_i)}{d\theta_i} = \frac{\beta}{\beta - 1} \Delta_\phi(\theta_i)[(n - 1)\frac{f(\theta_i)}{F(\theta_i)} \frac{W(\theta_i)}{dW(\theta_i)/d\theta_i} - 1]
\] (22.1)
it can be noticed that a sufficient condition for \( dp^*(\theta_i)/d\theta_i > 0 \) is that:

\[
\Delta_0(\theta_i) \frac{1 - E_0[e^{-rT_i(\theta_i)}]}{r} F^{(n)}(\theta_i) + f^{(n)}(\theta_i) > 0 \quad \text{for all } \theta_i \in [\theta^l, \theta^u]
\] (22.2)

This condition is satisfied by Assumption 6, which represents the minimum level of adjustment required for meeting the monotonicity condition in the presence of opportunistic sellers contemplating a potential early-exit.\(^{27}\)

The following proposition captures this result.

**Proposition 4** Under Assumption 6, for any finite \( n \) it will always exist an equilibrium in symmetric strictly increasing strategies \( s(\theta_i) \) with a non-decreasing optimal expected service period in \( \theta_i \), that is, \( dS(\theta_i)/d\theta_i > 0 \).

**Proof.** See Appendix A.4.

Note that the regularity condition imposed by Assumption 6 is a sufficient (not necessary) condition for the equilibrium existence. For instance, should the condition not be met in a subset of the space \([\theta^l, \theta^u]\), the solution in Proposition 2 could still constitute an equilibrium, since there can be a value \( \tilde{\theta} \in [\theta^l, \theta^u] \) beyond which, despite \( dp^*(\theta_i)/d\theta_i \leq 0 \), \( dS(\theta_i)/d\theta_i > 0 \) for every \( \theta_i \in [\theta^l, \theta^u] \) (see Appendix A.4). In this case, even though not ensuring the longest service provision, the least-cost agent would provide the buyer with the highest total payoff, by compensating the shortening of service period with higher instantaneous benefits.

### 6.3 Competition and eligibility rules

Proposition 4 implies that a strong competition for winning the contract reduces the risk of not selecting the agent providing the highest potential payoff to the buyer. Otherwise, when competition is relatively low, the buyer could still get the highest expected payoff by restricting eligibility for conservation payments.

To clarify this statement, consider the following example. Suppose that the sunk cost for developing \( L \) for commercial use is \( K(\theta) = \theta^u - \theta \), the private cost of environmental proactivity

\(^{27}\)Sung (2005) presents a similar condition in a continuous-time agency model, with both adverse selection and moral hazard, where the optimal contract is linear in the effort. Note, however, that condition (22.2) is more general, since it accounts for the non-linearity of the optimal contract (i.e., the bid function) in the level of "effort" (i.e., the contract duration \( T \)).
is \( c(g, \theta) = (g^2/2) - g(\theta - \theta^d) \), and the instantaneous social benefit is \( v(g) = g \). Moreover, for the sake of simplicity, suppose that the agent-types are uniformly distributed between 0 and 1, that is, \( F(\theta) = \theta \) and \( f(\theta) = 1 \).

Hence, by Eq. (10), we get:

\[
g(\theta) = 1 + \theta \geq 1 \tag{23}
\]

As shown in Appendix A.4, for the case of a trendless geometric Brownian motion, Assumption 6 is implied by the following condition:

\[
\Delta g(\theta)(F(\theta)/f(\theta)) > r(n - 1) \tag{24}
\]

where \( \Delta g(\theta) = r - g(\theta) < 0 \) is decreasing in \( \theta \).

Substituting Eq. (23) into condition (24) and rearranging we obtain:

\[
Q(\theta; n) \equiv (1 + \theta - r)\theta - r(n - 1) < 0 \tag{24.1}
\]

Notice that, for any \( n \), \( Q(\theta; n) \) is increasing in \( \theta \) in the interval \([0, 1]\), with \( Q(0) = -r(n - 1) \) and \( Q(1) = 2 - rn \).

Denoting by \( \hat{\theta} \) the solution of the equation \( Q(\hat{\theta}; n) = 0 \), this is given by:

\[
\hat{\theta} = \sqrt{\left(\frac{1 - r}{2}\right)^2 + r(n - 1) - \frac{1 - r}{2}}
\]

where \( \hat{\theta} \geq 1 \) for \( n \geq 2/r \).

Hence, as long as the number of bidders is relatively high (i.e. \( n \geq 2/r \)), no restrictions on the eligibility rules are needed in order to ensure that \( dS(\theta_i)/d\theta_i > 0 \) over the entire range \([0, 1]\).

On the other hand, if competition is relatively low (i.e. \( n < 2/r \)), the buyer would increase the total payoff by restricting the range of eligible agents, namely, by excluding from award those belonging to the subset \([\hat{\theta}, 1]\).

### 7 Final remarks

Like other public procurement initiatives, contracts providing payments for preserving and enhancing natural assets generally require long-term commitments. Contractors, however, can find it
profitable to breach the provision of environmental services when conservation compliance costs increase. While this does not necessarily have to lead to early-exit, this possibility can be exacerbated by the lack of sufficiently strong incentives against premature terminations of supply.

The question addressed in the paper is how nested termination options can affect bidding behavior and the parties' individual payoffs in procurement auctions where contractors are selected on the basis of both the promised output and the required payments.

The novelty of our model, with respect to the previous literature on scoring auctions, is that agents can adjust their bidding strategies by exploiting the managerial flexibility spurred by the lack of incentives against contract infringements. This implies that, unlike when the project deadline is enforceable, bidding strategies will be affected by endogenous timing considerations.

A first result of the paper is that procurement contracts embedding early-exit options do not translate into higher expected payoffs for the sellers. This is because, in a competitive environment, the potential benefits, stemming the opportunity to terminate the contract, are outweighed by the stronger bid competition spurred by the lack of enforcement of time commitments. Hence, besides increasing the risk of failure of conservation programs, the weak enforcement of contract deadlines may not even be in the contractors' best interest.

A second result is that failure to account for the risk of opportunistic behavior could lead to the choice of contractors who will not ensure the highest payoff to the buyer. In the specific case of conservation contracts, this possibility relies on the correlation between the cost of undertaking conservation activities and the opportunity cost of not exploiting land for commercial uses.

If costs are negatively correlated, that is, if agents that are able to more efficiently exploit the asset for commercial use are also able to undertake conservation activities at lower costs, the most efficient bidders, while providing the buyer with the highest instantaneous net benefits, can be more prone than others to early exit. Hence, the combination of higher instantaneous payoffs and shorter service periods can turn out not being the one giving the buyer the highest total value.

The paper makes suggestions which may contribute to avoid such potential bias in contract allocation. A main policy implication of our paper is that when, for whatever reasons, contracting agencies cannot rely on sufficiently strong and credible incentives against premature terminations of contracts, they should "internalize" the risk of early-exit.

In practice this means exploiting the information gathered through the auction process, in
order to assess, and include in bid evaluation, the bidders’ actual prospective compliance. As shown in the paper, this would not affect the auction’s allocative efficiency, so long as the number of bidders is sufficiently high to downsize the most efficient agents’ information rents. Otherwise, when competition is relatively low, it might be profitable for the buyer to restrict the range of eligible agents, by excluding from award those with relatively high opportunity costs of compliance with conservation requirements.

A Appendix

A.1 Proposition 1

In spite of being quite standard in the auction literature, we include the following proof for the reader’s convenience. Consider a common prior cumulative distribution $F(\theta)$ with continuously differentiable density $f(\theta)$ defined on a positive support $\Theta = [\theta^l, \theta^u] \subseteq R_+$, where the lowest value $\theta^l$ is such that $\theta^l = \inf \{\theta : f(\theta) > 0\}$ and the highest value is $\theta^u = \sup \{\theta : f(\theta) > 0\}$. Now consider the agent $i$’s bidding behavior. It is immediate to note that the maximization of the objective in (9) is equivalent to maximize the instantaneous expected net payoff, i.e. $[(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))]\Pr(\text{of win}/s_i)$, and that $(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))$ must be non-negative to guarantee a positive expected payoff (otherwise, winning the auction would never be profitable).

Assume that all other bidders use a strictly monotone increasing bid function $s(\theta_j)$, i.e., $s(\theta_j) : [\theta^l, \theta^u] \rightarrow [s(\theta^u), s(\theta^l)] \forall j \neq i$. Since, by assumption, $s(\theta_i)$ is monotone in $[\theta^l, \theta^u]$, the probability of winning by bidding $s(\theta_i)$ is $\Pr(s(\theta_i) > s(\theta_j) \mid \forall j \neq i) = \Pr(\theta_j < s^{-1}(s(\theta_i)) \mid \forall j \neq i) = F(s^{-1}(s(\theta_i)))^{n-1} = F(\theta_i)^{n-1} \equiv F(n)(\theta_i)$.

The type reported, $\tilde{\theta}_i$, by agent $\theta_i$ solves the following problem:

$$
\tilde{W}(\theta_i; \tilde{\theta}_i) = \max_{\theta} [(b(\tilde{\theta}_i) - c(g(\tilde{\theta}_i), \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - e^{-rT}}{r} \Pr(\text{of win}/s_i)
$$

$$
= \max_{\theta} [(b(\tilde{\theta}_i) - c(g(\tilde{\theta}_i), \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - e^{-rT}}{r} F(n)(\tilde{\theta}_i),
$$

where, by Che (1993, Lemma 1, p. 672), $g_i \equiv g(\tilde{\theta}_i)$ is determined by Eq. (10). This in turn implies that:

$$
\tilde{W}(\theta_i, \tilde{\theta}_i) = \max_{\theta} [(b(\tilde{\theta}_i) - c(g(\tilde{\theta}_i), \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - e^{-rT}}{r} F(n)(\tilde{\theta}_i), \quad (A.1.1)
$$
Finally, by using Eq. (A.1.5), we can easily prove the auction efficiency. That is, 

\[
\frac{\partial \bar{W}(\theta_i, \tilde{\theta}_i)}{\partial \tilde{\theta}_i} \bigg|_{\tilde{\theta}_i = \theta_i} = \left( \frac{db(\theta_i)}{d\theta_i} - c_g(g(\theta_i), \theta_i) \frac{dg(\theta_i)}{d\theta_i} \right) F^{(n)}(\theta_i) +
\]

\[
+ [(b(\theta_i) - c(g(\theta_i), \theta_i)) - (p_0 - r K(\theta_i))] f^{(n)}(\theta_i) = 0
\]

or

\[
\left( \frac{db(\theta_i)}{d\theta_i} - c_g(g(\theta_i), \theta_i) \frac{dg(\theta_i)}{d\theta_i} \right) F^{(n)}(\theta_i) = -[(b(\theta_i) - c(g(\theta_i), \theta_i)) - (p_0 - r K(\theta_i))] f^{(n)}(\theta_i). \tag{A.1.1b}
\]

Integrating Eq. (A.1.1b) on both sides yields:

\[
\int_{\theta_i}^{\theta_i} \left( \frac{db(x)}{d\theta} - c_g(g(x), \theta_i) \frac{dg(x)}{d\theta_i} \right) F^{(n)}(x) dx = -\int_{\theta_i}^{\theta_i} [(b(x) - c(g(x), x)) - (p_0 - r K(x))] f^{(n)}(x) dx
\]

Using integration by parts, one can easily show that:

\[
\bar{b}(\theta_i) = c(g(\theta_i), \theta_i) + (p_0 - r K(\theta_i)) - \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i)} dx, \tag{A.1.2}
\]

where $\Delta_\theta(x) = c_g(g(x), x) - r K_\theta(x) < 0$ for $x \in [\theta_i, \theta_i]$.

Eq. (A.1.2) can be used in order to define the agent’s expected payoff. That is:

\[
\bar{W}(\theta_i) = -\int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{1 - e^{-r F}}{r} F^{(n)}(x) dx + \bar{W}(\theta_i',) \tag{A.1.3}
\]

where, by Eq. (8), $\bar{W}(\theta_i') = 0$. Finally, by differentiating Eq. (A.1.2) with respect to $\theta_i$ and rearranging, we obtain:

\[
\frac{d\bar{b}(\theta_i)}{d\theta_i} = c_g(g(\theta_i), \theta_i) \frac{dg(\theta_i)}{d\theta_i} + (n - 1) \frac{f(\theta_i)}{F(\theta_i)} \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i)} dx. \tag{A.1.4}
\]

Eq. (A.1.4) implies that $\bar{b}(\theta_i)$ is increasing in the cost of producing higher quality, $g(\theta_i)$, and decreasing in the information rent, i.e. $-\int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i)} dx$, to be paid.

By differentiating Eq. (2) with respect to $\theta_i$ and using Eq. (10) and Eq. (A.1.4), we can immediately prove the assumed monotonicity of the optimal strategy $s(\theta_i)$:

\[
\frac{ds(\theta_i)}{d\theta_i} = -(n - 1) \frac{f(\theta_i)}{F(\theta_i)} \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i)} dx > 0. \tag{A.1.5}
\]

Finally, by using Eq. (A.1.5), we can easily prove the auction efficiency. That is,

\[
\frac{d\bar{S}(\theta_i)}{d\theta_i} = \frac{d\bar{S}(\theta_i)}{d\theta_i} 1 - e^{-r F} > 0. \tag{A.1.6}
\]

This concludes the proof.
A.2 Proposition 2

Separability - Let’s first show that the following lemma holds:

**Lemma 1** With a first-score auction, the equilibrium output \( g(\theta_i) \) is chosen such that

\[
g(\theta_i) = \arg \max \left[ v(g_i) - c(g_i, \theta_i) \right], \text{ for all } \theta_i \in [\theta^l, \theta^u]. \tag{A.2.1}
\]

This lemma can be easily shown by adapting to our case the proof provided by Che (1993, Lemma 1). Suppose that any equilibrium bid, \((b_i, g_i)\), with \( g_i \neq g(\theta_i) \), is dominated by an alternative bid, \((b'_i, g'_i)\) where \( b'_i = b_i + v(g'_i) - v(g_i) \), and \( g'_i = g(\theta_i) \). It follows that:

\[
b_i - c(g_i, \theta_i) = b'_i - c(g'_i, \theta_i) + \left[ (v(g_i) - c(g_i, \theta_i)) - (v(g'_i) - c(g'_i, \theta_i)) \right]
\]

\[
< b'_i - c(g'_i, \theta_i), \tag{A.2.2}
\]

which in turn implies \( p^*(b_i, g_i, \theta_i) < p^*(b'_i, g'_i, \theta_i) \) and \( E_0 \left[ e^{-rT_i} \right] > E_0 \left[ e^{-rT'_i} \right] \).

Given Eq. (A.2.2), it can be easily shown that:

\[
S_i = (v(g_i) - b_i) \frac{1 - E_0[e^{-rT_i}]}{r} = (v(g'_i) - b'_i) \frac{1 - E_0[e^{-rT'_i}]}{r} < (v(g'_i) - b'_i) \frac{1 - E_0[e^{-rT'_i}]}{r} = S'_i, \tag{A.2.3}
\]

or, equivalently, that \( \Pr(\text{of win}/S'_i) > \Pr(\text{of win}/S_i) \). Note also that \( \Pi(b_i, g_i; \theta_i) - \Pi(0, 0; \theta_i) \) (see Eq. (16)) is increasing in \( h(b_i, g_i; \theta_i) = b_i - c(g_i, \theta_i) \) as:

\[
\frac{\partial[\Pi(S_i) - \Pi(0, 0; \theta_i)]}{\partial h} = 1 - \left( \frac{p_0}{\bar{p}_0} \right) \frac{\beta}{\frac{d\bar{p}}{d\bar{p}}} > 0.
\]

This in turn implies that:

\[
W(b_i, g_i; \theta_i) = \{[(b_i - c(g_i, \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - E_0[e^{-rT_i}]}{r} + (p^* - p_0) \frac{E_0[e^{-rT_i}]}{r}\} \Pr(\text{of win}/S_i)
\]

\[
< \{[(b'_i - c(g'_i, \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - E_0[e^{-rT'_i}]}{r} + (p^* - p_0) \frac{E_0[e^{-rT'_i}]}{r}\} \Pr(\text{of win}/S'_i)
\]

\[
= W(b'_i, g'_i, \theta_i). \tag{A.2.4}
\]

Finally, given Lemma 1, the equilibrium quality, \( g(\theta_i) \), can be determined using a standard set of optimality conditions. Note that the problem is equivalent to (10). Hence, the same set of first- and second-order conditions holds also in this case (see condition (10.1)).

**Bayes-Nash equilibrium** - Let’s now consider agent \( i \)'s bidding behavior. Assume that all other bidders use a strictly monotone increasing bid function \( s(\theta_j), \) i.e., \( s(\theta_j) : [\theta^l, \theta^u] \rightarrow \)
[s(θ_i^n), s(θ_i^l)] \forall j \neq i$. Since, by assumption, $s(θ_i)$ is monotone in $[θ_i^l, θ_i^n]$, the probability of winning by bidding $s(θ_i)$ is $Pr(s(θ_i) > s(θ_j) \mid \forall j \neq i) = Pr(θ_j < s^{-1}(s(θ_i)) \mid \forall j \neq i) = F(s^{-1}(s(θ_i)))^{n-1} = F(θ_i)^{n-1} = F^{(n)}(θ_i)$.

It follows that agent $i$ chooses his report $\tilde{θ}_i$ by solving the following problem:

$$W(θ_i, \tilde{θ}_i) = \max_θ \{[(b(\tilde{θ}_i) - c(g(\tilde{θ}_i), θ_i)) - (p_0 - rK(θ_i))] \frac{1 - E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} + (p^*(\tilde{θ}_i, θ_i) - p_0) \frac{E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} \} \Pr(\text{of win}/s_i)$$

$$= \max_θ \{[(b(\tilde{θ}_i) - c(g(\tilde{θ}_i), θ_i)) - (p_0 - rK(θ_i))] \frac{1 - E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} + (p^*(\tilde{θ}_i, θ_i) - p_0) \frac{E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} \} \Pr(s(\tilde{θ}_i) < \max_{j \neq i} s(θ_j))$$

$$= \max_θ \{[(b(\tilde{θ}_i) - c(g(\tilde{θ}_i), θ_i)) - (p_0 - rK(θ_i))] \frac{1 - E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} + (p^*(\tilde{θ}_i, θ_i) - p_0) \frac{E_0[e^{-rT_i(\tilde{θ}_i, θ_i)}]}{r} \} F^{(n)}(\tilde{θ}_i), \tag{A.2.5}$$

where, by Lemma 1, $g_i \equiv g(\tilde{θ}_i)$ is still determined by solving Eq. (10).

In order to derive the equilibrium strategies we could solve the ordinary differential equation that follows from the maximization problem in Eq. (A.2.5). However, differently from the problem in Eq. (A.1.1), this ordinary differential equation, due to the presence of the optimal trigger $p^*(\tilde{θ}_i, θ_i)$ (and the stopping time $T_i(\tilde{θ}_i, θ_i)$), does not have a closed-form solution. We then determine the integral equation that describe the utility $W(θ_i)$ by using the generalized Envelope Theorem provided by Milgrom and Segal (2002, Theorem 2).\footnote{See Board (2007, p. 329) for a similar application.}

Suppose that each agent sets $y \in Y$ to maximize $ψ(y, ω)$, where $ω \in [0, 1]$ and $Y$ is arbitrary. Denote the set of maximizers by $\overline{ψ}(ω) := \arg\max_y ψ(y, ω)$ and let $Ψ(ω) = \sup_{y \in Y} ψ(y, ω)$.

**Lemma 2** (Milgrom and Segal, 2002, Theorem 2). Suppose a) $ψ(y, ω)$ is differentiable and absolutely continuous in $ω$ ($∇y$); b) $|∂ψ(y, ω)/∂ω|$ is uniformly bounded ($∇y(∇ω)$, and c) $\overline{ψ}(ω)$ is nonempty. Then, for any selection $y^*(ω) \in \overline{ψ}(ω)$,

$$Ψ(ω) = \int_0^ω \frac{∂ψ(y^*(x), x)}{∂ω} dx + Ψ(0). \tag{A.2.6}$$

Now, let set $ω = θ_i$ and $y = (\tilde{θ}_i, T_i)$ (or equivalently $(\tilde{θ}_i, p^*_i)$), and apply Milgrom and Segal’s theorem to the problem in Eq. (A.2.5). Note that: (i) $W(θ_i, \tilde{θ}_i)$ is always differentiable and
continuous in $\theta_i$; (ii) the derivative of $W$ is bounded because of Assumption 1; (iii) Eq. (15.1) says that the stopping time attains its maximum for given $b_i$ and $\theta_i$. Since, by the revelation principle, $\tilde{\theta}_i = \theta_i$, it follows that:

$$W(\theta_i) = -\int_{\theta'}^{\theta_i} \Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) dx + W(\theta'),$$  \hspace{1cm} (A.2.7)

where, by Eq. (8), $W(\theta') = 0$.

Now, we can use Eq. (A.2.7) in order to define the equilibrium payment. The equilibrium payment, $b(\theta_i)$, solves the following implicit equation:

$$[(b(\theta_i) - c(g(\theta_i), \theta_i)) - (p_0 - rK(\theta_i))] \frac{1 - E_0[e^{-rT_i(\theta_i)}]}{r} + (p^*(\theta_i) - p_0) \frac{E_0[e^{-rT_i(\theta_i)}]}{r} = -\int_{\theta'}^{\theta_i} \Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) F^{(n)}(\theta_i) dx,$$  \hspace{1cm} (A.2.8)

which can be rearranged as in Eq. (19). Although it is not possible to express $b(\theta_i)$ in a closed form, we may easily show some of its properties. In fact, by totally differentiating Eq. (A.2.8) with respect to $\theta_i$ and rearranging, we obtain:

$$\frac{db(\theta_i)}{d\theta_i} = c_g(g(\theta_i), \theta_i) \frac{dg(\theta_i)}{d\theta_i} + (n - 1) \frac{f(\theta_i)}{F(\theta_i)} \int_{\theta'}^{\theta_i} \Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) F^{(n)}(\theta_i) dx.$$  \hspace{1cm} (A.2.9)

We notice that the payment, $b(\theta_i)$, is increasing in the cost of producing higher quality and decreasing in the information rent to be paid. Finally, by using Eq. (A.2.9), it is easy to show that the bid function $s(\theta_i)$ is strictly monotone and increasing in $\theta_i$:

$$\frac{ds(\theta_i)}{d\theta_i} = -(n - 1) \frac{f(\theta_i)}{F(\theta_i)} \int_{\theta'}^{\theta_i} \Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) F^{(n)}(\theta_i) dx > 0.$$  \hspace{1cm} (A.2.10)

It remains to show that Eq. (A.2.7) provides the unique local maximum for the problem in Eq. (A.2.5). As usual, the uniqueness follows from the fact that the function in Eq. (A.2.7) is increasing in $\theta_i$ and has Eq. (8) as boundary condition. For showing that it is a maximum, it suffices to prove that $s_i$ is incentive compatible:

$$W(\theta_i, \tilde{\theta}_i) \geq W(\theta_i, \theta_i),$$  \hspace{1cm} (A.2.11)

for all $(\theta_i, \tilde{\theta}_i) \in [\theta^l, \theta^h] \times [\theta^l, \theta^h]$. We prove this by following the approach in adopted by Milgrom (2004, Ch.4).

**Lemma 3.** The equilibrium strategy $s(\theta_i)$ is incentive compatible if and only if equation (A.2.7) holds and if $s(\theta_i)$ is strictly increasing.
where $M_i$ is strictly increasing then by the Monotonic Selection Theorem (Milgrom, 2004, Theorem 4.1), the function $W(\theta_i, \bar{\theta}_i)$ satisfies the strict single-crossing difference condition.

**Necessity.** Since $s(\theta_i)$ is strictly increasing then by the Monotonic Selection Theorem (Milgrom, 2004, Theorem 4.1), the function $W(\theta_i, \bar{\theta}_i)$ satisfies the strict single-crossing difference condition. Finally, by direct inspection of $\Delta_\theta(x)(1 - E_0[e^{-rT_i(x)}])$ and $\Delta_\theta(x)(1 - e^{-rT})$, it is also immediate to show that if $T_i(x) > T$

\[ W(\theta_i) > W(\theta_i). \]  

This concludes the proof.

**A.3 Proposition 3**

By rearranging Eq. (19) we obtain:

\[ b(\theta_i) = c(g(\theta_i), \theta_i) + (p_0 - rK(\theta_i)) - \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F_n(x)}{F_n(\theta_i)} dx - M(\theta_i) = \bar{b}(\theta_i) - M(\theta_i), \quad (A.3.1) \]

where

\[ M(\theta_i) = \frac{-\int_{\theta_i}^{\theta_i} \Delta_\theta(x)(E_0[e^{-rT_i(x)}] - E_0[e^{-rT_i(\theta_i)}]) \frac{F_n(x)}{F_n(\theta_i)} dx + E_0[e^{-rT_i(\theta_i)}](p^*(\theta_i) - p_0).}{1 - E_0[e^{-rT_i(\theta_i)}]} \]

Now, note that:

\[ p^*(\theta_i) - p_0 = \bar{p}^*(\theta_i) - p_0 - \frac{\beta}{\beta - 1} M(\theta_i), \quad (A.3.2) \]

where

\[ \bar{p}^*(\theta_i) = \frac{\beta}{\beta - 1} (\bar{b}(\theta_i) - c(g(\theta_i), \theta_i) + rK(\theta_i)) = \frac{\beta}{\beta - 1} (p_0 - \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{F_n(x)}{F_n(\theta_i)} dx). \]

Substituting Eq. (A.3.2) into $M(\theta_i)$ yields:

\[ M(\theta_i) = -\frac{(\beta - 1) \int_{\theta_i}^{\theta_i} \Delta_\theta(x)(E_0[e^{-rT_i(x)}] - E_0[e^{-rT_i(\theta_i)}]) \frac{F_n(x)}{F_n(\theta_i)} dx + E_0[e^{-rT_i(\theta_i)}]p^*(\theta_i)}{(\beta - 1) + E_0[e^{-rT_i(\theta_i)}]} > 0. \quad (A.3.4) \]

This in turn implies that:

\[ b(\theta_i) < \bar{b}(\theta_i), \quad (A.3.5) \]

\[ p^*(\theta_i) < \bar{p}^*(\theta_i). \quad (A.3.6) \]

It follows that:

\[ s(\theta_i) = (v(g(\theta_i) - b(\theta_i)) > (v(g(\theta_i) - \bar{b}(\theta_i))) = \bar{s}(\theta_i). \quad (A.3.7) \]

Finally, by direct inspection of $\Delta_\theta(x)(1 - E_0[e^{-rT_i(x)}])$ and $\Delta_\theta(x)(1 - e^{-rT})$, it is also immediate to show that if $T_i(x) > T$

\[ W(\theta_i) > W(\theta_i). \quad (A.3.8) \]

This concludes the proof.
A.4 Proposition 4

By deriving \( p^*(\theta_i) \) with respect to \( \theta_i \) we obtain:

\[
\frac{dp^*(\theta_i)}{d\theta_i} = -\frac{\beta}{\beta - 1} (\Delta_\theta(\theta_i) + \frac{ds(\theta_i)}{d\theta_i}). \tag{A.4.1}
\]

As can be easily seen, substituting for \( ds(\theta_i)/d\theta_i \) yields:

\[
\frac{dp^*(\theta_i)}{d\theta_i} = \frac{\beta}{\beta - 1} \left[ -\Delta_\theta(\theta_i) + (n - 1) f(\theta_i) \int_{\theta_i}^{\theta_i} \Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} \frac{F^{(n)}(x)dx}{F^{(n)}(\theta_i)} \right] \\
= \frac{\beta}{\beta - 1} \Delta_\theta(\theta_i) [(n - 1) f(\theta_i) W(\theta_i)/dW(\theta_i) - 1] \tag{A.4.2}
\]

Since \([\beta/(\beta - 1)]\Delta_\theta(\theta_i) < 0\), it follows that \( dp^*(\theta_i)/d\theta_i > 0 \) when the term into square brackets in Eq. (A.4.2) is positive or, equivalently, if:

\[
W(\theta_i) < F^{(n)}(\theta_i) \tag{A.4.3}
\]

Rearranging condition (A.4.3), we obtain:

\[
-\int_{\theta_i}^{\theta_i} (\Delta_\theta(x) \frac{1 - E_0[e^{-rT_i(x)}]}{r} F^{(n)}(x) + f^{(n)}(x))dx < 0 \tag{A.4.4}
\]

which, by Assumption 6, is always satisfied. Further, note that, by the Jensen’s inequality, \( E_0[e^{-rT_i(x)}] > e^{-rE_0[T_i(x)]} \). This implies that:

\[
\frac{n - 1}{1 - E_0[e^{-rT_i(x)}]} > \frac{n - 1}{1 - e^{-rE_0[T_i(x)]}}
\]

This means that Assumption 6 is implied by the following condition:

\[
-\Delta_\theta(x)(F(x)/f(x)) < \frac{n - 1}{1 - e^{-rE_0[T_i(x)]}} \tag{A.4.5}
\]

which, for the case of a trendless geometric Brownian motion, reduces to:\textsuperscript{29}

\[
-\Delta_\theta(x)(F(x)/f(x)) < r(n - 1) \tag{A.4.5}
\]

Let’s now check the properties of the scoring rule \( S(\theta_i) \). We notice that:

\[
\frac{dS(\theta_i)}{d\theta_i} = \frac{ds(\theta_i)}{d\theta_i} 1 - E_0[e^{-rT_i(\theta_i)}] r \frac{1}{r} - \frac{dE_0[e^{-rT_i(\theta_i)}]}{r} \frac{d\theta_i}{d\theta_i} \frac{dp^*(\theta_i)}{d\theta_i}. \tag{A.4.6}
\]

\textsuperscript{29}By (1), \( E_0[T_i(x)] = \infty \). See (Dixit, 1993, pp. 54-57).
This implies that $S(\theta_i)$ is increasing in $\theta_i$ when the following condition holds:

$$(n - 1) \frac{f(\theta_i)}{F(\theta_i)} > \beta \frac{s(\theta_i)}{\theta_i} \int_{\theta_i}^{\theta} \Delta_\theta(x) \frac{1-E_0[e^{-rT_i(x)}]}{r} \frac{E_0[e^{-rT_i(\xi)}]}{F(x)} \frac{dp^*(\theta_i)}{d\theta_i} \frac{\theta_i}{p^*(\theta_i)}.$$

(A.4.7)

This condition is always satisfied for $dp^*(\theta_i)/d\theta_i > 0$.

We conclude by highlighting that $dp^*(\theta_i)/d\theta_i \geq 0$ is only a sufficient (not necessary) condition for the existence of the equilibrium, i.e., $S(\theta_i)$ monotone in $\theta_i$. By direct inspection of Eq. (A.4.2), the sign of $dp^*(\theta_i)/d\theta_i$ depends on:

$$\text{sign}(dp^*(\theta_i)/d\theta_i) = \text{sign}[-\frac{f(\theta_i)}{F(\theta_i)} \int_{\theta_i}^{\theta} F^{(n)}(x) d(\Delta_\theta(x) \frac{1-E_0[e^{-rT_i(x)}]}{r} \frac{F(x)}{f(x)}) dx] > 0.$$

(A.4.8)

Since, by Assumption 6, $-\Delta_\theta(x) \frac{1-E_0[e^{-rT_i(x)}]}{r} (F(x)/f(x))$ is increasing, there exists $\theta^l \leq \xi \leq \theta_i$ such that:

$$\text{sign}(dp^*(\theta_i)/d\theta_i) = \text{sign}\left[\frac{f(\theta_i)}{F(\theta_i)} F^{(n)}(\theta_i) \Delta_\theta(\xi) \frac{1-E_0[e^{-rT_i(\xi)}]}{r} \frac{F(\xi)}{f(\xi)} - \Delta_\theta(\theta_i) \frac{1-E_0[e^{-rT_i(\xi)}]}{r} \frac{F(\theta_i)}{f(\theta_i)}\right]$$

(A.4.9)

from which it follows that $\lim_{\theta_i \leq \theta^l} dp^*(\theta_i)/d\theta_i > 0$. Thus, even if it may exist a value of $\hat{\theta}$ beyond which $dp^*(\theta_i)/d\theta_i \leq 0$, $S(\theta_i)$ is still increasing for all $\theta_i \in [\theta^l, \theta^u]$.

This concludes the proof.
References


Construction Management and Economics, 24, 475-484.