Economic optimisation of silvicultural regimes for Scots pine using dynamic programming

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Stand treatment prescriptions in Lithuania are based on silvicultural traditions and biometric models, whereas application of economic models still is in its very infancy. The forest yield model of Kuliesis, pine assortment tables, observed timber prices, and costs of silvicultural treatments constitute sufficient information for optimizing silvicultural regimes in pine stands, using economic criteria, namely, forest rent, net present value, and soil expectation value. The dynamic programming approach enables simultaneous optimisation of intermediate and final felling throughout the rotation.

Results obtained confirm that rotation ages differ according to site productivity. Optimal rotations range from 80 years in most productive stands to over 130 on the poorest sites, when the forest rent criterion is selected. The choice of thinning regimes is less obvious due to high sensitivity to timber prices, interest rates, and other factors.

keywords: silvicultural regimes, economic criteria, dynamic programming, Scots pine.

Introduction

Central European traditions of forest management and a half century period under a central planning economy predetermined focus on biological regularities, when choosing certain silvicultural regimes in Lithuania. Economic goals were expressed indirectly, when the main aim was to grow healthy stands, producing timber of the most valuable tree species on appropriate sites. Important silvicultural decisions such as, timing and intensity of thinnings, and rotation length continue to be strictly prescribed by rules and standards, directed towards the achievement of the maximum possible accumulation of valuable timber, until the determined rotation age is reached. While such prescriptions are not necessarily wrong, it is of great interest to compare them with silvicultural regimes obtained using economic criteria.

First attempts to calculate forest rent for pine stands in Lithuania were made in the 1970's (Rtycnfdbx/c B.> Rektibc F. 1977). Average annual returns were maximized by calculating the value of timber, obtained from the final felling. Costs of planting and precommercial thinnings as well as revenues from commercial thinnings were ignored. In the recent studies (Brukas 1997, Brukas and Brodie 1999) the entire cash flow throughout the rotation is included into the calculation of rotation ages maximising forest rent, net present value (NPV), and soil expectation value (SEV). The timing and intensity of thinnings are set at the average of those, observed in silvicultural practice. In this paper, the dynamic programming (DP) algorithm is used to estimate timing and intensity of the intermediate felling and the optimal rotation age for Scots pine (*Pinus sylvestris*) stands in Lithuania.
In early DP applications in even-aged forest management, dating back to 1960's to late 1970's, residual stand volume served as a decision variable (Brodie et al. 1978). Later studies extended to the projection of stand basal area and residual number of trees (Brodie and Kao 1979), incorporation of diameter distribution in stands (Haight et al. 1985), modification of the DP algorithm to reduce the computational burden of arduous forest management problems (Paredes and Brodie 1987). These examples stand for just a modest subset of studies, looking for efficient DP solutions in even-aged forest management. Notably, the tendency toward computational sophistication did not induce a trend to a more extensive use of dynamic optimisation for solving practical problems.
The main task of this paper is to examine DP solutions for Scots pine stands in Lithuania. This should gain a twofold benefit. First, simultaneous optimisation of stand treatments could provide new insights for making silvicultural and investment decisions. Second, it is of theoretical interest to disclose, what solutions would be generated in a country, where stand growth conditions, forest management practices, costs of labor and materials, and other relevant factors are quite distinct from those, observed in countries where the DP algorithm was repeatedly applied.

Materials and Methods

The forest yield model by Kulieðis (1993) serves to set up the network of stand parameters. An advantageous feature of this model is its ability to predict the dynamics of main stand parameters (height, diameter, basal area, stocking index, volume) for both thinned or self-thinned part and for remaining part of a stand according to stand age, stocking index, and site productivity. Site productivity is expressed as the mean height at so-called basal age (100 years for pine). The nature of the model allows flexible adjustment of growth dynamics in response to climatic or anthropogenic changes via alterations in the main diameter growth submodel.

According to Kulieðis model, stocking index is assumed to be constant throughout the rotation. This assumption is not adequate in seeking to simulate thinning regimes. The following function was derived from Kulieðis yield table relationships simulated and then estimated by multiple regression by the authors, to predict the stocking index after a 10 years period:

$$S_{10} = 0.1585 + 0.8838 \cdot S_0 + 0.0032 \cdot Pr - 0.0023 \cdot A - 0.0029\, S_0 \cdot Pr + 0.0023 \cdot S_0 \cdot A - 0.0506 \cdot S_0^2$$

where:
S_{10} - stocking index after 10 years,
S_0 - current stocking index,
Pr - site productivity, expressed as the mean height of the stand at age 100,
A - age of stand.

The increase in stocking index during the period of ten years ranges from less than 0.01 in mature high stocked stands on poor sites to 0.13 in low stocked young stands on rich sites.

Continuous price curves (Figure 1), describing the dependence of the net revenue (Litas/m^3) on mean cutting diameter from intermediate and final felling were constructed using the data on timber prices, announced by the Centre of Forest Market Economy (Centre of For… 1998), price lists for logs used in forest enterprises, assortment tables for pine stands in Lithuania (Kulieðis et al. 1997) and costs normative for intermediate and final felling in forest enterprises. Costs of planting and precommercial thinnings were differentiated according to the site productivity. Planting costs tend to be higher on poorer sites due to the increased planting density, while costs of precommercial thinnings vary depending on the intensity of treatments. The joint costs range from 2600 to 4000 Litas/ha, when no discounting is applied.

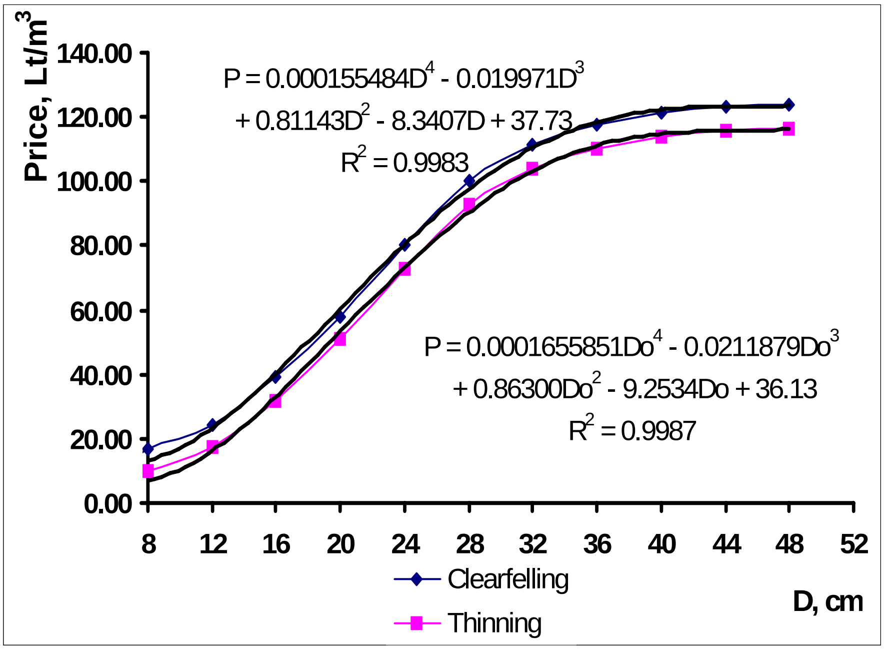

Figure 1. Prices curves for pine.

Remark: 1 Litas ≈ 0.25 US $.

Three criteria were used to optimize thinning regimes and the length of rotation. Forest rent simply maximizes average annual cash flow during rotation:

$$Forest\ rent = \frac{\sum_t (Ro_t \cdot Vo_t) + R \cdot V - \sum_t C_{th} - C_p}{RA};$$

where:

Ro - net revenue (Litas/m^3) from thinnings;

Vo - volume of thinning;

R - net revenue (Litas/m^3) from clear felling;

V- volume of clear felling;

C_p, C_{th} - costs of planting and pre-commercial thinning, respectively;

RA - rotation age;

t - subscript, indicating age, when a silvicultural decision is taken;

Forest rent does not reflect time preference of an investor, while using NPV, value of cash flow in the future is reduced using a discount factor. The effect of discounting increases with rotation age and discount rate r:

$$NPV = \max \sum_{t} \frac{Ro_t \cdot Vo_t - C_{th}}{(1+r)^t} + \frac{R \cdot V}{(1+r)^{RA}} - C_p$$

where:

r - discount rate.

SEV criterion is a modification of the NPV, accounting for the fact, that the rotation length predetermines the frequency with which rotation cycles are repeated, i.e. shorter rotations allow for more frequent regeneration and final felling. Optimal path is found using the following formula:

$$SEV = \max \sum_{t} \frac{(Ro_t \cdot Vo_t - C_{th}) \cdot (1+r)^{RA-t}}{(1+r)^t - 1} + \frac{R \cdot V - C_p \cdot (1+r)^{RA}}{(1+r)^{RA} - 1}$$

Derivation and more comprehensive discussion of the described economic criteria can be found in most Western textbooks for forest management and investment analysis, for example (Davis and Johnson 1987).

Given the above characterized data, dynamic programming finds an optimal path of silvicultural treatments when any of these objective functions are maximised. The DP algorithm is explained in most introductory books on operations research (for example, Hillier and Liebermann, 1996). The essence of DP can be comprehended considering the following example. Let us imagine a map of a country, containing cities, connected with each other via a road network. Our task is to travel from city A in the West (initial stage) to city Z in the East (final stage) minimizing the total travel distance. On the way we should pass other cities, for example, from city A we can travel to B, C or D, from B, C or D we can choose either E, or F, or H and so on. At each stage we should take an optimal decision, in our case, we should find the shortest total distance to each city at each stage. For example, we could get to the city F, using three alternatives: A→B→F, A→C→F and A→D→F. The number of possible paths increases rapidly with increasing number of stages, and number of alternatives (called states in DP jargon) in each stage. Fortunately, we do not have to check all possible paths at each stage and state. The amount of computation decreases significantly due to the so-called principle of optimality. It states, that, given the current state, an optimal policy for the remaining stages is independent of the policy decisions adopted in previous stages. If we discover a recursive relationship identifying the optimal policy to get from stage n to stage n + 1 (which would be the minimization of the travel distance in our trivial example), we can compute the optimal path without exhaustive enumeration of all possible combinations.

As is usually the case in DP applications in forest management, the age of the stand was chosen as a stage variable in the DP network. According to growth conditions and common silvicultural practices in Lithuania, a 10 years interval was selected between time points when a silvicultural decision (no treatment, thinning of certain intensity or clear felling) is taken. The initial stage is set at age 20, since a reliable growth model for younger stands is not available.

In contrast to other studies, stocking index (the ratio of actual stocking level with the stocking level of a normal stand, standing for a maximally stocked stand at given site and age) was chosen as the main descriptor variable. Stocking index traditionally serves in Lithuanian forestry, as an intuitively appealing measure, indicating what intensity of thinning, if any, is appropriate. The minimum stocking level is constrained to not allow reduction of the stocking index below 0.4, which, according to forest management rules in Lithuania, is the lowest stocking to register the inventory area, as a forest stand.

Mean height, diameter, volume, and basal area of the main and of the thinned part of stand are also stored in each stage and state. The mean diameter of thinning is calculated using the following formula.

$$D_O = \sqrt{\frac{N_B \cdot D_B^2 - N_A \cdot D_A^2}{N_O}}$$

where:

D_O - mean diameter of thinned trees;
N_B and N_A - number of trees before and after thinning, respectively;
D_B and D_A - mean diameter of stand before and after thinning, respectively;
N_O - number of thinned trees.

The recursive relationship between two stages originates from the previously presented objective functions. Using the forest rent criterion, the following expression represents the dynamic equation for all stages except the last, when Ro_t and Vo_t, and t are replaced by R, V, and RA, respectively.

$$f_t(S_1,\ldots,S_n) = \max \frac{(Ro_t \cdot Vo_t)}{t} + f_{(t-x)}(s_1,\ldots,s_n)\,;$$

where:

$f_t(S_1,\ldots,S_n)$ - optimal value function defined as best path from initial stage to age t;
$(s_1,\ldots,s_n)$ - set of states at a given stage, in our study represented by a set of all feasible stocking levels at given age;
x - the age interval between two neighboring stages, i.e. the number of years between thinning and rotation decisions (10 years in our case);

Similarly devised equations express the relationship between adjacent stages, when NPV or SEV is optimized:

$$f_t(S_1,\ldots,S_n) = \max \frac{(Ro_t \cdot Vo_t)}{(1+r)^t} + f_{(t-x)}(s_1,\ldots,s_n)\,;$$

$$f_t(S_1,\ldots,S_n) = \max \frac{(Ro_t \cdot Vo_t)\cdot(1+r)^{RA-t}}{(1+r)^t - 1} + f_{(t-x)}(s_1,\ldots,s_n)\,;$$

Results

As expected, both rotation ages, and thinning regimes on a given site are highly dependent on the selected economic criterion and the interest rate. Table 1 shows silvicultural regimes on a site of average productivity, when the average height at age 100 equals 24 meters.

Table 1. Impact of optimisation criterion and interest rate on silvicultural regimes for pine (H100 = 24, initial stocking index Sb_{20} = 0.9).

Age,	Stand befor felling				Stand after felling				Felled part			NPV*,
years	Db, cm	Nb, trees	Sb	Vb, m^3	Da, cm	Na, trees	Sa	Va, m^3	Do, cm	Mo, m^3	Do/Db	Lt
0-19	-	-	-	-	-	-	-	-	-	-	-	-2801
20	7.0	4526	0.90	72	7.0	4526	0.90	72	-	-	-	0
30	11.1	2320	0.93	128	11.1	2320	0.93	128	-	-	-	0
40	14.7	1527	0.95	182	14.8	1498	0.94	180	10.7	2	0.73	23
50	18.0	1129	0.96	231	18.0	1129	0.96	231	-	-	-	0
60	20.8	905	0.97	276	20.8	905	0.97	276	-	-	-	0
70	23.3	764	0.98	315	23.3	764	0.98	315	-	-	-	0
80	25.4	670	0.99	350	25.5	658	0.98	346	18.7	4	0.74	165
90	27.4	593	0.99	376	27.5	583	0.98	372	20.2	4	0.74	207
100	29.1	536	0.99	397	29.3	527	0.98	393	21.6	4	0.74	246
110	30.6	493	0.99	415	-	-	-	-	30.6	415	1.00	44228
									Forest Rent =			**382**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2752
20	7.0	4526	0.90	72	7.0	4526	0.90	72	-	-	-	0
30	11.1	2320	0.93	128	11.1	2320	0.93	128	-	-	-	0
40	14.7	1527	0.95	182	14.8	1498	0.94	180	10.7	2	0.73	16
50	18.0	1129	0.96	231	18.0	1129	0.96	231	18.0	-	-	0
60	20.8	905	0.97	276	20.8	905	0.97	276	20.8	-	-	0
70	23.3	764	0.98	315	23.3	764	0.98	315	23.3	-	-	0
80	25.4	670	0.99	350	26.6	546	0.88	311	19.4	39	0.76	882
90	28.4	502	0.90	342	29.1	583	0.84	319	22.0	23	0.77	591
100	29.1	536	0.86	345	-	-	-	-	29.1	345	1.00	13611
									NPV(1%) =			**12348**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2752
20	7.0	4526	0.90	72	7.0	4526	0.90	72	-	-	-	0
30	11.1	2320	0.93	128	11.1	2320	0.93	128	-	-	-	0
40	14.7	1527	0.95	182	14.8	1498	0.94	180	10.7	2	0.73	16
50	18.0	1129	0.96	231	18.0	1129	0.96	231	-	-	-	0
60	20.8	905	0.97	276	20.8	905	0.97	276	-	-	-	0
70	23.3	764	0.98	315	23.3	764	0.98	315	-	-	-	0
80	25.4	670	0.99	350	26.6	658	0.88	311	19.4	39	0.76	882
90	27.4	593	0.90	342	-	-	-	-	27.4	342	1.00	13831
									SEV(1%) =			**20244**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2708
20	7.0	4526	0.90	72	7.0	4526	0.90	72	-	-	-	0
30	11.1	2320	0.93	128	11.1	2320	0.93	128	-	-	-	0
40	14.7	1527	0.95	182	14.8	1498	0.94	180	10.7	2	0.73	7
50	18.0	1129	0.96	231	20.3	625	0.67	161	14.6	70	0.81	423
60	23.0	563	0.73	207	25.0	344	0.50	142	19.8	65	0.86	577
70	27.3	333	0.57	183	-	-	-	-	27.3	183	1.00	2201
									SEV(3%) =			**616**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2670
20	7.0	4526	0.90	72	7.3	3808	0.82	66	5.1	6	0.73	26
30	11.4	2057	0.87	119	11.5	1974	0.85	117	8.4	3	0.74	6
40	15.1	1359	0.89	170	16.3	936	0.71	136	12.0	34	0.79	116
50	19.5	775	0.77	185	22.3	317	0.41	99	17.2	87	0.88	476
60	25.0	326	0.50	142	26.0	251	0.40	114	22.2	28	0.88	174
70	28.3	257	0.48	154	-	-	-	-	28.3	154	1.00	977
									SEV(4%) =			**-859**

* Entries are NPV in Lithuanian Litas appropriately discounted; Forest Rent and SEV's are transformed appropriately.

Using the forest rent theory, optimal rotation is 100 years. The optimal path indicates several thinnings of negligible intensity. The stocking index is reduced just by 0.01, which practically implies a silvicultural regime without intermediate treatments. In accordance with the theory, the rotation using the NPV criterion exceeds SEV rotation if the obtained SEV value is positive. This difference is not high when low to moderate interest rates (up to 4%) are used on sites of average and high productivity.

The optimal rotation age decreases and the intensity of thinnings increases with the rising interest rate (Figure 2). Using the interest rate of 3%, the optimal rotation is 70 years, heavy thinnings are obtained at age 40 and 50. This is a logical result, since higher interest rate reflects higher preference for revenues at present as compared with future. Maximum internal rate of return, when discounted revenues equal discounted costs, on average site is approximately equal to 3.5 %, which falls below the industrially competitive rates of investment in Lithuania.

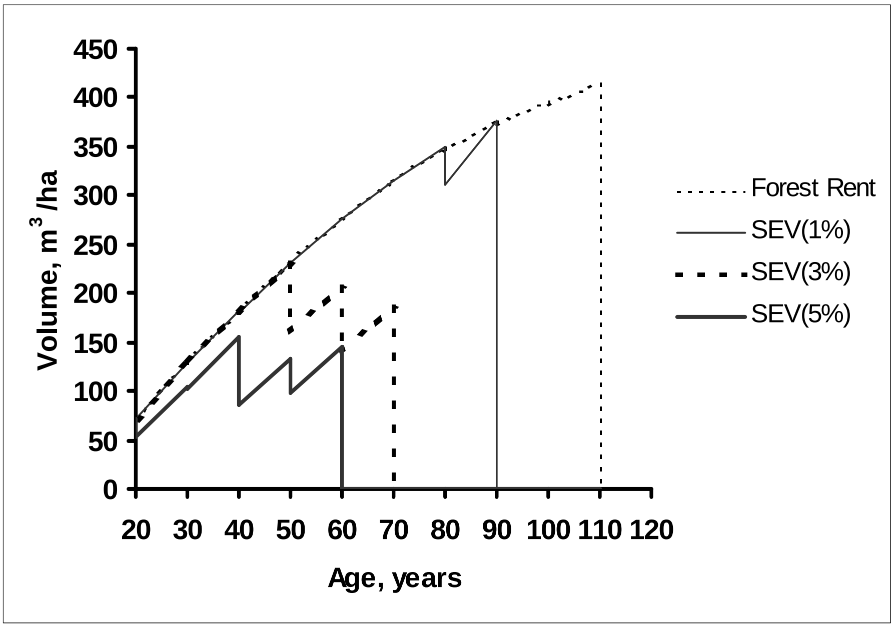

Figure 2. Comparison of thinnings and harvest, when interest rate is varied (H100 = 24, stocking $index_{20}$ = 0.9).

Site productivity also heavily affects the silvicultural regimes. Using forest rent theory, the optimal rotation ranges from 80 years on the most productive sites (H100 = 33) to 130 years or more on the poorest sites (H100 = 15). The amount of intermediate felling remains negligible on the entire range of site productivity. Using the SEV criterion with 3% interest rate thinning regimes substantially differ on various sites (Table 2).

Table 2. Comparison of optimal solutions according to site productivity, when SEV(3%) is used (stocking $index_{20}$ Sb = 0.9).

Age,	Stand befor felling				Stand after felling				Felled part			NPV,
years	Db, cm	Nb, trees	Sb	Vb, m^3	Da, cm	Na, trees	Sa	Va, m^3	Do, cm	Mo, m^3	Do/Db	Lt
0-19	-	-	-	-	-	-	-	-	-	-	-	-3952
20	4.6	7935	0.90	43	5.5	3594	0.57	27	3.7	16	0.80	111
30	8.9	2169	0.66	58	8.9	2118	0.65	57	7.0	1	0.79	2
40	11.8	1552	0.72	90	11.8	1519	0.71	89	9.3	1	0.79	3
50	14.3	1213	0.76	121	14.7	1043	0.69	109	11.5	11	0.80	37
60	16.9	907	0.74	139	18.5	516	0.50	94	14.6	45	0.86	202
70	20.4	497	0.56	120	21.5	329	0.41	88	18.1	32	0.89	176
80	23.2	328	0.46	109	23.7	283	0.40	94	20.6	14	0.89	75
90	25.2	274	0.44	112	25.2	274	0.44	112	25.2	274	1.00	675
	H100 = 18m										**SEV =**	**-2884**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2670
20	7	4526	0.90	72	7.0	4526	0.90	72	7.0	-	-	0
30	11.1	2320	0.93	128	11.1	2320	0.93	128	11.1	-	-	0
40	14.7	1527	0.95	182	14.8	1498	0.94	180	10.7	2	0.73	7
50	18	1129	0.96	231	20.3	625	0.67	161	14.6	70	0.81	423
60	23	563	0.73	207	25.0	344	0.50	142	19.8	65	0.86	577
70	27.3	333	0.57	183	27.3	333	0.57	183	27.3	333	1.00	2201
	H100 = 24m										**SEV =**	**616**
0-19	-	-	-	-	-	-	-	-	-	-	-	-2670
20	9.2	3038	0.90	103	9.2	3038	0.90	103	9.2	-	-	0
30	14.2	1610	0.93	178	14.2	1610	0.93	178	14.2	-	-	0
40	18.6	1080	0.95	252	19.0	980	0.90	239	13.7	13	0.74	93
50	22.8	764	0.93	310	27.7	248	0.44	146	20.0	163	0.88	1983
60	30.8	260	0.54	212	32.3	186	0.41	161	27.2	51	0.88	755
70	34.8	202	0.50	222	34.8	202	0.50	222	34.8	4	1.00	3272
	H100 = 30m										**SEV =**	**3928**

[insert Table2 here]

A rotation of 100 years is obtained on particularly poor sites (H100 = 15), it decreases to 70 years on sites of average and high productivity. Figure 3 shows the higher intensity of thinnings in more productive stands.

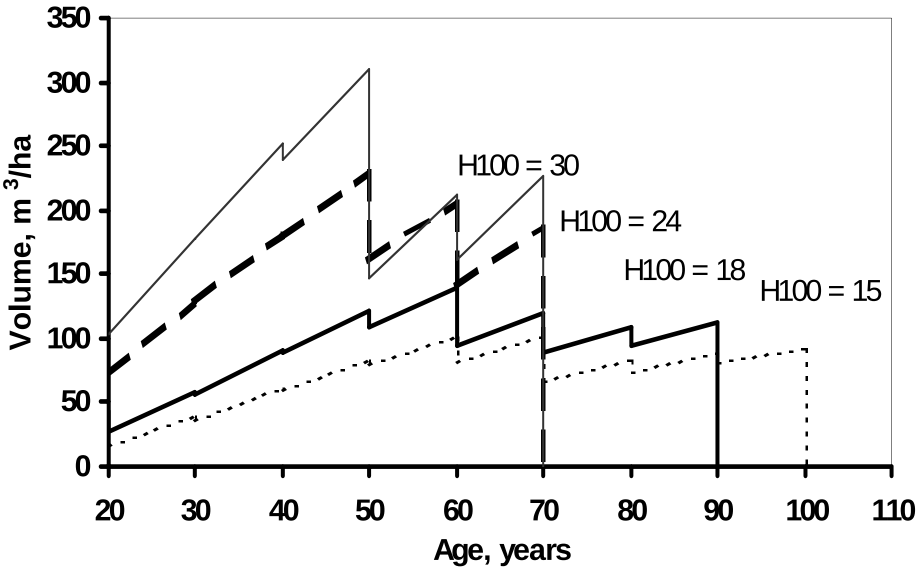

Figure 3. Comparison of optimal paths on different sites, when SEV(3%) criterion is used (stocking index_{20} = 0.9).

Initial stocking index at age 20 does not effect the choice of rotation age. The optimal paths on average sites are compared in Figure 4. Naturally, higher initial stocking favors more intensive intermediate treatments.

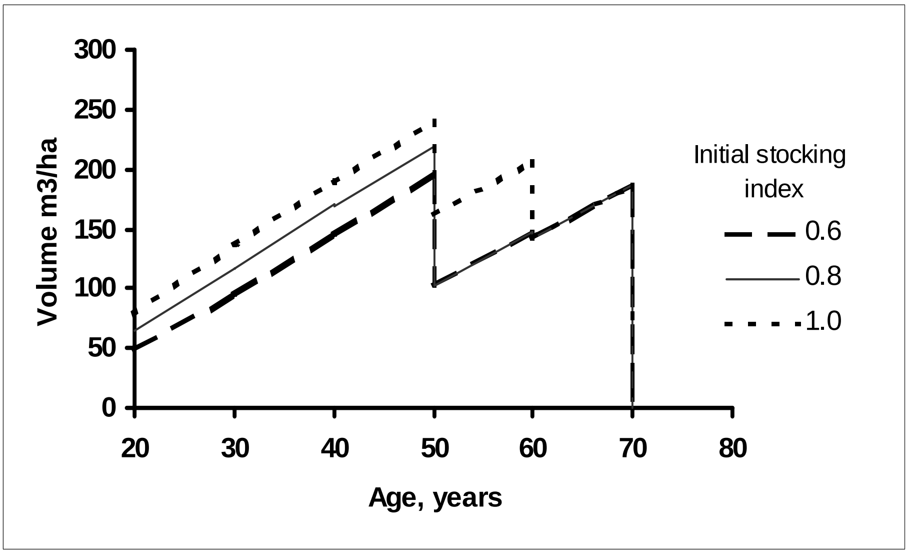

Figure 4. Comparison of optimal paths, when initial stocking level at age 20 is varied. (H100 = 24, SEV(3%) maximisation).

Beside the selected optimisation criterion, interest rate, and site productivity, the highly influential factors for resulting solutions are the shape of stand growth prediction function and the distribution of prices for various logs. Substantially different solution was obtained when the stocking index prediction function was replaced be the prediction of constant stocking index after 10 years from felling. The resulted optimal paths on various sites and with various initial stocking levels indicated no thinning or weak thinning treatments using SEV(3%).

The constructed price curves are based on timber prices observed since 1994, when a free timber market became fully established and when timber prices started to be observed by the Centre of Forest Market Economy in Lithuania. According to available observations, small sized assortments, mainly sold as pulpwood, are particularly inclined to prices changes. For that reason, two price levels (60 and 80 Lt/m^3) were used to construct alternative price curves in addition to the original price curve, when a pulpwood price of 70 Lt/m^3 was used. The resulting thinning regimes are shown in Figure 5. A change of 10 Litas in pulpwood price does not effect the length of the rotation, yet timing and intensity of thinnings appeared to be highly sensitive. Higher prices on small sized timber logically require more intensive intermediate treatments.

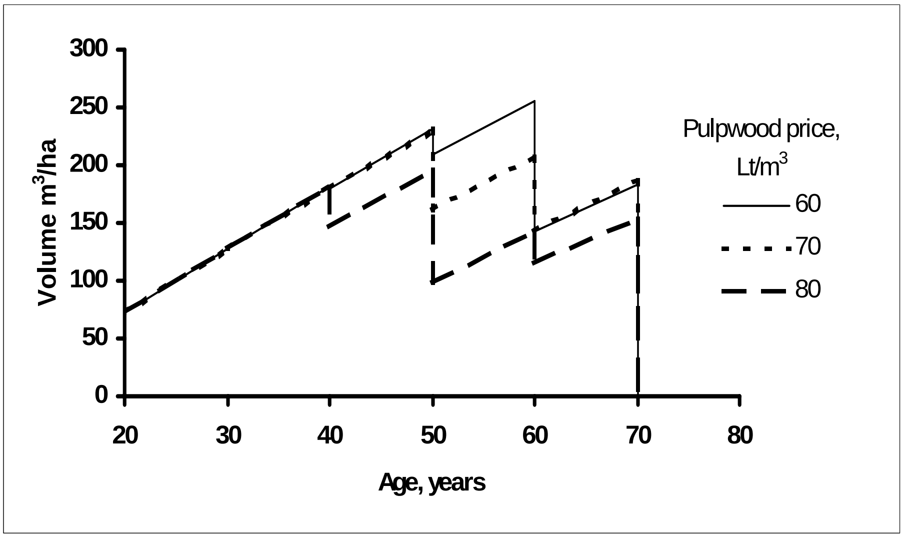

Figure 5. Impact of pulpwood price on optimal silvicultural regimes, using SEV(3%) criterion (H100 = 24, stocking level$_{20}$ = 0.9).

Discussion and Conclusions

An interesting question is how the obtained results compare with actual silvicultural treatments. Forest rent on sites of average productivity yields an optimal rotation length close to currently prescribed rotation ages, with negligible intermediate treatments. The SEV criterion with moderate interest rates renders 20 to 30 years lower rotations with heavy thinnings in stands, older than 40 years. In 20 to 40 years old stands light thinnings if any are indicated, despite high stocking levels. What are the main reasons for these appreciable differences between our solutions and the practise? The sensitivity analyses show that decisive forces in our model are the chosen rate of time preference, the shape of price curves, and the shape of the stocking index prediction function. In reality, highly stocked young stands are vulnerable to windthrows and snowbreaks, they often fall under the growth depression. Therefore, the assumption about their ability to sustain high stocking level is tentative. More information is needed to incorporate the stochastic nature of the development of highly stocked young stands in the stocking index prediction function. Such consideration would obviously induce heavier thinnings in young stands. We do not simulate stand development when planting density and regimes of precommercial thinnings are varied. Such analysis could be easily included into dynamic optimisation, but the required empirical data are absent. The current planting density (from 6 to 11 thousand trees per ha according to site conditions) is based mainly on tradition rather than on a sound biological or economic justification. Such a high density clearly is one of the main causes of the frequently observed growth depression in young stands. Lesser density bolsters diameter increment and increases the stability of a stand, but a risk of reduced timber quality is present, especially in highly understocked pine stands. Intensified growth would also increase the price premium and more intense thinnings in young stands could become optimal from an economic viewpoint.

The preceding discussion reveals the need for the further refinement of the model to more accurately emulate biological regularities of stand development. The economic assumptions also require further examination, first of all the choice of interest rates should be justified. However, the dynamic model provides valuable insights, how economic optimisation might impact silvicultural regimes, thus giving a starting point for economic modelling of stand treatments. An appealing feature of the DP model is its extensibility to a broader set of forest management decisions (Brodie and Haight 1985), for example, the initial planting density or fertilization. Production functions of non-timber forest outputs, such as mushrooms, berries, and resin can be incorporated into objective functions.

The high sensitivity of thinning regimes on changes in timber prices calls for more flexibility in forest management and planning. Forest privatization will inevitably lead to market oriented decisions of forest landowners. It will hardly become possible to force owners to strictly manage their land according to various rules and regulations. Market oriented decisions are induced on market based incentives. It is the right time now to start making efforts for creating demand on small sized timber in the Lithuanian market. Otherwise, a high share of overstocked, depressed young stands might become a severe issue in Lithuania, as has already happened, for example, in Germany.

The most significant finding of this study is that, regardless of the chosen optimisation criterion, the minimum allowable rotation age for pine stands should be differentiated according to the site productivity. The dynamic simulation discloses an insignificant dependence of rotation age on thinning regimes, when a certain interest rate is chosen. This study confirms the appropriateness of the rotation ages obtained using the static model (Brukas and Brodie 1999). Adoption of the proposed rotation ages would incite significant savings without any additional investments. The net benefits on productive sites would amount up to 3000 Lt/ha throughout the rotation.

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