Statistical analysis of possible trends for extreme floods in northern Sweden

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Abstract
With ongoing climate change, analysis of trends in maximum annual daily river flow is of interest. Flow magnitude and timing during the year were investigated in this study. Observations from 11 unregulated rivers in northern Sweden were analysed, using extreme-value distributions with time-dependent parameters. The Mann–Kendall test was used to investigate possible trends. The extreme-value statistics revealed no significant trends for the stations considered, but the Mann–Kendall test showed a significant upward trend for some stations. For timing of maximum flow (day of the year), the Mann–Kendall test revealed significant downward trends for two stations (with the longest records). This implies that the day of the maximum flow is occurring earlier in the year in northern Sweden.

KEYWORDS
extreme values, GEV distribution, river discharge, stationarity, trend detection

1 | INTRODUCTION

In environmental science, statistical analysis of extremes is important for several reasons. Within hydrology, there are implications for engineering design and regulations, and for insurance and finance. Upper quantiles of quantities of interest are usually estimated as conventional risk measures and hence suitably deduced probability distributions are necessary.

Research on aspects of climate change are intense and has a long history. A statement of Liljequist (1949) is still highly relevant today:

The recent climatic fluctuation, which seems to embrace the whole earth, has been the object of many discussions and investigations.

In statistical modelling of extreme values, climate change poses challenges (see Katz (2010) for overview and discussion). Much analysis in the literature has focused on mean quantities, but considerably less on how extremes are changing over time. Trends in extremes are of a different nature than those driven by the centre of the distribution.

This study examined hydrological extremes in the form of annual daily maximum river flows in northern Sweden. More precisely, it investigated the possible presence of trends for two parameters: change in flow magnitude and change in peak flow timing (during the year) based on data from 11 Swedish sites on unregulated rivers with a series of observations spanning almost a century in some cases. In statistical modelling, time-dependent parameters in the limiting extreme-value distributions were used. See the theoretical background in Coles (2001) and Rydén (2011) for an example of application to extreme temperatures in Sweden. Testing was performed for the possible statistical significance of such parameters. Moreover, the conventional Mann–Kendall test and a recently developed algorithm for finding breakpoints in time series (Killick & Eckley, 2014) were employed.

The impact of climate change on the magnitude and frequency of river floods in Europe was studied recently by Mangini et al. (2018), who investigated 629 gauging stations across Europe from the period 1965–2005. Two statistical approaches, the classical method of (annual) block maxima and the peaks-over-threshold method, were evaluated in that study. The former approach resulted in no apparent change in flood magnitude.

In a comprehensive study by Blöschl et al. (2019), covering more than 3,000 gauging stations for the period 1960–2010, trends in
magnitude were analysed. For each series, annual maxima were extracted and trends were estimated by the Theil–Sen slope estimator, followed by a Mann–Kendall test. The results showed that trends were less pronounced in northern Scandinavia.

More detailed regional studies are also of interest. For Swedish rivers, the annual daily maximum flow was investigated by Arheimer and Lindström (2015) using a dataset of observed time series from 69 gauging sites throughout Sweden in the past century. No significant trend was found for the past 100 years. Those authors also used a hydrological model, based on precipitation and temperature data, to predict future scenarios. Annual runoff volumes and annual and seasonal flood peaks in Sweden were analysed by Lindström and Bergström (2004) in a study that included a total of 61 discharge series, with emphasis on the period 1901–2002. Those authors identified wet decades in the 20th century and assessed the reliability of older data.

The aim of the present study was two-fold: (a) to continue the analysis by Lindström and Bergström (2004) in order to cover the almost two decades that have passed since its publication, using data for unregulated rivers in the extreme north of Sweden; and (b) to apply contemporary statistical methodology to statistics of extremes (time-dependent parameters in extreme-value distributions), an analysis which has not been performed previously. In data selection, the longest time series possible was chosen (cf. Mangini et al. (2018), Blöschl et al. (2019)).

The remainder of this paper is organised as follows. Section 2 describes the data sources in more detail and briefly reviews the statistical methodology used. Section 3 presents the results from the analysis of possible trends in the magnitude of extreme flows and their seasonal occurrence in two unregulated Swedish rivers. Section 4 discusses the results and presents some conclusions.

2 | DATA AND METHODOLOGY

2.1 | Annual maxima from unregulated rivers

A substantial portion of electricity production in Sweden stems from rivers in the north, so these have been regulated to a great extent. To study natural conditions as closely as possible, unregulated rivers were chosen in this work. Moreover, time series longer than 50–60 years were used, as is recommended to take into account natural variability (Chen & Grasby, 2009, Yue, Kundzewicz, & Wang, 2012). Hence the study object was long series of data from gauging stations on two unregulated rivers. Such data are provided online by the Swedish Meteorological and Hydrological Institute (SMHI: http://vattenwebb.smhi.se/station/).

Table 1 shows the data studied in this work. In all, data from 11 gauging stations on the unregulated rivers Torne and Kalix (river ID 1000 and 4000, respectively) were considered. These gauging stations were selected based on the length of records and data quality (no large gaps in the series). Gaps of one or a few days were accepted and handled by simple linear interpolation. Time series illustrating the annual maxima for the Torne River and the Kalix River are shown in Figures A1 and A2, respectively, in an Appendix A to this paper.

Table 1 shows an excerpt of a time series covering 6 years at Station 2012 on the Torne River. The plot reveals the typical major peak in spring, following snowmelt, with an occasional tendency for a few minor peaks in autumn. Marked variability can be seen after the major peak, with some years showing a smooth decrease in magnitude after the early flood and other years having considerable rainfall-based flood events in summer (e.g., 1974).

Table 1 shows a list of the 11 stations included in this study.

2.2 | Extreme-value distributions

The extreme-value analysis relates to the behaviour of tails of distributions. The conventional method, with roots in the landmark book by Gumbel (1958), is to fit a generalised extreme-value (GEV) distribution to a sample of independent annual maxima. The distribution function for the GEV distribution is given by the equation:

<table>
<thead>
<tr>
<th>Station</th>
<th>Name</th>
<th>River ID</th>
<th>River</th>
<th>Area (km²)</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Junosuando</td>
<td>1000</td>
<td>Torne</td>
<td>4,348.0</td>
<td>1968</td>
<td>2019</td>
</tr>
<tr>
<td>957</td>
<td>Övre Abiskojokk</td>
<td>1000</td>
<td>Torne</td>
<td>566.3</td>
<td>1986</td>
<td>2019</td>
</tr>
<tr>
<td>2012</td>
<td>Pajala pushmus</td>
<td>1000</td>
<td>Torne</td>
<td>11,038.1</td>
<td>1970</td>
<td>2019</td>
</tr>
<tr>
<td>2,357</td>
<td>Abisko</td>
<td>1000</td>
<td>Torneträsk</td>
<td>3,345.5</td>
<td>1985</td>
<td>2019</td>
</tr>
<tr>
<td>2,395</td>
<td>Kallio 2</td>
<td>1000</td>
<td>Muonio älv</td>
<td>14,477.1</td>
<td>1988</td>
<td>2019</td>
</tr>
<tr>
<td>16,722</td>
<td>Kukkolankoski övre</td>
<td>1000</td>
<td>Torne</td>
<td>33,929.6</td>
<td>1911</td>
<td>2019</td>
</tr>
<tr>
<td>11</td>
<td>Männikkö</td>
<td>4000</td>
<td>Tärendö</td>
<td>5,856.2</td>
<td>1976</td>
<td>2019</td>
</tr>
<tr>
<td>17</td>
<td>Räktfors</td>
<td>4000</td>
<td>Kalix</td>
<td>23,102.9</td>
<td>1937</td>
<td>2019</td>
</tr>
<tr>
<td>1,456</td>
<td>Kaalasjärvi</td>
<td>4000</td>
<td>Kalix</td>
<td>1,472.5</td>
<td>1975</td>
<td>2019</td>
</tr>
<tr>
<td>2,159</td>
<td>Källingi</td>
<td>4000</td>
<td>Kalix</td>
<td>2,345.5</td>
<td>1976</td>
<td>2019</td>
</tr>
<tr>
<td>2,358</td>
<td>Tärendö 2</td>
<td>4000</td>
<td>Kalix</td>
<td>13,000.0</td>
<td>1985</td>
<td>2019</td>
</tr>
</tbody>
</table>
The time-dependent location parameter

Let us now introduce a model where the location parameter varies linearly with time, that is:

\[ \mu(t) = \beta_0 + \beta_1 t, \quad (2) \]

where \( \beta_0 \) and \( \beta_1 \) are regression coefficients. Models with time-dependent parameters are discussed in Coles (2001). Note that other parameters could also be considered time-dependent and that implementations for estimation exist for practical purposes, but in practice, only the location parameter is investigated.

When performing hypothesis testing for the parameters in Equation (2), so-called Wald tests are employed. These follow the same principles as in conventional inference for regression models. For instance, for \( \beta_1 \), with the null hypothesis \( \beta_1 = 0 \), the test quantity \( \hat{\beta}_1 / \text{s.e.}(\hat{\beta}_1) \) is computed, where s.e. (\( \hat{\beta}_1 \)) is the standard error of the estimated parameter. Computation of p-values follows from asymptotic normality.

2.3 Mann–Kendall test

A non-parametric standard test, often used in the analysis of environmental data, is the Mann–Kendall test; see for example, Chandler and Scott (2011) for a review. For a time series \( \{X_t\} \), \( t = 1, 2, \ldots, N \) the test statistic is given by the following:

\[ S = \sum_{k=1}^{N-1} \sum_{j=k+1}^{N} \text{sgn}(X_j - X_k), \quad (3) \]

where

\[ \text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases} \quad (4) \]

With no ties, and the values \( X_1, \ldots, X_N \) randomly ordered, we have \( E[S] = 0 \) and \( V[S] = N(N - 1)(2N + 5)/18 \). Moreover, \( S \) is approximately normally distributed for large \( N \). The sign of the test statistic indicates the tendency of trend; a negative sign is related to a negative slope and vice versa. Kendall’s tau is often reported, wherein the case of no tie, \( \tau_K = S / [N(N - 1)/2] \). Note that if there is a weak trend, the Mann–Kendall test will probably fail to detect this, since its power is low in that situation (Sheng, Pilon, Phinney, & Cavadias, 2002). Implementation in R, testing for a monotonic trend, provided in the package trend (Pohlert, 2020) was used in this article.
2.4 | Detection of possible changepoints

For situations where the Mann–Kendall test results show a significant trend, it is of interest to follow up with an analysis of possible changepoints. The approach of Pettitt (1979) is commonly applied to detect a single changepoint in hydrological series or climate series with continuous data. It tests the null hypothesis that the variables follow distributions that have the same location parameter (no change), against the alternative that a changepoint exists.

The test statistic \( K_N \), say, is given by \( K_N = \max(U_{tN}) \) where:

\[
U_{tN} = \sum_{t=1}^{N} \sum_{j=t+1}^{N} \text{sgn}(X_j - X_t).
\]

(5)

Note that pairwise differences build up the test statistic, analogous to the Mann–Kendall test. Pettitt’s test was used in this study, as implemented in the R package trend (Pohlert, 2020). As an alternative, procedures provided in the R package changepoint (Killick, Haynes, & Eckley, 2016, described in Killick & Eckley, 2014) were employed. For instance, the methodology is provided to find changepoints for the mean and variance of time series. The default option for finding (at most) one changepoint in a data sequence was used, based on a likelihood-ratio test; see Killick et al. (2016) for details.

3 | RESULTS

The results of fitting various statistical models to the time series of data selected for this study are presented below.

3.1 | Trends in extreme flows

The null hypothesis \( \beta_1 = 0 \) for a GEV distribution with a time-dependent location parameter (cf. Equation [2]) was investigated here. Estimation was performed by maximum likelihood (R package extRemes) and hypothesis testing by Wald tests (cf. Section 2.2.1). There was no significant result \( p < .05 \) for any of the 11 stations considered (Table 2). Note that for the different stations there were positive and negative estimates of \( \hat{\beta}_1 \) (although not significant).

Estimates of the shape parameter obtained with a stationary model are presented in Table A1. In all cases, the estimate was negative, sometimes referred to as a type III extreme-value distribution (reverse Weibull). However, it should be noted that for the majority of cases, the Gumbel distribution could not be rejected (using a Wald test).

The standard methodology assumes independent observations, cf. Section 2.2. The issue of independence was assessed by investigating the sample auto-correlation function \( r(t) \), computed by the R implementation acf. This also returns 95% confidence limits. The auto-correlation function was computed up to a time lag of 15 (years). Figure 2 shows the sample auto-correlation function for Station 16,722 (Torne River), which had the longest time series. No excursions outside the 95% confidence limits were found, and hence there was no concern about dependence on this station. Among all stations considered, Station 2,395 (Torne River) had an excursion outside the limit at lag 4 (value: \(-0.43\)). Thus overall, the independence assumption was assumed to hold.

Results obtained with the Mann–Kendall test are presented in Table B1. There was evidence of a (positive) trend at Station 17, and
more or less at Station 11. These two stations were investigated for changepoints, and their time series are presented in Figure 3. The algorithm in the R package changepoint identified changepoints in the mean in 1992 for Station 11 and in the mean in 1947 for Station 17. However, Pettitt’s test yielded no statistically significant changepoints ($p = .087$ and $p = .085$ for Stations 11 and 17, respectively (changepoint years 1992 and 1992, respectively). Note that Station 17 had the longest time series on the Kalix River (1937–2019).

3.2 | Trends in timing of annual maximum flood

Extreme flow magnitudes were not studied here, so GEV analysis was obviously not performed. Results for the Mann–Kendall test are given in Table 4. For Station 16,722 (Torne River), the Mann–Kendall test results indicated a significant downward trend, that is, with the day of maximum flow occurring earlier in the year. Note that this station had the longest series of observations on the Torne River (1911–2019). Follow-up testing of changepoint in mean using the R package changepoint algorithm returned the year 1952 (Figure 4). No changepoint in variance was found. Pettitt’s test identified a significant change-point in the year 1979 ($p = .013$).

Among stations on the Kalix River, Station 17 (with the longest series of records on that river) showed a significant trend (again downward). A test for changepoint in mean using the R package changepoint identified the year 1974 (Figure 4). Pettitt’s test identified a statistically significant change-point in the year 1980 ($p = .046$).

Note that for Station 16,722 (Torne River), there was a clearly visible peak in 1951, with the related outcome 192 (which means July 11). One could suspect that this sudden observation influenced the algorithm, so the effect of replacing this observation with the mean of all observations 1911–1951 and re-running the procedure was tested.

| TABLE 3 | Magnitude of annual maximum daily flow. Results after applying the Mann–Kendall test |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| **Torne**            |                      |                      |                      |                      |                      |                      |
| Station              | 4                    | 957                  | 2012                 | 2,357                | 2,395                | 16,722               |
| $\tau_K$             | 0.10                 | 0.048                | 0.038                | 0.013                | 0.17                 | 0.11                 |
| p-value              | .30                  | .70                  | .70                  | .92                  | .18                  | .094                 |
| **Kalix**            |                      |                      |                      |                      |                      |                      |
| Station              | 11                   | 17                   | 1,456                | 2,159                | 2,358                |                      |
| $\tau_K$             | 0.20                 | 0.18                 | 0.084                | 0.021                | 0.11                 |                      |
| p-value              | .055                 | .016                 | .42                  | .85                  | .37                  |                      |

| FIGURE 3 | Time series of daily annual maximum flow at two stations with possible changepoints. Left: Station 11 (Kalix River). Right: Station 17 (Kalix River) [Color figure can be viewed at wileyonlinelibrary.com] |

| TABLE 4 | Timing (during the year) of annual maximum daily flow. Results after applying the Mann–Kendall test |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| **Torne**            |                      |                      |                      |                      |                      |                      |
| Station              | 4                    | 957                  | 2012                 | 2,357                | 2,395                | 16,722               |
| $\tau_K$             | -0.027               | -0.19                | -0.10                | 0.041                | 0.0041               | -0.22                |
| p-value              | .78                  | .12                  | .30                  | .74                  | .99                  | .00074               |
| **Kalix**            |                      |                      |                      |                      |                      |                      |
| Station              | 11                   | 17                   | 1,456                | 2,159                | 2,358                |                      |
| $\tau_K$             | -0.05                | -0.17                | 0.062                | 0.022                | 0.088                |                      |
| p-value              | .59                  | .027                 | .56                  | .84                  | .47                  |                      |
However, the outcome was the same for that artificial dataset, that is, the algorithm returned a breakpoint.

4 | SUMMARY AND DISCUSSION

The statistical methodology was applied for the analysis of trends in observations of extremes. For comparison, the conventional Mann–Kendall test was applied for analysis of the same trends. The results for trends were similar in magnitude to those reported by Arheimer and Lindström (2015) (concerning Sweden) and Blöschl et al. (2019) (concerning Europe and concluding that trends are less pronounced in northern Scandinavia). For the methodology based on time-dependent GEV distributions, no trends were found for the 11 stations considered in the present study. The Mann–Kendall test showed signs of an increasing trend for a few stations. In the analysis of possible trends in flow timing, that is, the day of the annual maximum, the Mann–Kendall test identified significant trends for two stations (with the longest records). These trends were downward, that is, with the day of maximum flow occurring earlier in the year. This is an interesting finding and in line with the statement by Arheimer and Lindström (2015) that “the temporal pattern in future daily high flow in some catchments will shift in time, with spring floods in the northern-central part of Sweden occurring about 1 month earlier than today.”

However, from a hydrological point of view, the future situation is more complex. A warmer climate will lead to less snowmelt at high latitudes and hence lower snowmelt peaks in flow. On the other hand, more precipitation could be expected to occur in intense rainfall events, hence causing an increase in flows. This topic warrants more research.

In this study, a decision was taken to analyse as long a series as possible for each station, but the total number of stations was quite small (11). This approach was adopted in order to take advantage of some of the longer series available. It should be borne in mind that the use of the GEV distribution is based on a limit argument, so long series are desirable. On the other hand, when investigating a large number of stations, the best strategy might be to analyse a common time period. For example, in the study by Blöschl et al. (2019), 37,387 stations in Europe were analysed for the period 1960–2010. Other aspects are raised in Hall et al. (2014), where for instance so-called historical floods are discussed.

From the viewpoint of statistical methodology, other approaches could be employed for studying the phenomena examined in this study. For instance, one could consider the occurrences of maxima as events in a point process and investigate possible changes in its intensity. Moreover, it might be interesting to examine more closely the seasonal features illustrated in Figure 1, in particular, to check for tendencies over time in the appearance of the time series of daily flow in terms of for example, “wigginess” on a year-to-year basis. A possible option could be to apply the technique of rainfall counts and rainfall filtering employed for example, in analysing fatigue of materials (Lindgren & Rydén, 2002; Rychlik, 1987). The peaks-over-threshold (POT) methodology, used for example, by Mangini et al. (2018), might be another option to investigate. However, as the results in this study were in line with findings in the literature, no further modelling was undertaken.

Another possible extension of the present work could be to employ the methodology presented by Burauskaite-Harju, Grimvall, and von Brömssen (2017) as a test for trends in a network of gauging stations. One could consider the stations in a certain part of Sweden as a particular network and dependencies between the stations could be taken into account.

Finally, a related view on modelling extremes in nature is by taking into account long-range dependence in the series. For instance, temporal multifractal properties of long daily river discharge and precipitation records are presented in Rybski, Bunde, Havlin, and Kantelhardt (2011), along with a review of the methodology. This could be the topic of a future study on flows in unregulated Swedish rivers.

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DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available at a site governed by SMHI (the Swedish Swedish Meteorological and Hydrological Institute): http://vattenwebb.smhi.se/station/.

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REFERENCES

A.1 Visualisations of time series

**Figure A1** Time series for stations with river ID 1000 (Torne) [Color figure can be viewed at wileyonlinelibrary.com]
FIGURE A2  Time series for stations with river ID 4000 (Kalix) [Color figure can be viewed at wileyonlinelibrary.com]
### A.2 | Estimates of shape parameter in a GEV distribution

**Table A1** Maximum-likelihood estimates of the shape parameter in the GEV distribution (stationary model)

<table>
<thead>
<tr>
<th>Station</th>
<th>Estimate</th>
<th>Rejection of Gumbel</th>
</tr>
</thead>
<tbody>
<tr>
<td>957</td>
<td>−0.10</td>
<td>No</td>
</tr>
<tr>
<td>2.395</td>
<td>−0.23</td>
<td>No</td>
</tr>
<tr>
<td>2012</td>
<td>−0.30</td>
<td>Yes</td>
</tr>
<tr>
<td>16,722</td>
<td>−0.16</td>
<td>No</td>
</tr>
<tr>
<td>2,357</td>
<td>−0.16</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>−0.029</td>
<td>No</td>
</tr>
<tr>
<td>17</td>
<td>−0.27</td>
<td>No</td>
</tr>
<tr>
<td>1,456</td>
<td>−0.13</td>
<td>No</td>
</tr>
<tr>
<td>2,358</td>
<td>−0.12</td>
<td>No</td>
</tr>
<tr>
<td>2,159</td>
<td>−0.11</td>
<td>No</td>
</tr>
<tr>
<td>11</td>
<td>−0.16</td>
<td>No</td>
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</tbody>
</table>