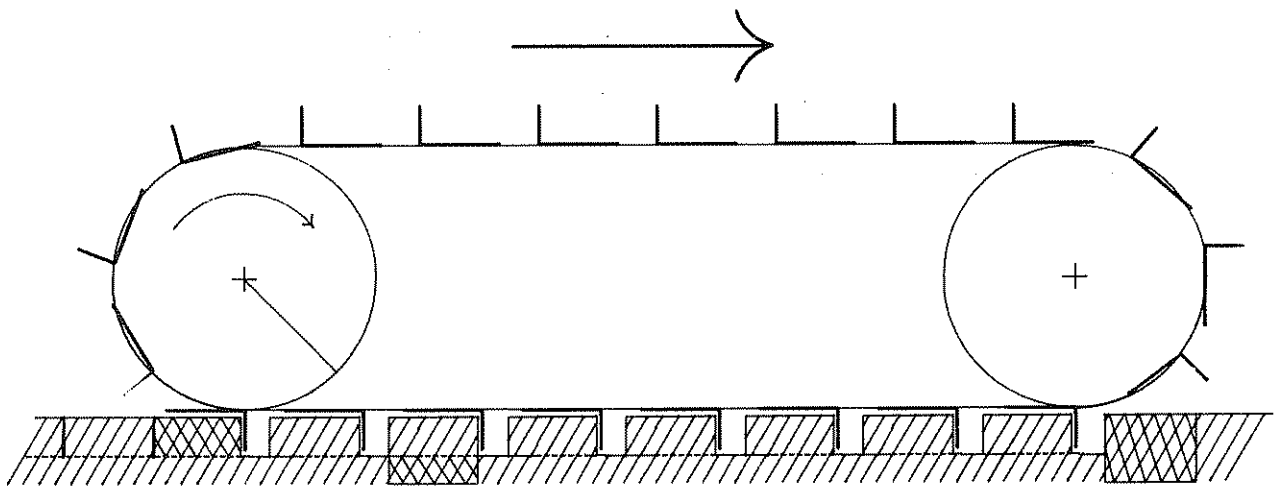


**SVERIGES  
LANTBRUKSUNIVERSITET**

## **Effect of Shear Stress on Soil Compaction**

**H.J. Olsen**



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**Institutionen för  
lantbruksteknik**

**Swedish University of Agricultural Sciences  
Department of Agricultural  
Engineering**

**Rapport 124  
Report**

**Uppsala 1988**

ISSN 0283-0086

ISBN 91-576-3332-0

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DOKUMENTDATABLAD för rapportering till SLU:s lantbruksdatabas LANTDOK, Svensk lantbruksbibliografi och AGRIS (FAO:s lantbruksdatabas)

Institution/motsvarande		Dokumenttyp	
Institutionen för lantbruksteknik		Rapport	
		Utgivningsår	Ärendebeteckning
		1988	
Författare/upphov			
Hans Jørgen Olsen			
Dokumentets titel			
Effect of Shear Stress on Soil Compaction			
Referat			
<p>The report presents the results from theoretical considerations and practical tests concerning the influence of slip generated shear on the compaction of agricultural soil. The slip mechanics is reviewed and its influence on the stress field below a wheel or track is investigated. It is shown that shear stresses in the interface between wheel and soil have great influence on the magnitude of the major principal stress acting in the upper soil layer. Field tests with a simple shear plate indicated that the shear strain affected only the upper 2 cm of the soil below the shear plate.</p>			Målgrupp
Ämnesord (AGROVOC)			
Soil compaction, soil shear			
Andra ämnesord			
Övriga bibliografiska uppgifter			
Serie-/tidskriftstitel och volym/nr		ISBN	
Sveriges Lantbruksuniversitet		91-576-3332-0	
Institutionen för lantbruksteknik		ISSN	
		0283-0086	
Språk	Smf-språk	Omfång	Antal ref.
Engelska	Engelska	36	18
Projektnamn			
Slirningens betydelse för jordpackningen			

Postadress

SVERIGES LANTBRUKSUNIVERSITETS BIBLIOTEK  
 Ultunabiblioteket  
 Förvävssektionen/LANTDOK  
 Box 7071  
 S-750 07 UPPSALA  
 Sweden

Besöksadress

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Telex

76062 ULTB18LS

## ABSTRACT

This report presents the results from theoretical considerations and practical tests concerning the influence of slip generated shear on the compaction of agricultural soil. The slip mechanics is reviewed and its influence on the stress field below a wheel or track is investigated. It is shown that shear stresses in the interface between wheel and soil have great influence on the magnitude of the major principal stress acting in the upper soil layer. Tests with shear displacement introduced by means of a torsional shearing device indicated increases in soil density due to shear at moderate normal stress. By means of field tests with a simple shear plate it was shown that the shear strain only affected the upper 2 cm of the soil below the shear plate.

## PREFACE

The present paper constitutes the final report on the project:  
Significance of slip on soil compaction.

The project has mainly been financed by funds made available by The Swedish Council for Forestry and Agricultural Research. Contributions have also been received by Dorthi and Nils Troedsson's Donations fond.

The work was carried out at The Swedish University of Agricultural Research, Department of Agricultural Engineering, Uppsala.

The author is greatly indebted to agr. dr. I Håkansson at The Department of Soil Sciences for interesting and encouraging discussions. Thanks are also due to Prof. N. Möller and agr. lic. H. Jönsson for discussions on the practical and theoretical aspects of the work.

Finally the author wish to express his grattitude to Ö. Bergwall and A. Wesslen for editing and printing the manuscript.

Ultuna, Nov. 1987.

H. J. Olsen

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## SUMMARY AND CONCLUSIONS

A theoretical study of the stresses introduced in the soil below a slipping wheel or track has been done. It has been shown that the shear stresses in the surface layer greatly influences the major magnitude of the major principle stress. Formation of a compacted soil layer that, at least to some degree follows the wheel surface, has been suggested. This will mean that the shear failure zone will be displaced some depth into the soil at high ground pressures. Density measurements in the soil below a shear annulus showed moderately increased densities. Tests with a simple shear plate indicated that shear displacements occurred down into the soil to a depth of only 2 cm.

Following conclusions may be drawn from this study.

- 1 The major principle stress is affected by the introduction of shear stress. It may be more than double the magnitude of the vertical normal stress.
  
- 2 In a wet sandy soil the shear stress introduced shear strain in the soil down to a depth of 2 cm below the bottom of the rut after a shear plate.

## INTRODUCTION

When soil is subjected to external forces it may fail. According to Bailey and Vandenberg (1968) the soil failure may be divided in four different types: compression, shear, tension and plastic flow. When concerning soil compaction the latter two failure types can be disregarded. Soil compaction due to compression have been subject to numerous studies. Soil compaction as an effect of shear failure have been more scarcely investigated.

When scanning the literature not very much is written about the connection between shear stress and soil compaction. However, there seems to be considerable disagreement as to which extent the compaction is affected by the shear stress. Davies et al. (1973) found that under certain conditions the wheel slip was more important in causing compaction than additional wheel loading. Contrasting to this Trowse (1982) stated without proof that the compaction effect of travel reduction is nearly negligible.

However, Ronai and Trowse (1982) recorded a somewhat increased compaction when operating at travel reductions between 15 and 30 percent.

Raghavan and McKyes (1977) found increases in dry bulk density between 20 and 40 kg/m<sup>3</sup> when sandy loam soil was sheared in a special shear box in the laboratory. They note, however, that considerable higher density increases was observed in the field. Later they (Raghavan et al., 1978) conducted similar studies on clay soil. Also under these conditions they found increases in soil density albeit not so pronounced as for the sandy soil. They note that the highest increases in density was observed for slip rates between 10 and 50 percent. Unfortunately the contact area length was not noted so that nothing exactly is known about the actual soil displacements.

## THEORETICAL CONSIDERATIONS

Consider a cubic element in the soil below a surface load. The stress- and strain-state of such an element may be described formally by the expression

$$\text{state of strain} = f(\text{state of stress}) \quad (1)$$

The function  $f$  is quite complicated and should be found experimentally. The independent variables for  $f$  on the right hand side consists of the principal stresses  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  as well as their directions and change with time both of their value and directions.

The left hand side should describe both the volume change of the element as well as its possible change of geometrical form. Usually, the volume strain is considered of greatest importance. However, also a distortion of the element at constant volume may have an adverse effect on the soil's impedance to root growth. In Fig. 1 is shown a possible distortion of an element. Suppose that this element contains a channel with a diameter originally sized so that a root exactly is able to pass through it. If the element is strained as shown in the right of the figure its volume is unchanged but the square section of the channel is decreased so that the root can no longer pass through.



Fig. 1. Section of a soil element with a channel embedded. Left: original shape. Right: shape after shear strain at constant volume.



### Soil compaction

Most soil compaction happens due to loaded wheels travelling on the surface. The mere fact that the wheel moves - with or without slip - causes the principal stress directions to change. This phenomenon introduces a kneading effect on an element in the soil. Concerning compaction this kneading is usually assumed to have only a marginal effect. However, by repeated passes it may have some effect for the collapse of the soil skeleton. Especially under nearly saturated conditions where the effective stress between the soil particles is moderate.

It is obvious that some final stress state that results in a certain compaction state as described by Eq. 1 may be obtained in many ways. However, according to Koolen and Kuipers (1983) the compaction state of a soil sample is merely dependent on the values of major and minor principal stress and not of their paths to this final stress state. Thus, Eq. 1 can be written

$$BWV = f( \sigma_1, \sigma_3 ) \quad (2)$$

where       $BWV$  = bulk weight volume i.e. reciprocal dry bulk density  
 $\sigma_1$  = major principle stress  
 $\sigma_3$  = minor principle stress

This equation expresses the compaction state as function of two normal stresses. It may, however, be readily converted to express the compaction state as function of mean normal stress and maximum shear stress because

$$\sigma_m = ( \sigma_1 + 2 \sigma_3 ) / 3 \quad (3)$$

$$\text{and } \tau_{\max} = ( \sigma_1 - \sigma_3 ) / 2 \quad (4)$$

where       $\sigma_m$  = mean normal stress  
 $\tau_{\max}$  = maximum shear stress

Hence, Eq. 2 may be written as

$$BWV = f(\sigma_m, \tau_{max}) \quad (5)$$

Based on data from various workers Koolen and Kuipers (1983) have shown that the influence of  $\sigma_3$  on the compaction is usually very limited. Therefore they propose that soil compaction can be expressed as a function of the major principal stress only and be written as

$$BWV = f(\sigma_1) \quad (6)$$

#### Soil physical behaviour during compaction

When soil is compacted its particles comes in closer contact with each other. If the compacting stress is hydrostatic the soil may be thought to compress in such a way that the particles remain in the same relative positions with respect to each other. Only the pores will decrease in size.

This scheme is not quite true because of irregularities within the soil. The soil is not isotropic when seen on a micro scale. However, the scheme serves well as contrast to the situation where shear stresses are present. When a soil element is deformed as shown in Fig. 1 the soil particles are forced to move with respect to each other. This condition is often illustrated by considering the soil particles as spheres as shown in Fig. 2. If the spheres are not in a very close configuration the applied shear stress may bring them closer together as shown in the figure by changing from state 'a' to state 'b'. In the case where a confining stress is present simultaneously the closer configuration may be more readily obtained.

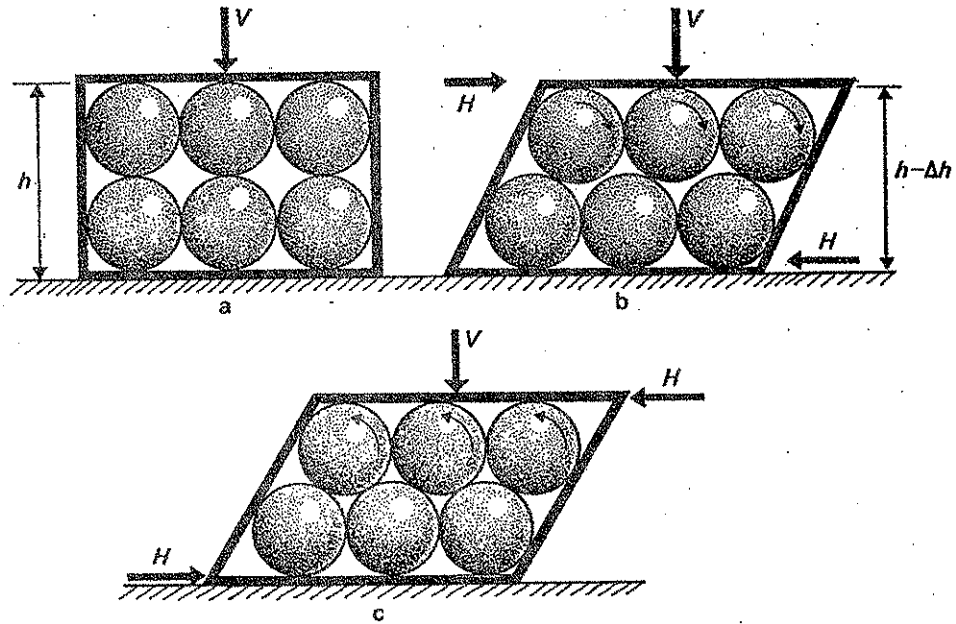


Fig. 2. Change in packing state of spheres by application of shear stress. After Sven Hansbo (1975).

In the case where the spheres of Fig. 2 are incompressible and initially in a close configuration a shear stress will cause an increase in the volume occupied by the spheres as shown by going from state 'c' back to state 'a'. This behaviour is often seen under shear tests on firm cohesive soil. Under such conditions the spheres represent aggregates of very high strength.

When soil is subjected to a shear stress of such a magnitude that failure occurs the actual behaviour of the soil particles or aggregates is a matter of considerable complexity. The actual rearrangement of soil particles is dependent on a number of factors of which the main ones are:

- degree of saturation
- hydraulic conductivity
- texture of the soil
- density of the soil
- aggregation of the soil
- degree of cementing between soil particles
- degree of interlocking between soil particles

Furthermore, the soil may be reinforced to a varying degree by means of plant roots and other debris that has been mixed into the soil.

In the realm of basic soil mechanics numerous tests have demonstrated the soil's basic behaviour when it is subjected to shear stress with accompanying strains. In Fig. 3 is shown the behaviour of a dense and a loose soil respectively after Lambe and Whitman (1969). The two soils are described by the initial void ratio. The upper part of Fig. 3 shows the typical peak in the shear stress which is seen for a soil that dilates when sheared. As seen in the centre diagram of Fig. 3 this soil increases its volume as the shearing proceeds eventually approaching a constant level. In the lower part of Fig. 3 this behaviour is shown as an increase in void ratio.

The loose soil with the higher void ratio in Fig. 3 behaves quite differently when sheared. Firstly, it does not show any pronounced peak stress and, secondly, its volume hardly changes with the shear strain. Thus, for the same type of soil the final shearing resistance and void ratio approaches identical values as the shear strain proceeds.

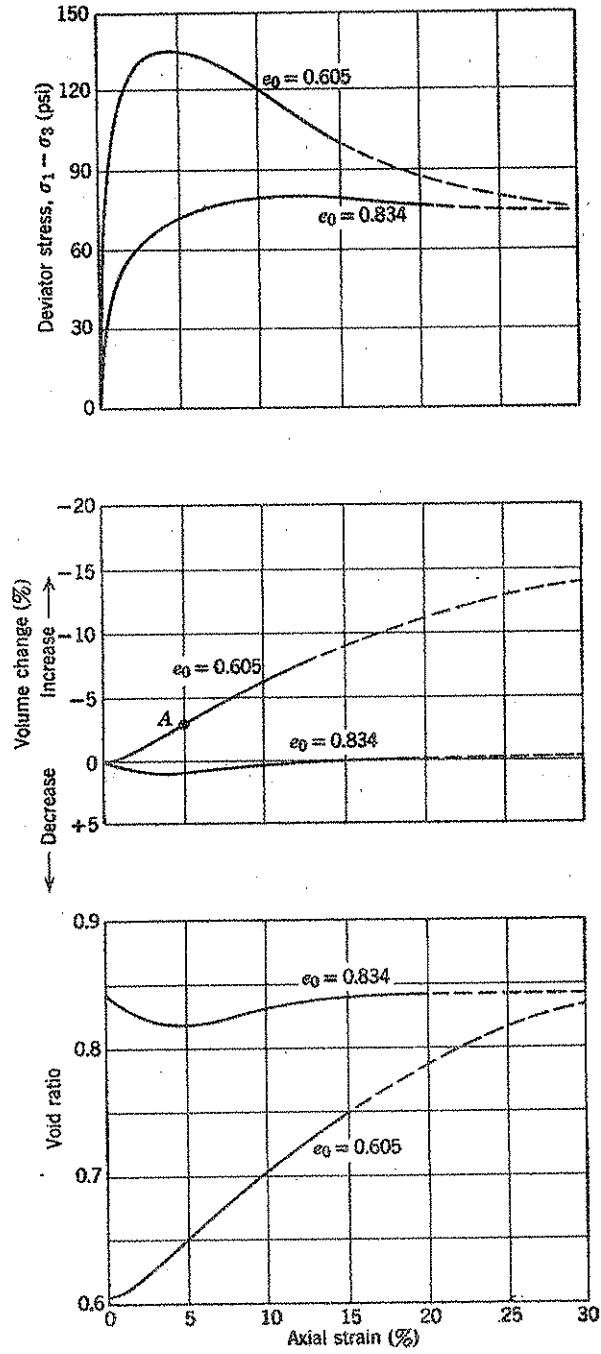


Fig. 3. Stress-strain curves for loose and dense soil specimens in a triaxial test. Medium-fine sand. Solid line: actual test data. Dashed lines: extrapolations based on other tests. (After Lambe and Whitman, 1969).

The results in Fig. 3 was obtained by means of a triaxial test apparatus, but the same basic behaviour is, of course, found independent of by which means the soil is sheared. Thus, Olsen (1986b) conducted field shear tests on a soil subjected to three different packing intensities and these results also indicated identical shear stress values at high shear strains.

Concerning the influence of the strain rate it has been demonstrated by a number of researchers, e.g. Stafford and Tanner (1983) and Dudzinski (1987), that the dynamic of the shearing process clearly influences the soil's shear strength. However, little is known about its influence on the soil's strain behaviour. This may be due to the considerable difficulties in accurately performing this kind of tests. Nevertheless, such information would be of great significance as the shear process occurring in practical agriculture operations usually is carried out at a much larger speed than the shear process in the laboratory experiments.

Even if a dense soil in a laboratory test undergoes a certain dilatation and thus enters a more loose state when sheared, it may be questioned whether this effect has any significance in practical agricultural soil tillage. In some experiments, e.g. Ljungars (1977), a slight decrease in soil density has been observed after passage of a tractor driven with a certain slip. It may, however, be argued that even if a dilatation has occurred its effect is limited to narrow zones in the soil. Once a dilatation process has started at a certain place the soil at this location immediately becomes weaker than the surrounding soil. Therefore, further dilatation will take place at the same location. Thus, after this kind of soil 'loosening' the density of the soil will vary considerably between the dilating locations and the places in between where the soil may be just as compact as before.

For saturated soil the behaviour under shear stress is often quite different from its behaviour when partly saturated. In the saturated condition the action of the soil is very dependent on the draining possibilities of the water.

If the water drains easily, as in coarse sand, it may be squeezed out quite rapidly and a certain compression of the soil may result. For a fine-grained soil, however, the water often drains so slowly that it does not have time to move any appreciable distance. Triaxial tests show that under such conditions a work-hardening soil will exhibit a dilative behaviour whereas a work-softening soil shows a contractive tendency. The changes in void ratio are, however, so limited that they hardly have any significance for agriculture. Thus, under practical conditions a saturated fine-grained soil can be considered incompressible.

However, this does not mean that the saturated soil is insensitive to shear stresses. It will usually be deformed plastically to a very high degree thereby losing its previous internal structure. Such a remoulding is hardly positive to the soil even if it occurs under constant volumetric conditions, cf. Fig. 1.

#### SOIL BEHAVIOUR BELOW A SLIPPING WHEEL OR TRACK

When a wheel that develops a drawbar pull runs over a loose soil surface a shear stress field is introduced in the soil material. This shear stress field will give rise to a strain field below the wheel. The question treated in this section is to which depth the strain field extends, how large strains that can be expected and how these strains affect the soil with respect to deteriorations of various kinds. In order to clarify things the behaviour of a slipping wheel is treated first whereafter the soil response is discussed.

#### Mechanics of a slipping wheel

Traction tyres for agricultural use are always equipped with a more or less aggressive lug pattern. The lugs are assumed to penetrate the soil surface in order to ensure that the soil's internal strength will be the limiting factor for the drawbar pull.

It should be mentioned that today's use of very large tyres with proportionally low ground pressures have, to some degree, altered this situation. On firm surfaces, e.g. a stubble field in the autumn, the ground pressure may be insufficient to force the lugs into the soil. In this situation the maximum drawbar pull that can be developed is determined by the adhesion and friction properties of the soil surface against the rubber material of the tyre.

In Fig. 4 is schematically shown what happens in the track-soil interface at various degrees of slip. The schematic is equally valid for a lugged wheel but it is easier visualized by means of a track. For clarity the soil is thought to be incompressible and locally very strong so that the soil blocks sheared off by the lugs will retain their original form. Likewise, the lugs are imagined to engage vertically into the soil and also be retracted straight upwards.

In the situation labelled 'a' the slip is zero and the lugs act in the soil like a cogwheel on a rack. Therefore, the soil mass originally in the crosshatched area A will be left intact in position B with respect to the vehicle when this have advanced the distance AB. In this case no shearing of the soil occur and the surface is left undisturbed apart from the impressions left by the lugs. It should be mentioned that this situation is highly unrealistic. If the soil is so strong that no slipping will occur the lugs will usually not be able to penetrate, especially for a rubber tyre because of the relatively wide lugs.

In part 'b' of Fig. 4 the situation is shown for a slip of 25 percent. Here the vehicle moves forward only three quarters of the distance that the circumferential movement of its running gears would suggest. Hence, when the front lug is about to engage the interlug space will be filled with soil only in its forward part. The soil mass about to be engaged is shown in the crosshatched area labelled C. It should be noted that the empty space just in front of each lug does not mean that any soil will disappear. At the rear end of the track the soil blocks are neatly pushed together and the surface re-established. As the process advance the upper part of the soil block at C will eventually be left at position E. When this happens, the vehicle have advanced so that the lower part of the soil block originally at C is now positioned at point



D. Thus, the upper part of the soil have slided the distance DE. As the slip usually is defined as a reduction in vehicle velocity this distance may be determined the following way. Referring to Fig. 4b the slip velocity is

$$v_j = r \cdot \omega \cdot i \quad (7)$$

where  $v_j$ : slip speed  
 $r$ : radius of the track sprocket  
 $\omega$ : angular velocity of track sprocket  
 $i$ : slip ratio

Integrating over time yields slip distance:

$$j = \int_0^t v_j dt \quad (8)$$

and as  $\omega = d\theta/dt$  where  $\theta$  is the angle of rotation

$$j = r \cdot i \int_0^{\theta_1} d\theta = x \cdot i \quad (9)$$

where  $x$  is the distance from the front of the track to the point of interest. Thus, if  $x$  is total track length the sliding distance becomes:

$$j = l \cdot i \quad (10)$$

where

$j$  is the sliding distance DE,  
 $l$  is the length of the contact area CE,

Thus, the distance that the soil entrapped between the lugs will be displaced with respect to the soil mass below depends only on the slip ratio and the length of the contact area in the direction of travel. For

a typical rear tyre of a tractor operating at 15 percent slip the soil displacement will amount to about 10 cm.

In the real situation the soil between the lugs will collapse thereby allowing the lugs to penetrate deeper into the soil. Therefore, the interlug space will be filled with soil but the mechanics of the filling is different from the situation in part 'a' of the figure.

Fig. 4c shows a situation similar to part 'b' as described above the only difference being that the slip in part 'c' is 50 percent. The soil originally at position F is here positioned at G and H for the lower and upper part respectively after passage of the vehicle. Thus, the sliding distance GH will equal half the length of the contact area for this situation.

Finally the situation shown in Fig. 4d illustrates a 100 percent slip situation. Theoretically, no soil will be found in the interlug space under this condition. The soil originally in the upper part at position I will be displaced to position J when the track has moved a distance equal to the contact length. While this happens the lower part at I will remain stationary.

It is obvious that the situations illustrated in Fig. 4 is only some of many possible conditions. In the real life a driving wheel may slip to any degree between 0 and 100 percent and often dig-in will occur which makes the situation far more complicated.

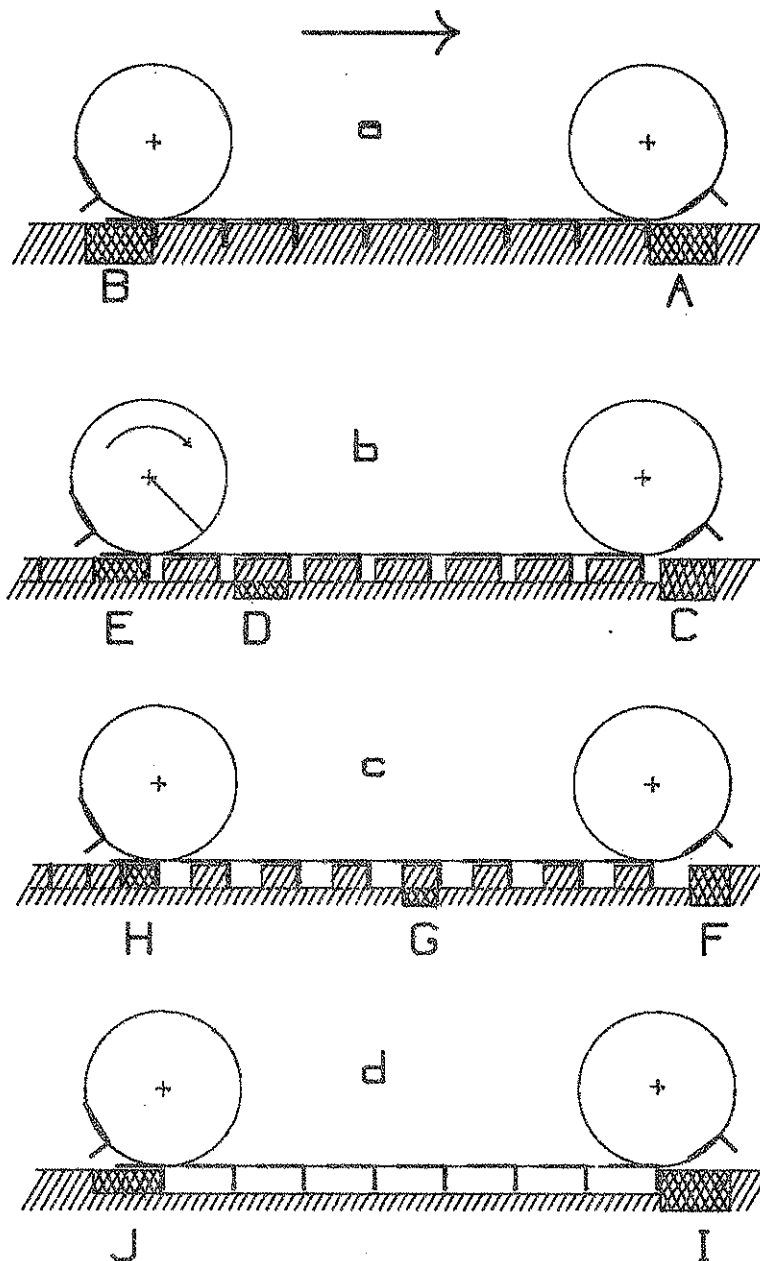


Fig. 4. Schematic drawing of the conditions below a track running on soft terrain with different degrees of slip. a: no slip, b: 25 percent, c: 50 percent and d: 100 percent slip. The arrow shows the direction of motion.

### Principal stresses below a slipping wheel

For a static load the isobars of the vertical normal stress field in the soil forms egg-shaped curves as shown by Söhne (1953). The highest normal stresses occur just below the load and decreases with depth. The same scheme applies for the major principal stress. Hence, the magnitude of the largest normal stress equals, in principle, the ground pressure of the load, i.e. wheel or track.

For a slipping wheel that introduces both shear and normal stress to the soil surface the shape of the isobars is far more complicated. No attempt to draw such isobars has been found in the literature. Nevertheless it is evident that the shear stress increases the magnitude of the major principal stress. This may be shown by means of Mohr's stress circle.

Figs. 5, 6 and 7 show three stress situations for a soil with known values of cohesion and angle of internal shearing resistance. In Fig. 5 the pure normal stress situation is shown. No shear stress is present and  $\sigma_3$  is, somewhat arbitrarily, shown as the half of  $\sigma_1$  which coincides with  $\sigma_z$  the applied normal stress on the soil surface. Hence, the stress state is described as the circle shown in the diagram.

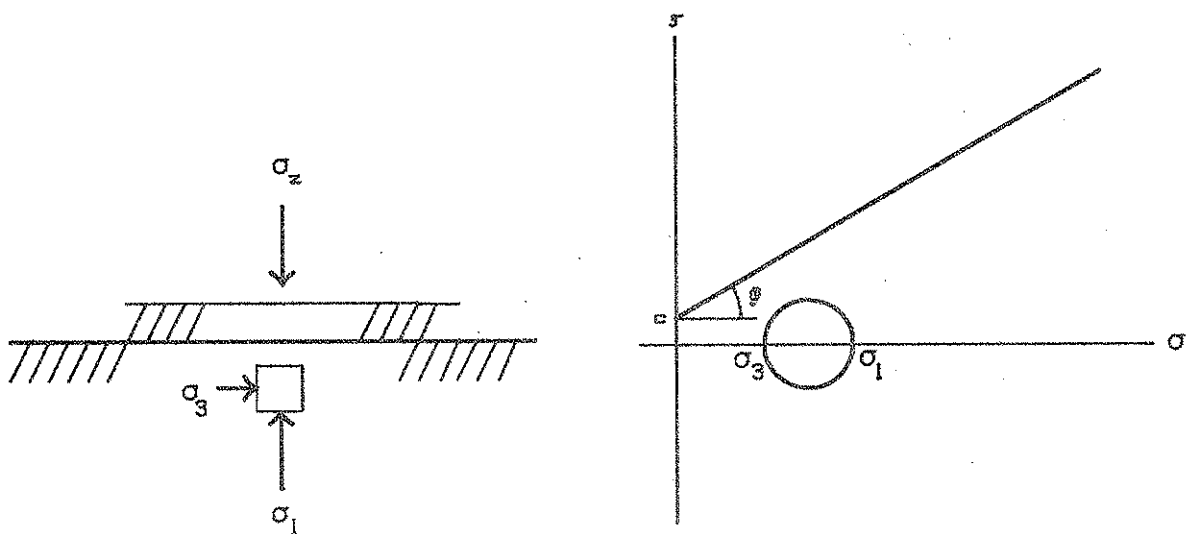


Fig. 5. Principle stresses below a vertical normal stress on the soil surface. No shear stress is present and  $\sigma_3$  is shown as half the magnitude of  $\sigma_1$  which, in turn, equals  $\sigma_z$ .

In the situation shown in Fig. 6 the normal stress acting on the surface is still  $\sigma_z$  but a shear stress of approximately half the soil's strength is applied. This means that the direction of the major principle stress is no longer perpendicular to the surface. In order to balance the combination of normal and shear stress  $\sigma$  and  $\tau$  it is somewhat inclined and increased in magnitude. As failure does not occur it is not possible to define a certain stress circle in the diagram.

It is possible, however, to set up limits for the major principle stress. Mohr's circle must pass through point T. Because failure does not occur the circle should not touch the line of Coulomb. Therefore, circle 1 constitutes a lower boundary for the stress state and  $\sigma_{1a}$  is a lower value for  $\sigma_1$ .

By observing the soils behaviour under this kind of load situation in a glass-sided box it is known that a soil element originally of shape  $rstu$  deforms into  $rs't'u'$ . According to Koolen and Kuipers (1983) such a deformation may be assumed as composed of two deformations. First the soil is compressed vertically with  $\theta=0$  and thereafter deformed by pure shear with  $\sigma_1$  in the  $su$ -direction and  $\sigma_3$  in the  $rt$ -direction so that  $\theta=45$  degrees. The stress circle for this deformation is shown as circle 2 which yields an upper bound for  $\sigma_1$  as  $\sigma_{1b}$ .

In order to find  $\sigma_1$  it is observed that since the Coulomb line is tangent to circle 1 it may be described by:

$$\tau = (\sigma - A)\tan(\phi) + R/\cos(\phi) \quad (11)$$

where A is the coordinate of the centre on the  $\sigma$ -axis and R denotes the radius.

Thus, from Eq. 11

$$c = R/\cos(\phi) - A \tan(\phi) \quad (12)$$

R is expressed by the circle equation as

$$R = ((\sigma_z - A)^2 + \tau_z^2)^{1/2} \quad (13)$$

By applying Eq. 13 to Eq. 12 A may be found from the quadratic

$$A^2 - \frac{2\sigma_z + 2c\sin(\phi)\cos(\phi)}{\cos^2(\phi)} A + \frac{\sigma_z^2 + \tau_z^2}{\cos^2(\phi)} - c = 0 \quad (14)$$

This equation yields two values for A of which only the smaller is relevant due to the physical constraints.

After A has been calculated the radius R is easily found from Eq. 13.

For a soil with the characteristics:

$$c = 15 \text{ kPa}, \quad \phi = 30 \text{ deg.}$$

and stressed with:

$$\sigma_z = 100 \text{ kPa}, \quad \tau_z = 35 \text{ kPa}$$

it may be calculated that

$$\sigma_{1a} = 116 \text{ kPa and } \sigma_{1b} = 135 \text{ kPa.}$$

Thus,  $116 < \sigma_1 < 135$  kPa and therefore the major principle stress may be up to 35 percent larger than the surface normal stress under these conditions.

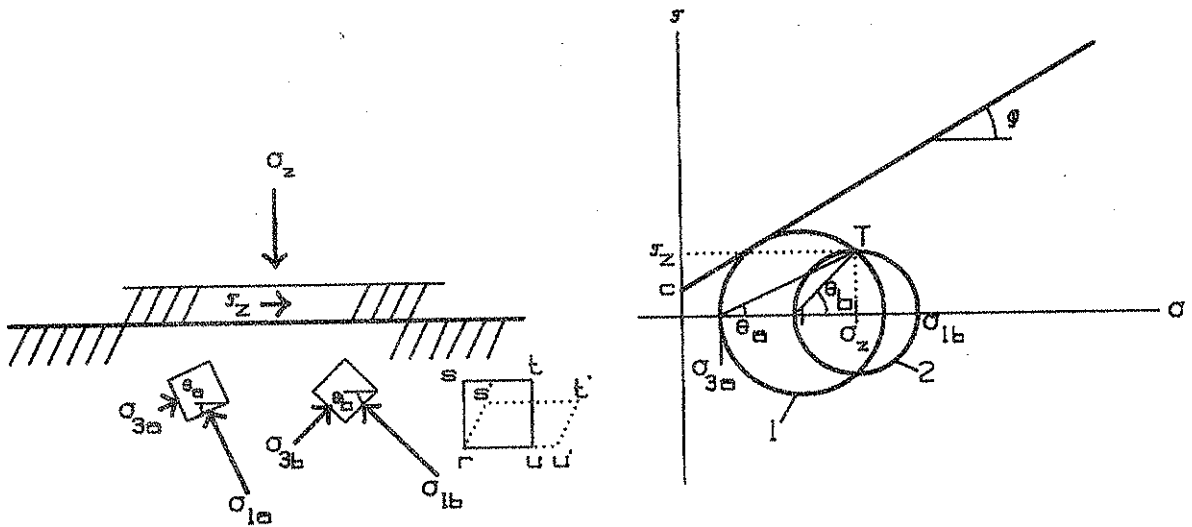


Fig. 6. Same conditions as in Fig. 5 with the addition of a shear stress of approximately half the soil's strength. Mohr circles are shown for two boundary conditions as explained in the text.

Fig. 7 shows the situation in the failure condition. Hence, from the Mohr-Coulomb failure theory it is known that the major principle stress plane slopes  $45 + \phi/2$  degrees to the failure plane. Assuming the same soil conditions and surface normal load as previously it may be found that  $\sigma_1 = 226$  kPa or more than double the magnitude of the surface normal stress. If  $\sigma_1$  is by itself responsible for soil compaction as proposed by Koolen and Kuipers (1983) it is understood that shear stress may affect the compaction by its influence on the major principal stress.

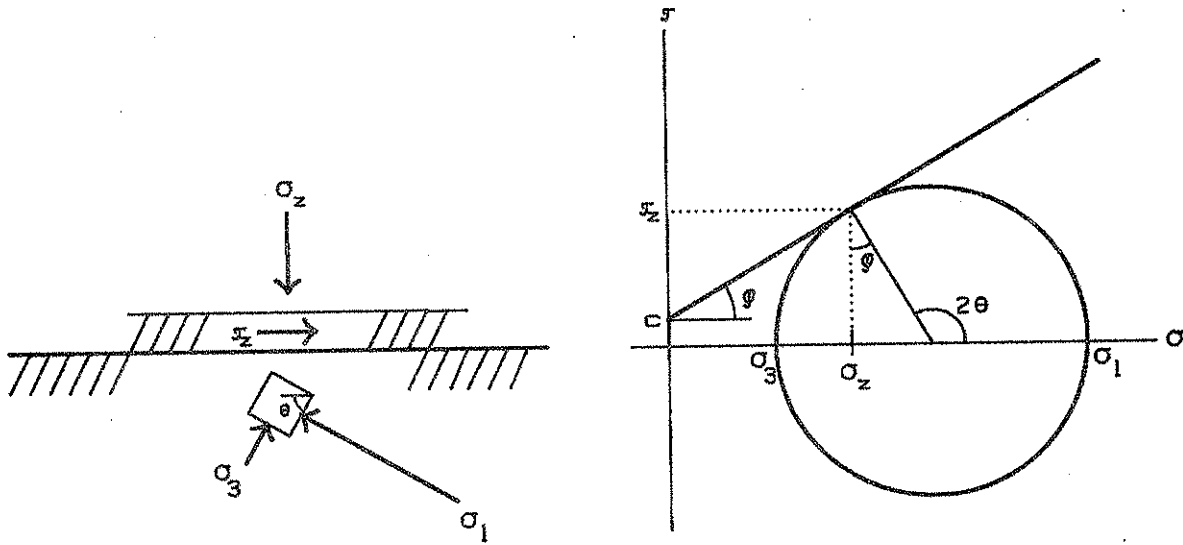


Fig. 7. Principle stresses at failure for the same soil and normal loading conditions as in Fig. 5 and 6.

The three stress conditions shown in Fig. 5, 6 and 7 are those existing below the very front edge, the middle and the rear part of the contact area of a slipping wheel. Consequently, it may then be concluded that reducing the slip will ameliorate the soil compaction situation. However, this is often not practically possible. Considering the slip mechanics as shown in Fig. 4 it will be understood that the slip rate should then often be reduced to unrealistic low values in order to avoid the situation of Fig. 7 at the rear of the contact area. But nevertheless, the lower the slip rate the shorter time is the soil subject to the large  $\sigma_1$ -value, a fact that may be significant in some cases.

#### Shear stresses below a slipping wheel

The stress field in the soil below a shear plate or a slipping wheel is a function of both the normal stress, arising from the normal load, and the shear stress introduced by the shearing or slipping. Therefore, the



one phenomenon cannot be analyzed completely separate from the other. For the analysis to be correct in a strict sense the surface load must be applied by an infinitely long strip. This is, of course, better approached by a track than a wheel but still the analysis supplies a qualitative description of the conditions even for wheels. However, in order to avoid any ambiguity the wheel or track is in the following denoted the 'loading body'.

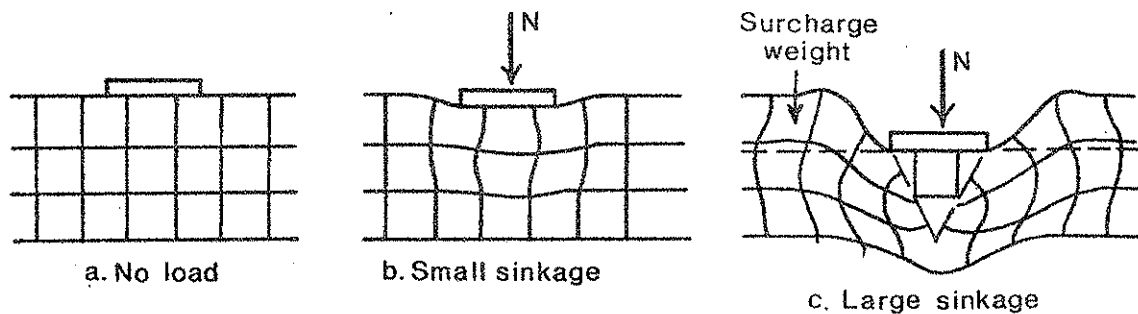


Fig. 8. Conceptual soil behaviour below a plate on the soil surface, under no load (a), under a limited load that does not cause the soil to fail (b) and under a load which fully mobilizes the soil's bearing capacity causing shear failure along the sides of the triangular wedge. After McKyes (1985).

When a distributed load is applied to the soil surface a normal stress field is set up in the soil mass as described by Söhne (1953). However, this analysis does not imply any kind of soil movement or change in density. On the other hand, it is very well known that a certain movement of the soil does occur. In Fig. 8 a concept by McKyes (1985) is shown. The soil is imagined, in some way, to be prepared with a grid in order to clarify the movements in the soil mass. In part 'a' of the figure no load is applied and the grid is undisturbed. Fig. 8b shows the situation when the soil must carry a moderate load. Some compaction of the soil has obviously occurred as the grid is somewhat disturbed but

the soil's bearing capacity is far from exceeded. In part 'c' of the figure the soil's bearing capacity has been fully mobilized. Here the grid is very deformed. A triangular wedge of compact soil has formed just below the loading plate. This wedge will follow the plate as it eventually sinks further and it behaves as if it was part of the plate.

Fig. 9 shows the stress pattern at fully developed bearing capacity in schematic form. With no surcharge on the soil beside the loading plate the triangular wedge will form with the sides sloping at angles  $\theta = 45 + \phi/2$  degrees to the horizontal as shown. As the plate sinks further into the soil shear stresses will develop along the wedge sides. Thus, during sinking the soil will be pushed sideways-upwards developing a shear failure zone on each side of the loading plate. The upward moving soil masses will eventually exert a surcharge on the surface as indicated in Fig. 8c.

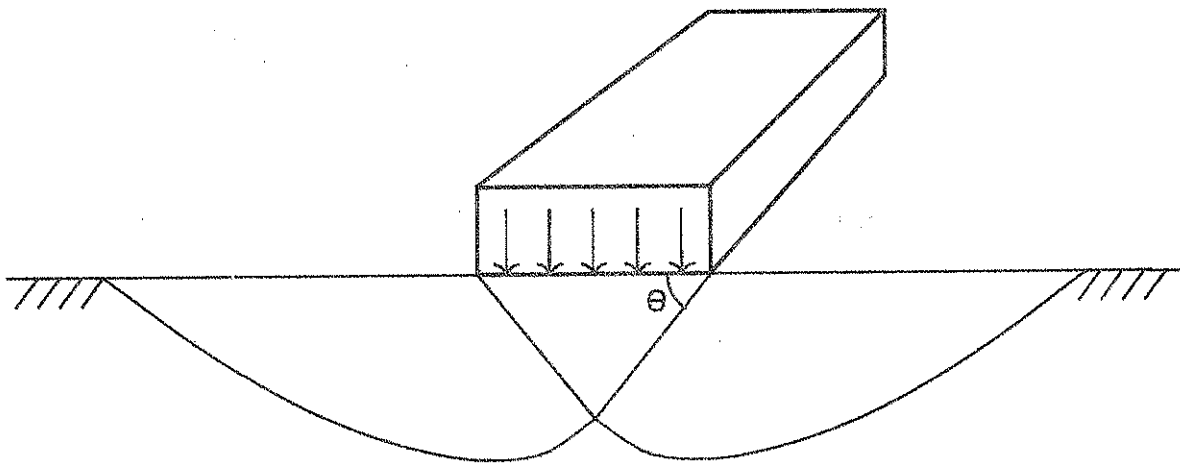


Fig. 9. Development of shear failure zones besides the central triangular wedge below a loaded strip on the soil surface. The wedge with sides sloping  $\theta = 45 + \phi/2$  degrees to the horizontal consists of compacted soil and follows the strip during eventual sinkage as if it was part of the strip itself.

In this situation the bearing capacity of the soil is mainly determined by the vertical components of the shear and normal stresses acting along and against the sides of the wedge. If the normal stress acting on each

side of the wedge is denoted  $\sigma_\theta$  the upward directed shear stress  $\tau_\theta$  at fully developed bearing capacity is:

$$\tau_\theta = c + \sigma_\theta \cdot \tan(\phi) \quad (15)$$

from the Coulomb equation. The situation is sketched in Fig. 10. The total bearing capacity of the soil is then twice the vertical components of and as only one side was considered.

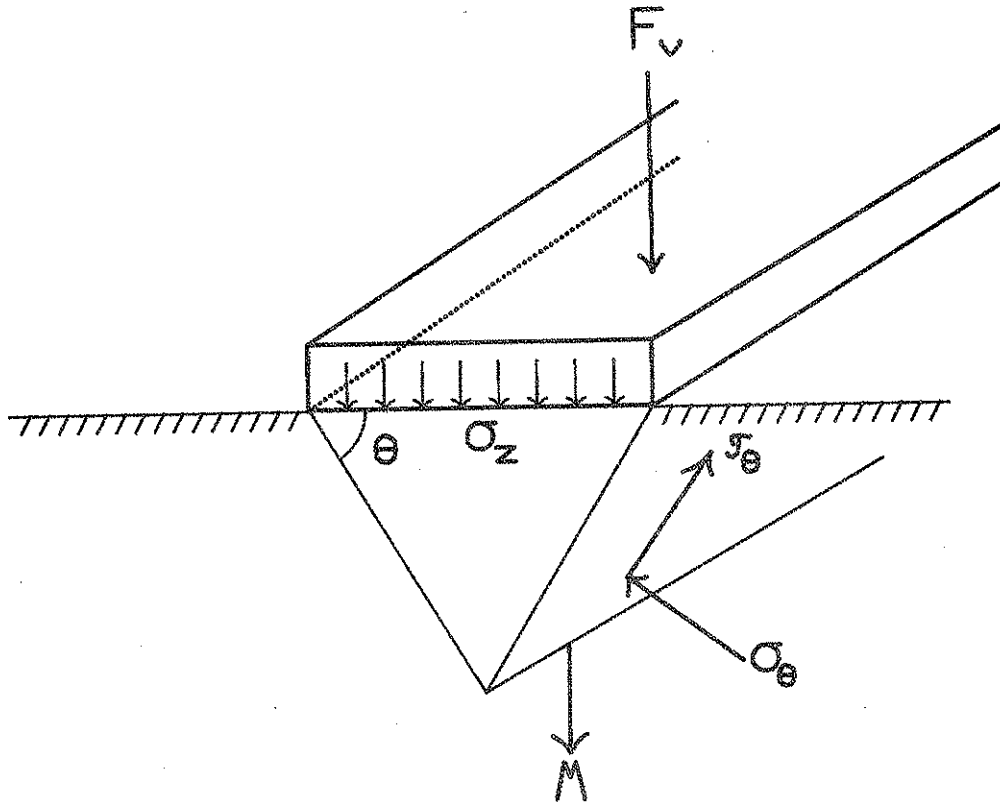


Fig. 10. Schematic of the stress situation on the side of the triangular wedge below a strip load.  $F_v$  is the vertical force exerted by the load giving rise to the surface normal stress  $\sigma$ , the normal stress  $\sigma_\theta$  and the shear stress  $\tau_\theta$  acting on the sides which slopes  $\theta = 45 + \phi/2$  to the horizontal.  $M$  signifies the vertical direction of motion.

Assuming that the wedge not only follows the loading plate in its downward motion but also - at least in the initial stage - will stick to the plate in the case of a horizontal motion it is possible to calculate stresses on the wedge sides.

Referring to Fig. 11 it is seen that as the horizontal force  $F_h$  is applied it will be counteracted by oppositely directed shear stresses along the sloped wedge-sides. At failure the shear stress along the surface will be:

$$\tau_z = c + \sigma_z \cdot \tan(\phi) \quad (16)$$

Thus, the horizontal shear stress along one of the sloped wedge sides will be

$$\tau_h = (c + \sigma_z \cdot \tan(\phi))/2 \quad (17)$$

The upward directed shear stress along the wedge side may be calculated to:

$$\tau_u = ((\sigma_\theta \cdot \tan(\phi))^2 - \tau_z^2/4)^{1/2} \quad (18)$$

which yields:

$$\tau_u = (\sigma_\theta^2 - (c/\tan(\phi) + \sigma_z)^2/4)^{1/2} \tan(\phi) \quad (19)$$

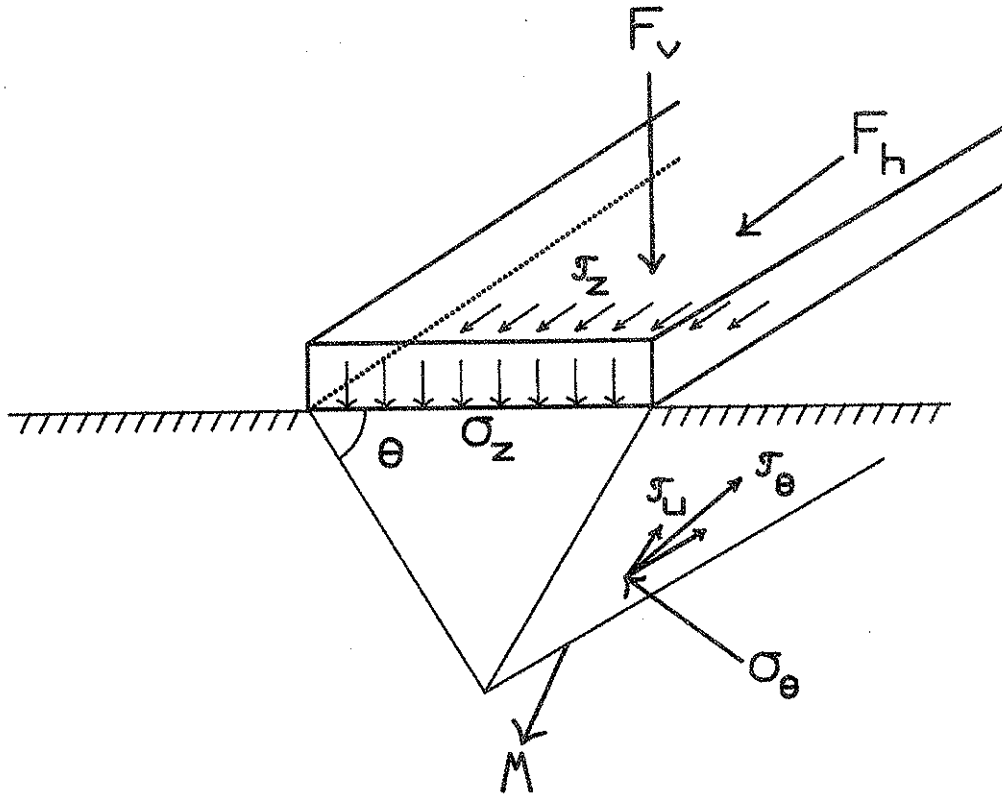


Fig. 11. Schematic of the same situation as in Fig. 10 with the addition of a horizontal force  $F_h$ . The magnitude of  $\tau_\theta$  is unchanged but it is tilted so that the upward directed shear stress component  $\tau_u$  is decreased as compared to the situation in Fig. 10.

This shows that the upward directed component of the shear stress decreases as a shear stress is applied and, therefore, the bearing capacity is reduced. Physically this is due to the fact that the upward directed shear stress  $\tau_\theta$  has a fixed maximum magnitude determined by the soil. As the horizontal force is applied  $\tau_\theta$  will be tilted in order to oppose the force and part of its contribution to the bearing capacity is used up.

It may, of course, be questioned whether the wedge below the strip load will follow the loading body even when it is moving in a horizontal direction. Reece (1964) performed some tests with a rectangular shear plate where he - in some of the experiments - fitted a suitable triangular wooden bar covered with sandpaper below the shear plate. The results indicated no difference whether the wooden bar was fitted or not. Even if this is not a strict proof, it is an indication that the philosophy outlined above is mainly correct.

From a compaction point of view the situation discussed above results in two different compaction mechanisms. The soil enclosed in the triangular wedge is compacted by a mainly hydrostatic stress until its strength is sufficient to force a shear failure situation to occur along its sloped sides. Just outside these sides the eventual compaction will be due to the shear compaction mechanism.

For agricultural operations the total bearing capacity of the soil is seldom mobilized. The preceding considerations may therefore be considered of limited value for agriculture. It is felt, however, that they constitute a good base for the understanding of the mechanics of the soil compaction process below a slipping wheel or track.

It is obvious that the development shown in Fig. 8 does not show all of the development in the soil from a moderate load in the centre figure to the full bearing capacity shown in the right part. The triangular wedge does obviously not develop suddenly at a certain threshold load. It is, however, difficult to obtain exact information on how the soil changes its compaction state as the load increases.

From the elastic theory it is well known that the highest normal stress is found in the top soil just below the loading body. From here the stress decreases with increasing depth and sideways distance as shown by e.g. Söhne (1953).

It is then possible to suggest the following development in the formation of the wedge below a loading body with increasing load: As the load increases the soil layer just below the body will be somewhat compacted. As this soil layer compacts its compressive strength

increases and it will therefore, to some degree, act as if it was part of the body.

Because of this behavior, soil just below the compacted zone will be subject to an increased normal stress field. The stress generating lower surface of the body has, so to say, been moved downwards due to the compacted layer just below the body. Therefore, the next soil layer will compact and - in turn - behave as fitted to the body and the process proceeds.

In the previous paragraphs the development has been described as a stepwise process. For a homogeneous soil it is probably not so. One can imagine that the compaction proceeds continuously down into the soil.

In the process described above the major principal stress is in the vertical direction and the intermediate and minor principal stress act in the horizontal plane. In the fundamental soil mechanics (see e.g. Lambe and Whitman, 1969) it is shown that the largest shear stress under such conditions acts along a plane sloping  $45 + \phi/2$  to the horizontal. When the shear stress along this plane exceeds the maximum value which the soil can sustain it will fail in shear along this plane. Because such a plane develops from both sides of the loading body the isosceles triangular wedge is formed.

Fig. 12 is an attempt to illustrate the compaction process outlined above. For a moderate loading the process may be imagined to cease at one of the curves inside the triangle.

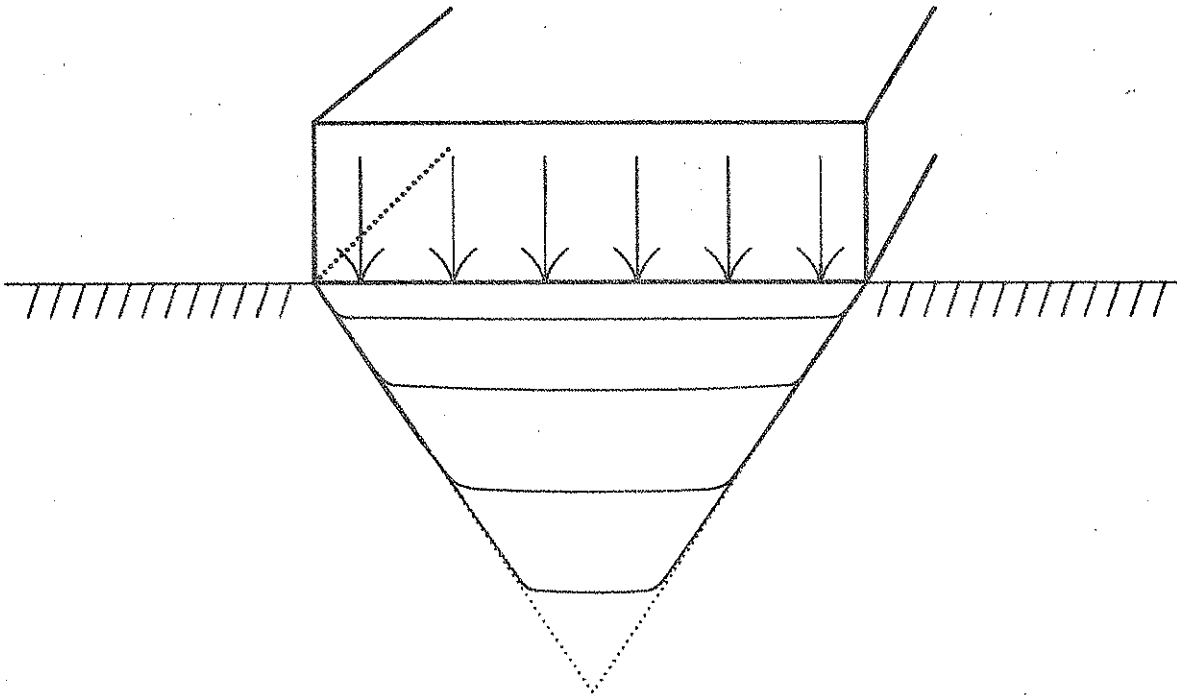


Fig. 12. The author's concept of the soil compaction process below a two-dimensional strip load. The curves within the triangle show progressive developments of the compaction process.

It should be emphasized that the compaction development described above is just a tentative supposition of the author. It has not been in any way proved in real soil.

For agricultural machines the loading body does usually not have the form of a strip load. The reason for considering this kind of body is that end edge effects can be neglected. However, the scheme should qualitatively work quite as well below a loaded wheel with the exception that the triangular wedge form should be somewhat deformed.

Considering Reece's (1964) argument that the compacted wedge will follow the loading body even in a shearing or slipping action this, together with the wedge forming shown in Fig. 12, implies that the soil shear will take place at a depth dependent on the vertical load. Thus, the higher the vertical load the deeper into the shearing will occur.



This behaviour seems reasonable from the following argumentation. It is well known (Olsen, 1986a) that the soil layer just below the loading body compacts first due to the normal stress against the soil surface. If the shear strength of the compacted soil becomes larger than the shear strength of the uncompacted soil in deeper layers the shear failure will occur along the interface between the compacted and uncompacted soil. This behaviour is illustrated in Fig. 13.

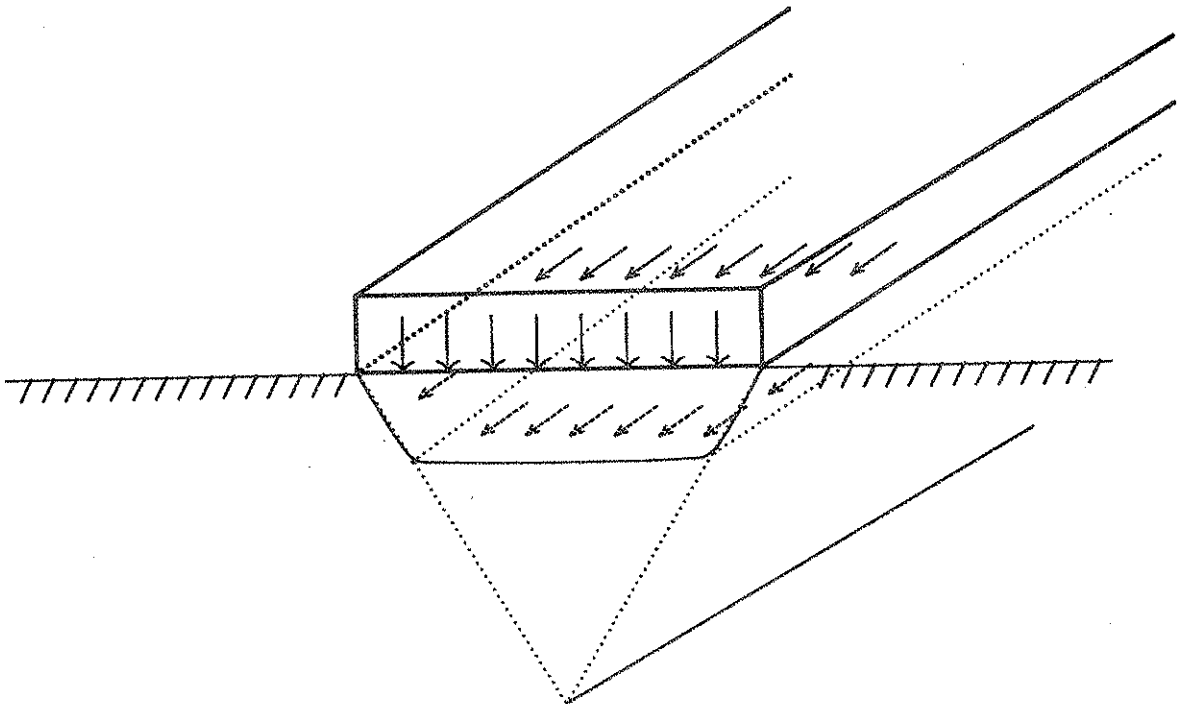


Fig. 13. Shear failure below a moderately loaded body. Because the upper soil layer has been compacted its shear strength has increased. Therefore, the shear failure mainly occurs at some depth in the soil.

If, however, the shear strength of the compacted soil is not considerably larger than that of the deeper layers the shear failure will take place along the interface between the loading body and the soil.

Unfortunately, not very much is known about the strength of newly compacted soil. One may assume its strength has only increased moderately because of some rearrangement of the particles which may consequently loose some of their cohesive interconnections. It is therefore difficult to say which failure mechanism will be dominant.

Some observations have been done in connections with torsional shear tests. During shear tests soil prisms will be enclosed between the base of the shear ring and the grousers. Studies of the lower boundary of these soil prisms which were sheared off during the tests gave no absolutely clear answer. There appeared to be a tendency so that at very low normal stresses, about 30 kPa, the lower surface was plane and thus appeared as being sheared off along the lower edge of the grousers. For higher normal stresses the lower surface was mostly very uneven as could have been the result of shear failure in the compacted layer but the picture was not quite clear. Furthermore, a torsional shearing device is not ideal for this kind of observations because of its circular motion.

In the practical situation it may well be possible that both shearing mechanisms occur simultaneously or that one is dominant in the front part of the contact area and another in the rear.

The next question concerns the behaviour of the soil just below the shear failure plane whether that be just below the loading body or below the compacted soil layer. In this zone the shear stress is smaller than the soil strength because shear failure has already occurred just on top of the zone. In particular, the interesting question is to which depth the soil is affected i.e. deformed, even if no large displacements have occurred.

## EXPERIMENTS

Practical experiments were carried out in order to find the influence of the shear strain on the soil density. Also some simple experiments were done with a shear plate to investigate the depth to which the soil is affected.

### Soil density measurements

Measurements of the change in dry bulk density was performed in the laboratory using a soil bin filled with a loam type of soil. The soil had been sieved through a 10 mm mesh so that even moderate stones were removed. A torsional shearing device described in Olsen (1984) was used to apply the normal and shear stresses. After the treatment soil samples were taken using rings of 72 mm diameter and 50 mm height. Three samples were taken in the foot print after the shear annulus when a test was completed.

Two normal stresses of 45 and 100 kPa were used and for each stress level three displacements of 5, 10 and 20 cm were applied. The soil's moisture content was 9 percent on dry basis.

The results revealed no difference in dry bulk density as function of shear displacement. Therefore, all density results for the various displacements were lumped together. Table 1 shows the results after this grouping.

Table 1. Change in soil's dry bulk density as a result of shearing. The difference in densities for the 45 kPa normal stress is significant on a 0.01 percent level. For 100 kPa there is no significant difference

Normal stress	Without shear	With shear
45 kPa	1326 kg/m <sup>3</sup>	1369 kg/m <sup>3</sup>
100 kPa	1363 kg/m <sup>3</sup>	1369 kg/m <sup>3</sup>

From Table 1 it is seen that the shearing does affect the soil's density at the low normal stress although to a quite moderate degree. At the high normal stress level no influence was found.

### Shear depth measurements

In order to obtain some information about the depth to which the shear stress extends some simple experiments were carried out. The tests were done in a field with sandy loam soil. The moisture content was 20 percent on dry base which was close to field capacity. The soil was well consolidated having been left undisturbed for 3 months during which period it received 172 mm rainfall. The cohesion was 15 kPa and the angle of internal shearing resistance was 33 deg. measured in the field by means of a torsional shearing device.

The soil was prepared by making a hole 8 mm in diameter and 80 to 100 mm deep. The hole was filled with labelled dried yellow peas so that pea no. 1 was in the bottom with the numbers incrementing upwards. For each pea its depth was recorded.

When the hole was filled to within about 15 mm from the surface a simple shear plate was placed over the pea column in such a way that the column was in the middle between two grousers. The shear plate was rectangular with dimensions 18x40 cm and 30 mm grouser height. Grousers were spaced 80 mm apart. The plate was loaded with an appropriate weight and pulled a distance of approximately 15 cm.

Fig. 14 shows the experimental set-up. The soil in front of the shear plate was removed to same depth as the grouser height in order to avoid too much piling up of soil during shearing. Pulling of the plate was done by means of a wire parallel with the plate base.

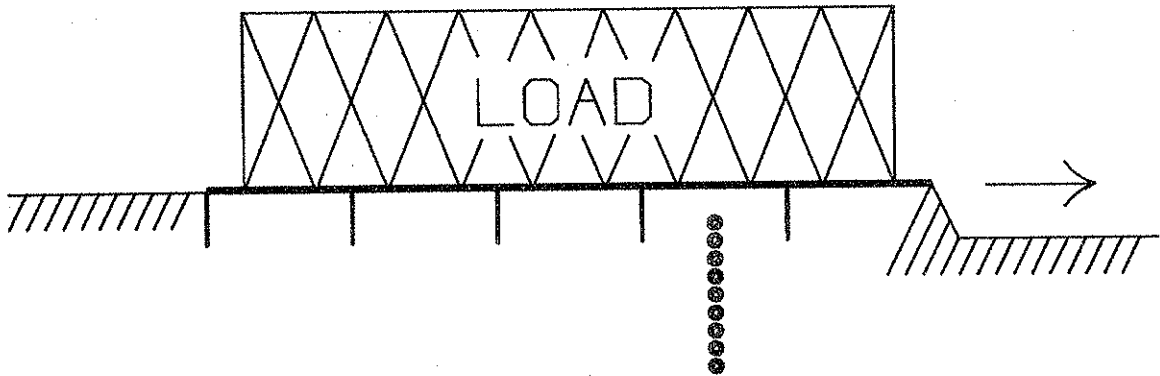


Fig. 14. Sketch of the experimental set-up for measuring the depth to which the soil is strained by shear. The circular spots show the initial position of the labelled peas.

After the shearing procedure the sinkage of the plate was recorded. Then the peas were carefully excavated and their new positions recorded. The excavation was continued until the pea-column was reached for which no displacement of the peas had occurred.

Because the experiments were done in the field with no special measuring equipment available the accuracy was limited. It is judged that the uncertainty of the measurements was about 2 to 3 mm. Likewise, the size of the peas limits the accuracy. Peas were about 6 to 7 mm in diameter. This means that movements smaller than this size was hard to observe.

The normal stresses calculated on the basis of the shear plate's dimensions ranged between 7 and 83 kPa. Typical results are shown in Fig. 15. The initial position of the peas is indicated at the left in the figures as filled circles and their final positions after shearing are shown as hollow circles. The dotted lines bind together the individual peas in their initial and final positions. However, it should be noted that this does not necessarily mean that the peas have moved along these lines. Nothing is known about the paths for the individual peas. The fat vertical lines indicate the depth of the grousers before and after the shearing process. It should be emphasized that only the vertical position of the grousers is indicated, in reality the pea-column was placed midway between two grousers as previously explained.

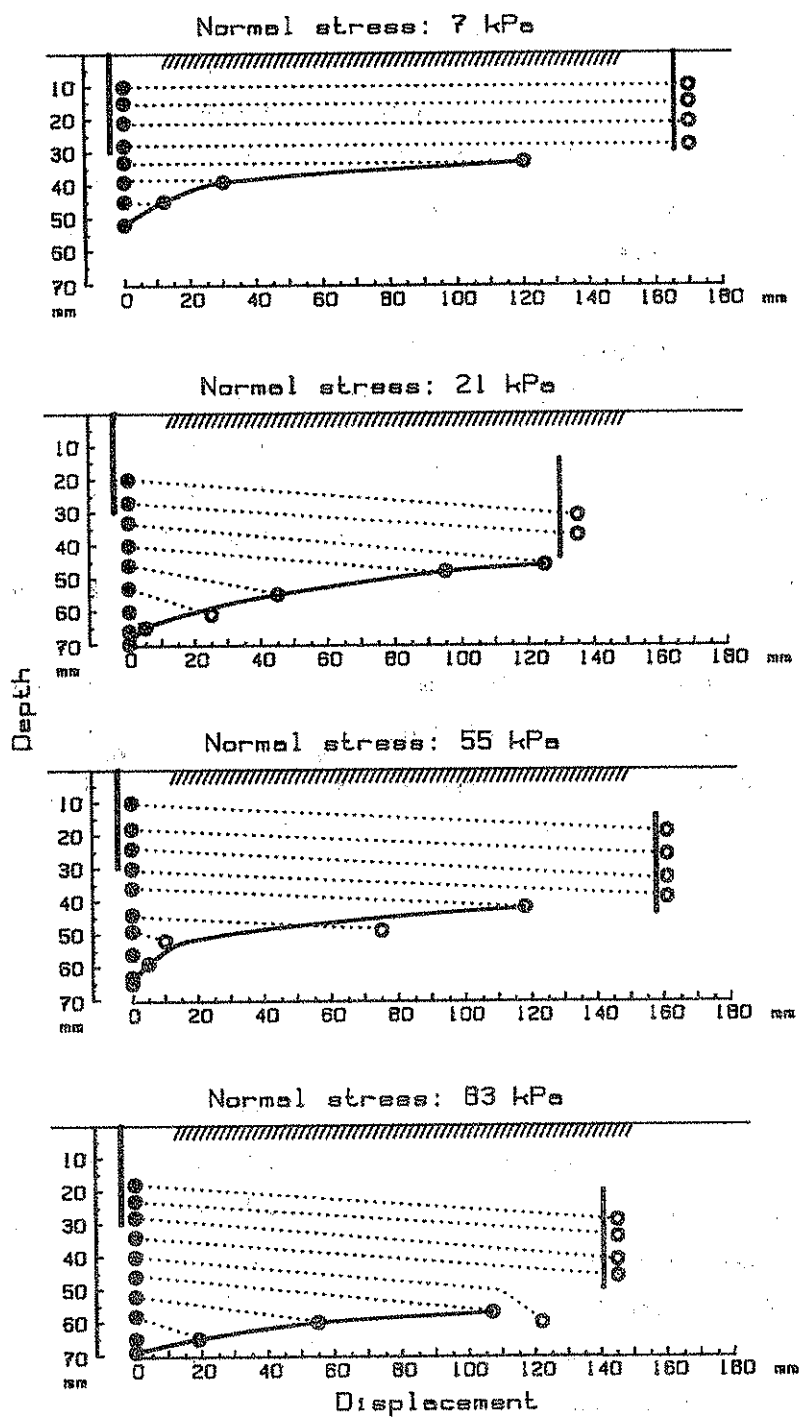


Fig. 15. Results from experiments with shearing of a soil containing labelled peas. The initial positions of the peas are shown as filled circles and their final positions as hollow circles. The fat vertical lines indicate initial and final depth of the shear plates grousers.

It is seen in Fig. 15 that the peas were displaced in very much the same way independent of the normal stress. It is especially interesting to note that the peas are displaced down to a depth of only 20 mm below the final depth of the grousers lower edge. This observation suggests that the shear stress effect does not reach very deep into the soil.

The somewhat odd displacement of pea no. 5 from the top in the lowest figure in Fig. 15 may be due to the formation of a compacted layer below the grousers as shown in Fig. 13. If this pea was trapped in such a layer it would follow the horizontal movement of the shear plate right to the end. Obviously, it did not do so which may be due to break-down of the front end of the layer just before the shearing stopped.

The same method was tried for a more clayey soil type. This was no success, however. It proved very difficult to excavate the peas because the soil loosened in large lumps often containing a pea so that the exact position of that pea was lost.

#### DISCUSSION

It is questionable to use a torsional shearing device for the study of the shearing's compaction effect. As shown Fig. 12 the formation of a wedge below the annulus may be expected at higher normal loads. Due to the rotational movement of the annulus this wedge will probably be very unstable. It may break down and build up again in an uncontrolled manner. Furthermore, the direction of the shear stress shifts in the horizontal plane as the rotation proceeds.

As to the results in Table 1 for the low normal stress it is impossible to state how much of the increase in density should be ascribed to the formation of a wedge during rotation and how much is due to rearrangement of soil particles close to the shear failure zone. Because the soil sampling rings have a diameter of about same size as the width of the shear annulus a soil sample will incorporate zones of both kinds of compaction mechanisms.

The fact that no difference in dry bulk density was found for the high normal stress may be due to full mobilization of the soil's bearing capacity. When this occurs the only thing that happens during shearing is that the shear direction shifts as illustrated in Figs. 10 and 11. This shifting is not likely to affect the compaction of the soil as the magnitude of the shear stress remains unchanged.

As previously noted the results from the shear plate tests indicate that shearing affects the soil to a rather limited depth. This makes density measurements by means of sample rings difficult. If only the upper two centimetres of the five cm high samples are affected a somewhat higher density of these two cm could easily be shaded by the natural variation in the soil.

It was not possible to perform this kind of measurements in a clayey soil type for reasons as noted above. It may, however, be assumed that the shear effect depth for this kind of soil will be shallower at least in very dry and very wet conditions. When relatively dry such a soil usually exhibit a considerable cohesion. Because of this the shear effect may be limited to a narrow zone around the shear failure plane. On the other hand, when this soil is near saturation the effective stress between the particles is of moderate size. When the soil becomes compacted part of the porespace collapses and the effective stress may nearly vanish. Under such conditions the shear stress will be very low and consequently affect the soil to a smaller depth.

#### REFERENCED LITERATURE

- Bailey, A. C. and VandenBerg, G. E., 1968.  
Yielding by compaction and shear in unsaturated soils.  
Trans. ASAE 11:307-311,317.
- Davies, D. B.; Finney, J. B. and Richardson, S. J., 1973.  
Relative effects of tractor weight and wheel-slip in causing soil compaction. Journal of soil science, 24:399-409.
- Dudzinski, P. A., 1987.  
Contribution to the studies on the dynamic shearing strength of soils. Proceedings of the 9th International Conference of the ISTVS. Barcelona.
- Hansbo, S., 1975.  
Jordmateriallära. Uppsala.



- Koolen, A. J. and Kuipers, H., 1983.  
Agricultural soil mechanics. Berlin.
- Lambe, T. W. and Whitman, R. V., 1969.  
Soil Mechanics. New York.
- Ljungars, A., 1977.  
Olika faktorerers betydelse för traktorernas jordpackningsverkan.  
(Importance of different factors on soil compaction by tractors, In  
swedish with english summary). Agricultural College of Sweden.  
Department of Soil Sciences. Rep. no. 52. Uppsala.
- McKyes, E., 1985.  
Soil Cutting and Tillage. New York.
- Olsen, H. J., 1984.  
A torsional shearing device for field tests. Soil and Tillage  
Research, 4:599-611.
- Olsen, H. J., 1986a.  
Tekniska möjligheter att undvika skadlig jordpackning.  
Sveriges Lantbruksuniversitet. Konsulentavdelningens rapporter.  
Allmänt 84. Uppsala.
- Olsen, H. J., 1986b.  
Soil mechanical behaviour of a heavy clay soil after three long-term  
compaction treatments. Soil and Tillage Research, 7:145-156.
- Raghavan, G. S. V. and McKyes, E., 1977.  
Laboratory study to determine the effect of slip-generated shear on  
soil compaction. Canadian Agricultural Engineering, 19:40-42.
- Raghavan, G. S. V.; McKyes, E. and Beaulieu, B., 1978.  
Clay soil compaction due to wheel slip. Trans. ASAE, 21:646-649,653.
- Reece, A. R., 1964.  
Problems of soil vehicle mechanics. Land Locomotion Laboratory.  
Report no. 8470. Michigan.
- Ronai, M. and Trowse, A. C., 1982.  
Influence of tires construction on soil compaction.  
Proceedings of the 9th conference of the International Soil Tillage  
Research Organization. Osijek. Yugoslavia.
- Söhne, W., 1953.  
Druckverteilung im boden und Bodenverformung unter Schlepperreifen.  
Grundlagen der Landtechnik, Heft 5:49-63.
- Stafford, J. V. and Tanner, D. W., 1983.  
Effect of rate on soil shear strength and soil-metal friction.  
Soil and Tillage Research, 3:245-260.
- Trowse, A. C., 1982.  
Effects of tires on recently tilled soils. Proceedings of the 9th  
conference of the International Soil Tillage Research Organization.  
Osijek. Yugoslavia.