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Estimating the accuracy of site index curves by means of simulation

Skattning av noggrannheten vid höjdbonitering med hjälp av simulering

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Abstract

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The aim of this study is to develop a method for estimating the accuracy of site index curves which is independent of the method used for the construction of the curves. Site index curves for Scots pine in Sweden are used to illustrate the method. The investigation deals mainly with prediction errors, i.e. errors caused by actual height development not following the site index curves. Permanent plot data is used to construct a stochastic, autoregressive simulation model, by means of which it is possible to study the variation of site index over age within stands over long periods. A number of simulations are performed and used for estimating prediction errors. Earlier investigations provide figures on sampling and measurement errors which, together with estimated prediction errors, are used to obtain recommendations on the number of plots required within a stand to reach a predefined level of accuracy at site index estimation.

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1 Introduction

Site index curves, describing the development of dominant height over age, are widely used for site classification. However, the accuracy of these curves is not always estimated in a reliable way. The reason for this is that the curves are often constructed using methods (e.g. graphical fitting) which do not allow estimation of accuracy by straightforward statistical methods.

The aim of this study is to develop a method for estimating the accuracy of site index curves, which is independent of the method used for constructing the curves. The study was undertaken because there was an urgent need to estimate the accuracy of recently constructed Swedish site index curves (Hägglund, 1972, 1973, 1974). One of these sets of curves (those for Scots pine) is used to illustrate the proposed method.

If unrestricted random sampling of plots within a stand is assumed the accuracy of a site index estimate can, expressed as variance, be written as follows.

$$S_{si}^2 = S_p^2 + (S_m^2 + S_s^2)/n$$

where

S_{si}^2 is the total variance of site index within a stand

S_p^2 is the variance caused by "prediction" errors

S_m^2 is the variance caused by measurement errors

S_s^2 is the variance caused by sampling errors

n is the number of sampling units within the stand.

Prediction errors might be caused by either bias in the site index curves or by abnormal weather conditions, diseases, vertical variations in soil fertility and other influences which make the actual height development differ from the site index curves.

In a stand $(S_m^2 + S_s^2)/n$ can be brought near 0 by means of a large number of sampling units, covering different parts of the stand, different operators, different altimeters and so on. S_m and S_s have been investigated in some earlier studies, referred to later.

The prediction errors, S_p, are independent of the number of sampling units within the stand as long as all measurements are made at one occasion. In the investigation carried out here, only prediction errors are studied. Results from the earlier investigations mentioned are used to obtain a more complete picture of the accuracy of site index estimation.

2 Definitions

The definitions used in this study are those used for constructing the Swedish site index curves. These curves describe the development of dominant height over age at breast height. Dominant height is defined as the arithmetic mean height of the 100 largest (by diameter) trees per hectare. The curves are attached to site indices, defined as the dominant height at a total age of 100 years, h_{100}.

In stand number i at age t years, an "observation" of h_{100} made without sampling and measurement errors is called h_{100} (i, t). This observation is obtained by means of the site index curves under investigation. Then, the "true site index" h_{100} (i), for stand i is defined as

$$h_{100}(i) = 1/(t_2 - t_1) \int_{t_1}^{t_2} h_{100}(i, t)\,dt$$

where t_1 is a low age, and t_2 is well beyond the normal rotation. In practice, an estimate of $h_{100}(i)$ is obtained by averaging a sufficiently large number of estimates of $h_{100}(i, t)$ performed at uniformly distributed ages during the whole rotation. The prediction error is defined as

$$S_p^2(i, t) = E((h_{100}(i, t) - h_{100}(i))^2)$$

where $h_{100}(i, t)$ is observed without sampling and measurement errors. The symbol E is for expectation. If h_{100} (i, t) is estimated by $\hat{h}_{100}(i, t)$, the sampling and measurement errors are

$$(S_m^2(i, t) + S_s^2(i, t))/n = \\ = E((\hat{h}_{100}(i, t) - h_{100}(i, t))^2)$$

The definitions are examples and not prerequisites for the method of estimating accuracy. For instance many authors (see e.g. Heger, 1973) define "true site index" as the actual height a stand reaches at a fixed age. This definition may be applied to the method proposed here.

3 Method

A brief description of the method for estimating accuracy follows below. The basic idea is that "true site index" and the variance of site index estimates made at different ages can easily be estimated from permanent plot data where the stand on each plot has been observed during the whole rotation. Observation series of such a length are rare, so shorter observation series are used to simulate series of the desired length. To use this method data from permanent plots consisting of estimates of site index, made in a way that is representative for the normal way of using the site index curves under investigation, must be available. From each plot at least three estimates of site index, made at different ages of the stand, are necessary. The total amount of data must completely cover the range of ages for which estimates of accuracy are of interest. This age range must be so wide that reliable estimates of "true site index" can be obtained from the simulated observation series.

The first step in processing the data is to compute estimated h_{100} at equidistant ages $t_1, t_2, \ldots\ldots, t_j, \ldots. t_m$ for each plot. The interval between t_j and t_{j+1} is of such a length that it is reasonable to assume that the variance about the "true site index" is stable within intervals. The plots or observation series are denoted $1, 2, \ldots., i, \ldots k$. It is now assumed that $h_{100}(i, t_j)$ can be described with the following model

$$h_{100}(i, t_j) = \beta_1(t_j) h_{100}(i, t_{j-1}) + \\ + \beta_2(t_j) h_{100}(i, t_{j-2}) + \varepsilon(i, t_j) \qquad (1)$$

β_1 and β_2 are constants, ε the "error"—the stochastic component. The mean of ε is 0, the variance is assumed to be constant within age intervals. The correlation between $\varepsilon(i, t_j)$ and $\varepsilon(i, t_{j+l})$ where l is an integer is assumed to be 0. The model (1) can be rewritten as follows

$$h_{100}(i, t_j) = h_{100}(i, t_{j-1}) + (\beta_1(t_j) + \\ + \beta_2(t_j) - 1) h_{100}(i, t_{j-1}) - \qquad (2) \\ - \beta_2(t_j)\,(h_{100}(i, t_{j-1}) - h_{100}(i, t_{j-2})) + \varepsilon(i, t_j)$$

Thus if β_1 is 1 and β_2 is 0, the process under study is stationary in the mean with independent increments. If $(\beta_1 + \beta_2)$ is 1 and β_2 is not 0, the increments are autocorrelated. If $(\beta_1 + \beta_2)$ is not 1, the process involves some trend in h_{100} over age. Formally (1) is a second-order autoregressive model. See for instance Anderson, 1971.

Assume that $(\beta_1 + \beta_2)$ is 1. Then the correlation between successive increments is positive if β_2 is negative and vice versa.

By means of regression analysis, β_1, β_2 and the standard error of ε are estimated as b_1, b_2 and the standard error of e. Then (1) is used to simulate successive estimates of h_{100}. The distribution of e is investigated, and the result of the investigation is used at simulation. By means of the simulation, long "observation series" of h_{100} are obtained. For each simulated series, "true site index" is estimated as

$$\hat{h}_{100}(i) = (1/m) \sum_{j=1}^{m} (h_{100}(i, t_j)) \qquad (3)$$

The accuracy in prediction of h_{100} at age t_j can now be estimated as

$$S_p(t_j) = \sqrt{(1/k) \sum_{i=1}^{k} (h_{100}(i, t_j) - \hat{h}_{100}(i))^2} \qquad (4)$$

Formula 4 gives the information wanted in this investigation. Some information about the nature of S_p—which part is due to "lack of fit" of the site index curves and which is "pure error"—can be obtained from the magnitudes of the constants b. However, in this case, the site index curves are previously tested for "lack of fit" (Hägglund, 1974). The results of these tests were such that the curves are assumed to be unbiased. The magnitude of S_p is therefore assumed to be entirely dependent on "pure error".

4 Site index curves used in the study

The set of site index curves for Scots pine in Sweden, the accuracy of which is estimated here, is reported in Hägglund, 1974. The curves describe the development of dominant height over age at breast height. To construct the curves stem analysis data from felled trees was used. The trees originate from 213 temporary sample plots. At the time of felling the trees were among the 100 largest per ha. However, this does not guarantee that the felled trees had been dominants during the whole life of the stand. This question was investigated with data from permanent sample plots. The heights of single trees, dominants at some occasion, were followed backwards in time and compared with dominant height. In this way a "rank effect" was detected and quantified with functions. The stem analysis data from single trees was corrected to dominant height development with these functions. A model was fitted to the corrected data set. The model (originating from "Chapman-Richards model"—see e.g. Richards, 1959) can be written as follows.

$$
\begin{aligned}
h(i, t_j) - 13 &= A(i) \cdot (1 - \exp(-c_1(i) \cdot \\
&\quad \cdot t_j(i)))^{1/(1-c_2(i))} \\
c_1(i) &= b_{10} + b_{11} \cdot A(i)^{b_{12}} \\
c_2(i) &= b_{20} + b_{21} \cdot A(i)^{b_{22}}
\end{aligned}
\tag{5}
$$

h is dominant height in dm for stand i at age t_j. A,b,c are constants to be estimated. Notice that every stand must have its own, unique A constant. The model (5) was fitted to data according to the least squares principle by means of a three-step process, involving iterative non-linear regression (Gauss-Newton method—see Hartley, 1961) and linear multiple regression. The following estimates of b were obtained

$b_{10} = 1.0002\ 10^{-4}$	$b_{20} = 6.6074\ 10^{-2}$
$b_{11} = 9.5953\ 10^{-6}$	$b_{21} = 4.4189\ 10^{5}$
$b_{12} = 1.3755$	$b_{23} = -2.9134$

The function is corrected for logarithmic bias by multiplying with 1.0075.

To relate (5) to h_{100}, the relationship between total age and age at breast height must be known. When this is the case, (5) can be used to estimate h_{100}. This can be done for example with graphical site index curves (figure 1) or with a simple step halving algorithm, programmed for computer.

The method for the construction of site index curves, briefly described above, is more thoroughly reported in Hägglund, 1972. It is evident that this method does not permit any straightforward estimating of accuracy. The reasons for this are for example that

— the errors of the observations of age and height from stem analysis are in this case dependent within trees. The reason for this is that the borings were made at points on the stems which were defined as percentages of total height. An error in the measurement of total height will therefore affect all the estimates of heights of boring points.
— the corrections for "rank effects" will probably affect accuracy.
— the fitting of the model (5) to data is complicated. From a statistical viewpoint objections can be made to the method.

One way to estimate the accuracy of the site index curves described is to use the "simulation method".

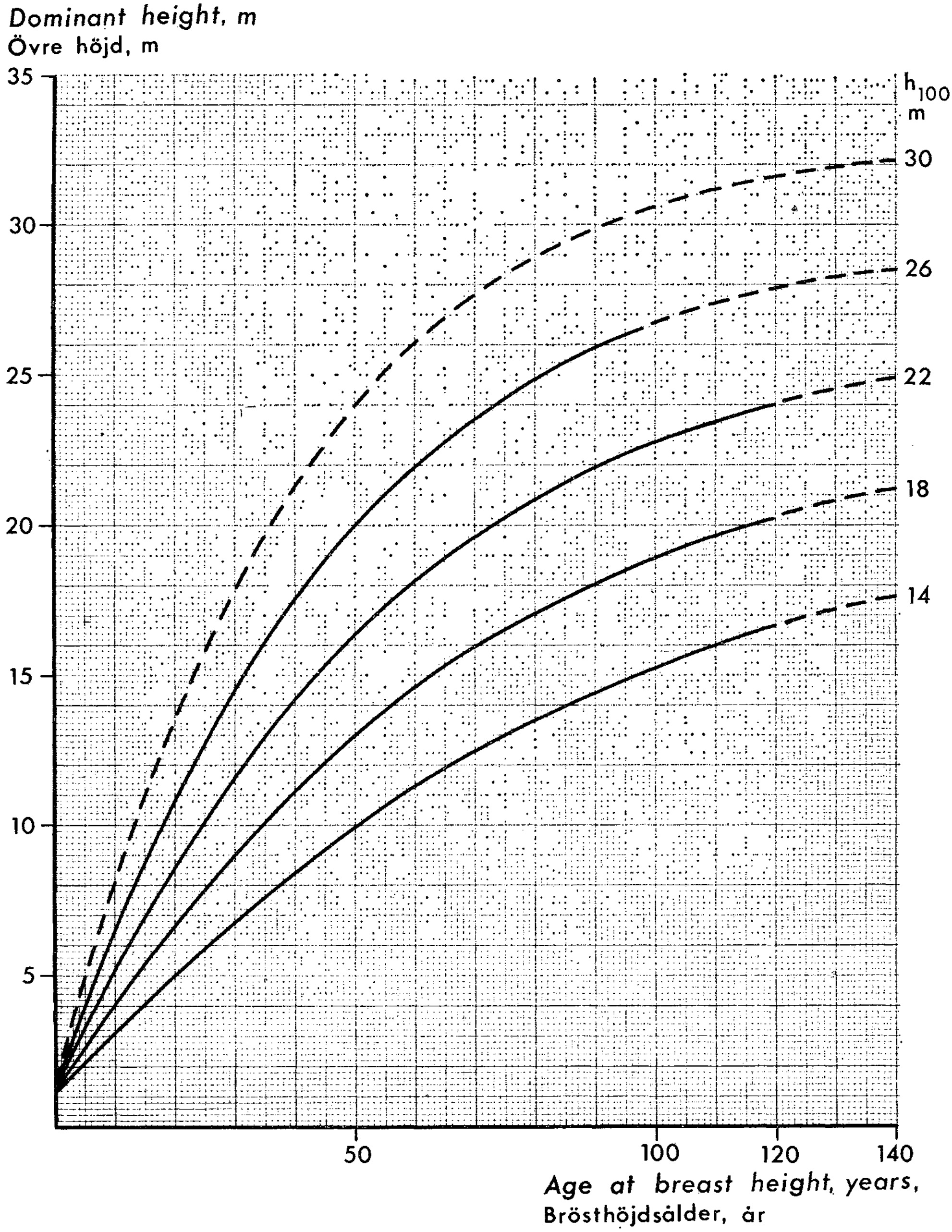


Figure 1. Site index curves for Scots pine in Sweden.

Höjdutvecklingskurvor för tall i Sverige.

5 Data

5.1 Description of data

The data for the study consists of observation series of total age and dominant height from 203 permanent sample plots. The data was transformed to age at breast height and h_{100} by means of function (5) and a relationship between total age and age at breast height. These transformed data are illustrated in figure 2. In the figure observation series from plots located in the same stand have been averaged. This averaging of data is only used to make figure 2 clearer. At the further processing no averaging is done.

The permanent plots are 0.1 ha on average. Thus, the dominant height (see "Definitions") is determined from 10 trees. The height measurements are normally performed with the Tirén altimeter. According to Eriksson, 1970, this instrument gives a standard error at single tree measurement of 1.1 % of tree height. This means that the standard error of the dominant height measurement is around 0.35 %. These figures only include random errors. In this investigation other measurement errors cannot be separated from prediction errors.

The plots used are in many respects more homogeneous than average Swedish pine stands. The soil fertility is uniform within plots, the stands are even-aged and of one species, the frequency of diseases is low etc. This might cause differences in site index prediction error between the data set used and "normal" Swedish pine stands. The applicability of the quantitative results from the investigation must therefore be limited to stands with the same properties as the permanent plots.

5.2 Processing of data

The first step at the data processing was to compute $h_{100}(i, t_j)$ at ages 5, 10, 15, ..., $5 \cdot m$ years by linear interpolation. The lowest age used here was 15 years, the highest 165 years. According to model (1) a complete "case" in the regression analysis should contain observations from ages t_j (dependent variable), t_{j-1} and t_{j-2}. However, when $h_{100}(i, 20)$ is the dependent variable, there are no observations of $h_{100}(i, 10)$. In this and only in this case the term $h_{100}(i, t_{j-2})$ is omitted from the model.

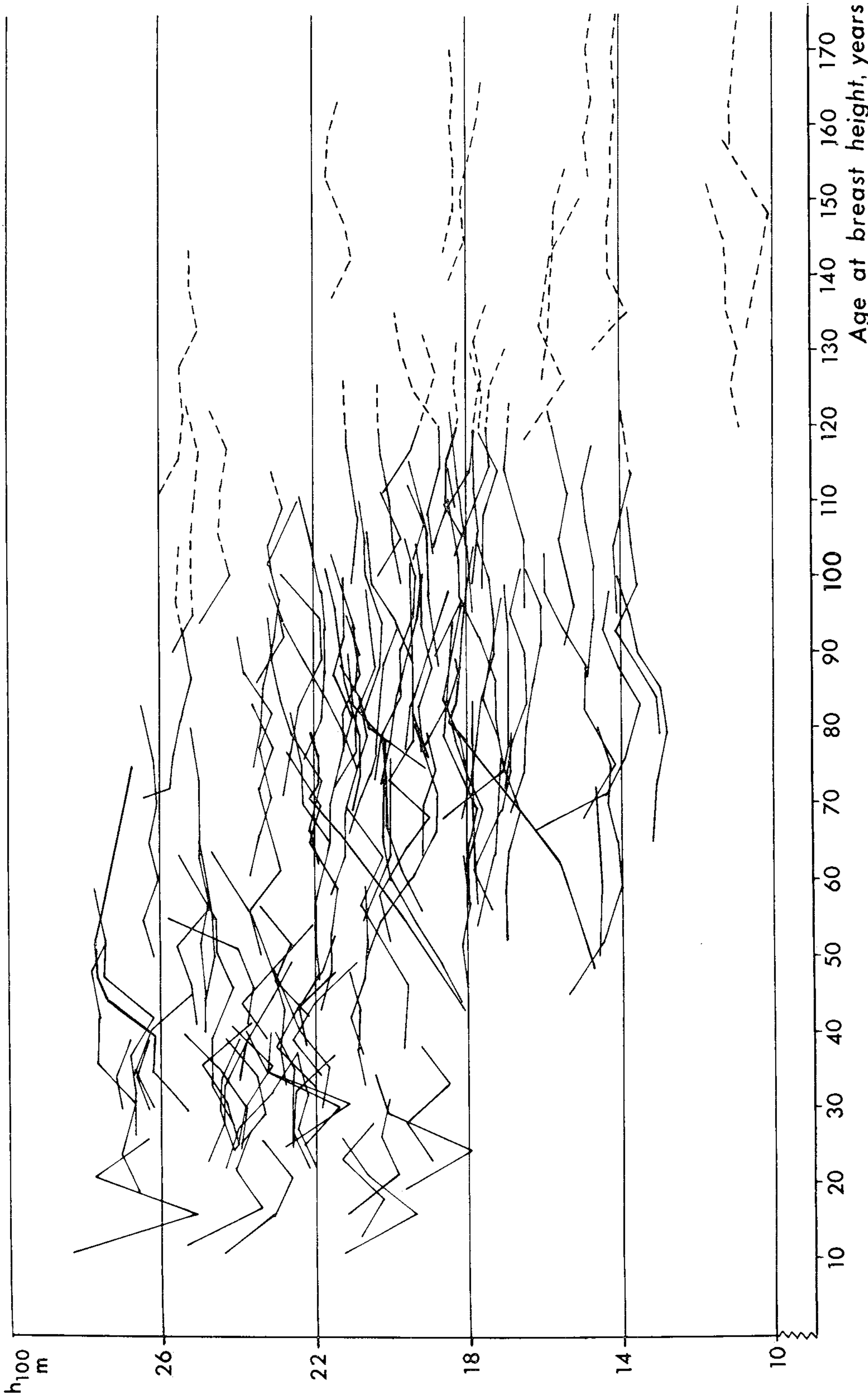
Some observation series, which were not long enough to produce a complete case, were omitted from the data set. The total number of cases used in regression analysis is 679. The average number of cases per age is 23, the minimum number 5 and the maximum number 47.

Model (1) was fitted to data for each age separately. The distributions of the residuals from the functions were tested for normality with χ^2-tests. The correlations between the residuals $e(i, t_j)$, $e(i, t_{j-1})$, $e(i, t_{j-2})$ and $e(i, t_{j-3})$ were studied. The number of observations, the regression coefficients, the standard errors of $h_{100}(i, t_j)$ and the χ^2-tests are reported for each age in Appendix 1.

For all ages the sum of the estimated regression coefficients b_1 and b_2 is close to 1. As mentioned earlier (see "Method") a deviation of $(b_1 + b_2)$ from 1 indicates some trend in

Figure 2. Data for the study. Observations of h_{100} from 203 permanent sample plots, some of them averaged in this figure.

Material till undersökningen. Observationer av h_{100} från 203 permanenta försöksytor. Observationsserier från näraliggande ytor har sammanslagits till medelförlopp i figuren.



h_{100} m
26
22
18
14
10
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
Age at breast height, years
Brösthöjdsålder, år

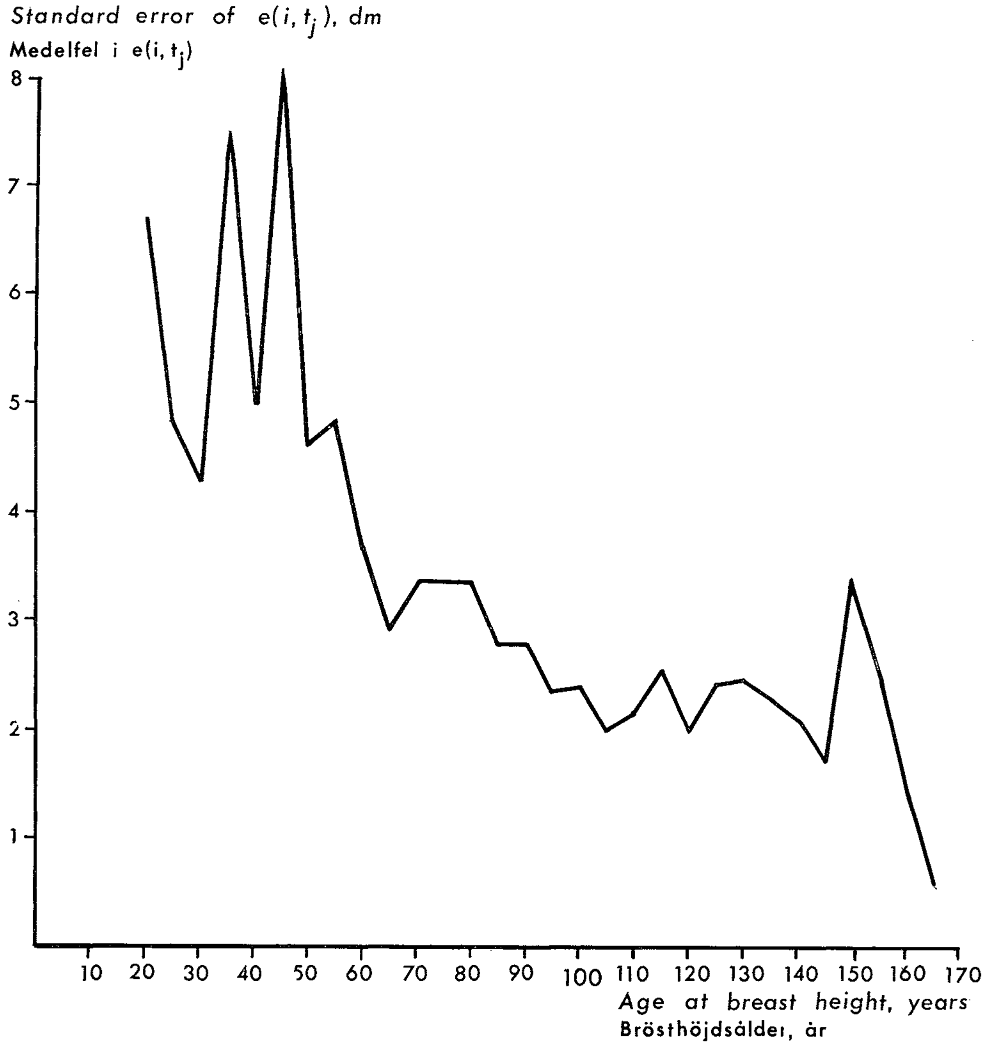


Figure 3. Standard error of e(i, t_j), estimated by regression functions.
Medelfelet i e(i, t_j), skattat med regressionsanalys.

h_{100} over age. As reported in Appendix 1 ($b_1 + b_2$) is somewhat greater than 1 for all intervals up to age 115 years. In a regression performed for the whole material ($b_1 + b_2$) is 1.00436, with a standard error of 0.0075. The deviation of ($b_1 + b_2$) from 1 is not significant, but a suspicion of a slight trend in h_{100} over age still remains. The character of the trend is such that h_{100} increases with age. The reason for this is probably that three plots included in the data (and specially reported in Hägglund, 1974) were extremely overdense at low ages. This resulted in a depressed dominant height growth. After thinning dominant height growth increased to a level "normal" for the site and age, which resulted in increasing h_{100}. The trend possibly brought into the data with these plots will, however, be included in the random variation when calculating prediction errors.

For most ages b_1 is positive and b_2 is negative. Hence, the correlation between successive increments is mostly positive. A minor part of this autocorrelation is probably caused by the linear interpolation used at estimating h_{100} for different ages.

The estimates of the standard errors of e(i, t_j) over age are illustrated in figure 3.

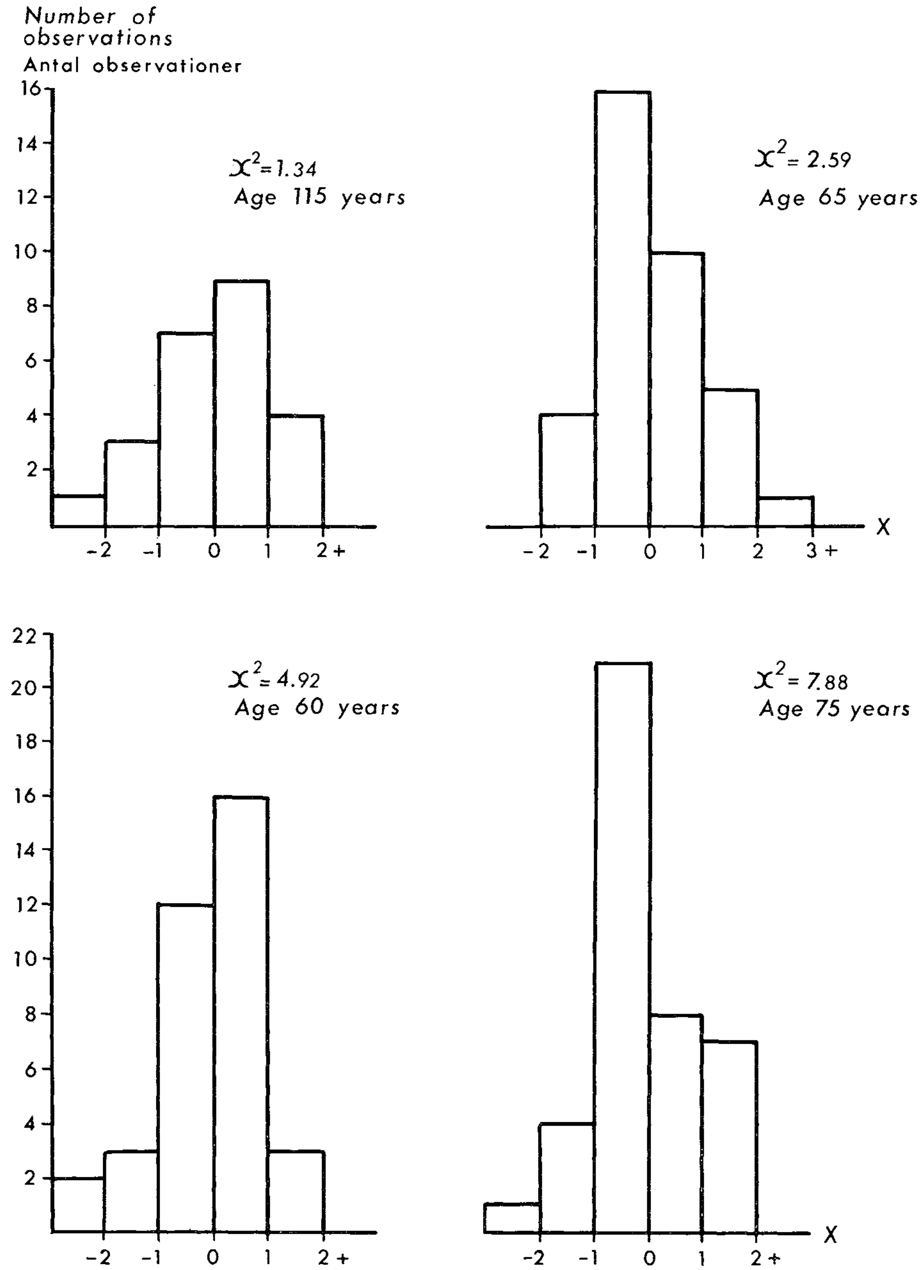


Figure 4. χ^2-tests, some examples. x = residual/standard deviation.
χ^2-tester, några exempel. x = residual/standardavvikelse.

As expected, estimated standard errors decrease over age. Probably because of the low number of observations, the standard error is very unstable for low ages.

The χ^2-tests were based on a division of residuals by normalized size (residual/standard deviation) in six classes. Thus, the number of degrees of freedom is 3. Significant deviations from normal distribution on the 5 %-level are indicated by a χ^2 greater than 7.81. For only one age was a χ^2 of that magnitude observed (age 75, $\chi^2 = 7.88$). Some examples of the shape of the distributions are illustrated in figure 4. However the amount of data for the

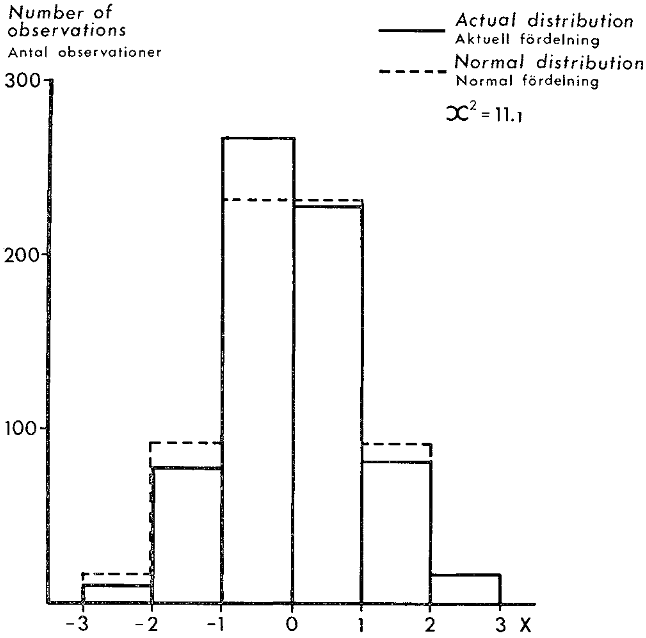


Figure 5. χ^2-test, whole material. x = residual/standard deviation

χ^2-test, hela materialet. x = residual/standardavvikelse.

χ^2-tests is small. If a χ^2-test is performed for the whole material, a significant deviation (5 %-level) from the normal distribution is obtained. This deviation is judged as being too small (see figure 5) to be taken into account. At simulation, it is assumed that the residuals $e(i, t_j)$ are normally distributed.

The correlations between the residuals $e(i, t_j)$ and respectively $e(i, t_{j-1})$, $e(i, t_{j-2})$ and $e(i, t_{j-3})$ were investigated in order to check the important assumption of no correlation between residuals. The correlations were respectively 0.02, −0.05 and −0.04, with non-significant t-values of 0.4, −0.6 and −0.3. Thus, there is no reason to suspect correlation between residuals.

6 Simulation of successive site index estimates

The regression functions reported in appendix 1 are used to simulate "observation" series of successive estimates of h_{100}. The simulation of a series starts by arbitrary chosing a value of h_{100} for age 15. Here the values chosen are 14, 16, 18, ... 28 m. After that, the first regression function is used to calculate a new h_{100} for age 20. A random deviation $e(i, t_j)$ is added to the estimate from the regression function. This deviation is calculated as a random number, picked from a normal distribution with mean 0 and a standard deviation in accordance with the standard error calculated for the regression function. Technically, this operation is performed by means of the FORTRAN IV function subroutine RNORM, developed at the Stockholms Datamaskincentral.

The $h_{100}(i, t_j)$ obtained at ages 15 and 20 are used as independent variables in the second regression function to calculate $h_{100}(i, t_j)$ at age 25 and so on. In this way observation series with a range from 15 to 165 years are calculated. For an IBM 360/75 computer the CPU-time needed to calculate one such observation series is about 10^{-4} minutes. Some simulated series are illustrated in figure 6. In appendix 2, 128 simulated series are reported. There is a tendency of rising h_{100} with age in these series. The reasons for this have been discussed earlier.

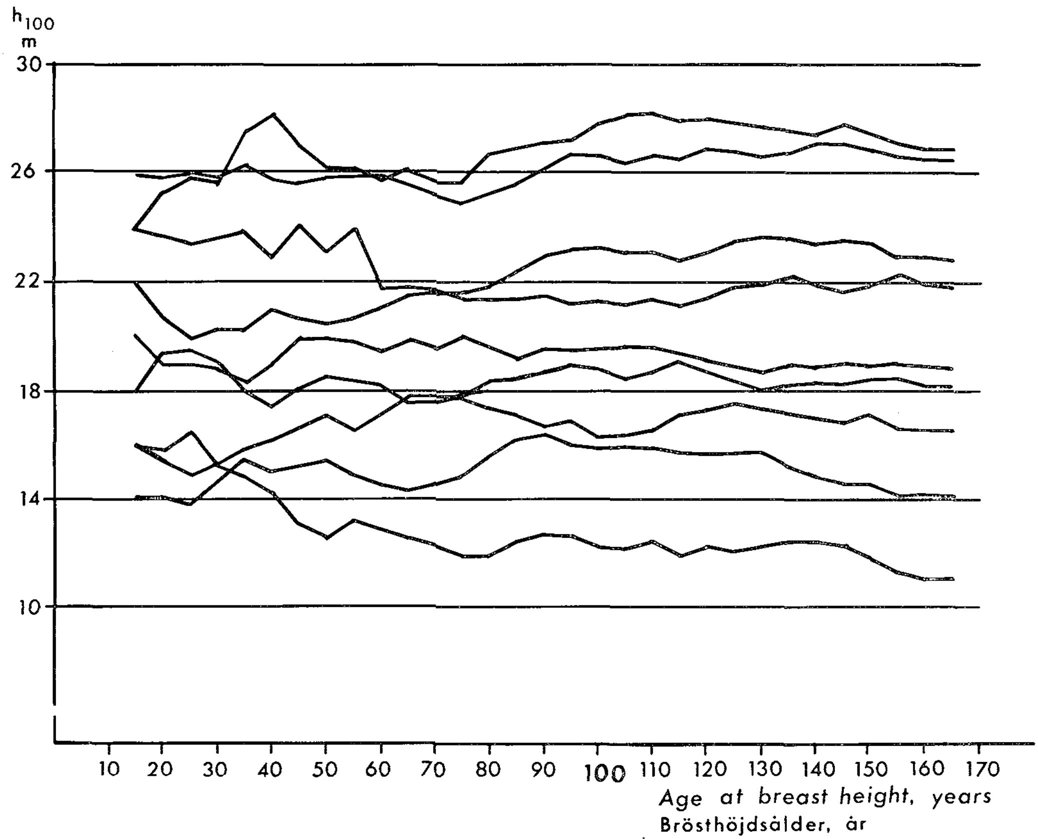


Figure 6. Some examples of simulated observation series.

Några exempel på simulerade observationsserier.

7 Estimating accuracy

From the simulated observation series, $S_p(t_j)$ were calculated by formula (4)

$$S_p(t_j) = \sqrt{(1/k) \sum_{i=1}^{k} (h_{100}(i, t_j) - \hat{h}_{100}(i))^2} \qquad (4)$$

$\hat{h}_{100}(i)$ is an estimate of the "true site index" for observation series no i. $S_p^2(t_j)$ might be divided in two parts, corresponding to "lack of fit" due to the site index curves and "pure error". To do this, the shape of some trend f(t) describing the bias of the site index curves must be known. In principle the b-coefficients can also be used to estimate bias. However, the data set must be large, if reliable estimates of bias are to be calculated from separate five-year intervals. Otherwise there is an evident risk of confusing bias of the site index curves with errors occurring from the fact that the data set is a sample. For example, in the present investigation the data set includes too large a proportion of stands with too slow early dominant height development. To ensure that no depreciation of S_p is done, these stands are included in the data set. Their variance must however be treated as random.

By performing a number of simulations, $S_p(t_j)$ is estimated for each age interval. The simulations start at age 15 and end at age 165. At the start, h_{100} is 14, 16, 18, ..., 28 m. The whole period from age 15 to age 165 is used for calculating "true h_{100}" by averaging the 31 values per series corresponding to each 5-year interval. It seems reasonable to found the calculation of "true h_{100}" on as long a series as possible, since h_{100} is more stable at higher ages. In the present case, the calculation of "true h_{100}" is based to some extent on extrapolated parts of the site index curves. According to figure 2, this extrapolation does not introduce any bias in the calculations.

At the calculation of $S_p(t_j)$, all observation series have the same weight. An alternative way of weighting is to give every series i a weight proportional to the frequency of $h_{100}(i)$ in the data set. Weighting can also be based on the frequency of $h_{100}(i)$ in the population where the results shall be applied.

Figure 7 shows an example of the behaviour of $S_p(t_j)$ for three ages (25, 50 and 100 years) when the number of simulations increases. In the example $S_p(t_j)$ seems to stabilize after 256 simulations. Here the values of $S_p(t_j)$ obtained after 8192 simulations are accepted as final. Figure 7 also shows the variation between four sets of 8192 simulations. The difference between the largest and the smallest value of $S_p(t_j)$ is 3 cm at age 25, 1.5 cm at age 50 and 0.5 cm at age 100.

As mentioned earlier, some measurement errors of a magnitude of 0.35 % of dominant height are included in the simulated series. These errors are, expressed as h_{100}, around 0.6 dm. The values of $S_p(t_j)$ obtained by simulation are corrected for these errors. Such corrected $S_p(t_j)$ are illustrated over age in figure 8 and table 1.

The curve in figure 8 has some irregularities. An attempt to smooth these out with a six-order polynomial did not give acceptable results. Therefore, the "raw" values are reported as the main results of the investigations. As can be seen, the prediction error S_p is at the level 20 dm at age 15—20 years. Then the error drops quickly with increasing age. A minimum of around 7 dm is reached at age 80—85. After that, the error increases slowly with increasing age and is around 11 dm at age 155—165.

The relationship between S_p and age applies to average conditions in the data. No attempt was made to differentiate S_p for different h_{100}. Evidently the shape of the relationship depends to some extent on the age interval on which the calculation of "true h_{100}" is based. The minimum of S_p will always occur close to the middle of that interval.

As mentioned earlier, the total error of a

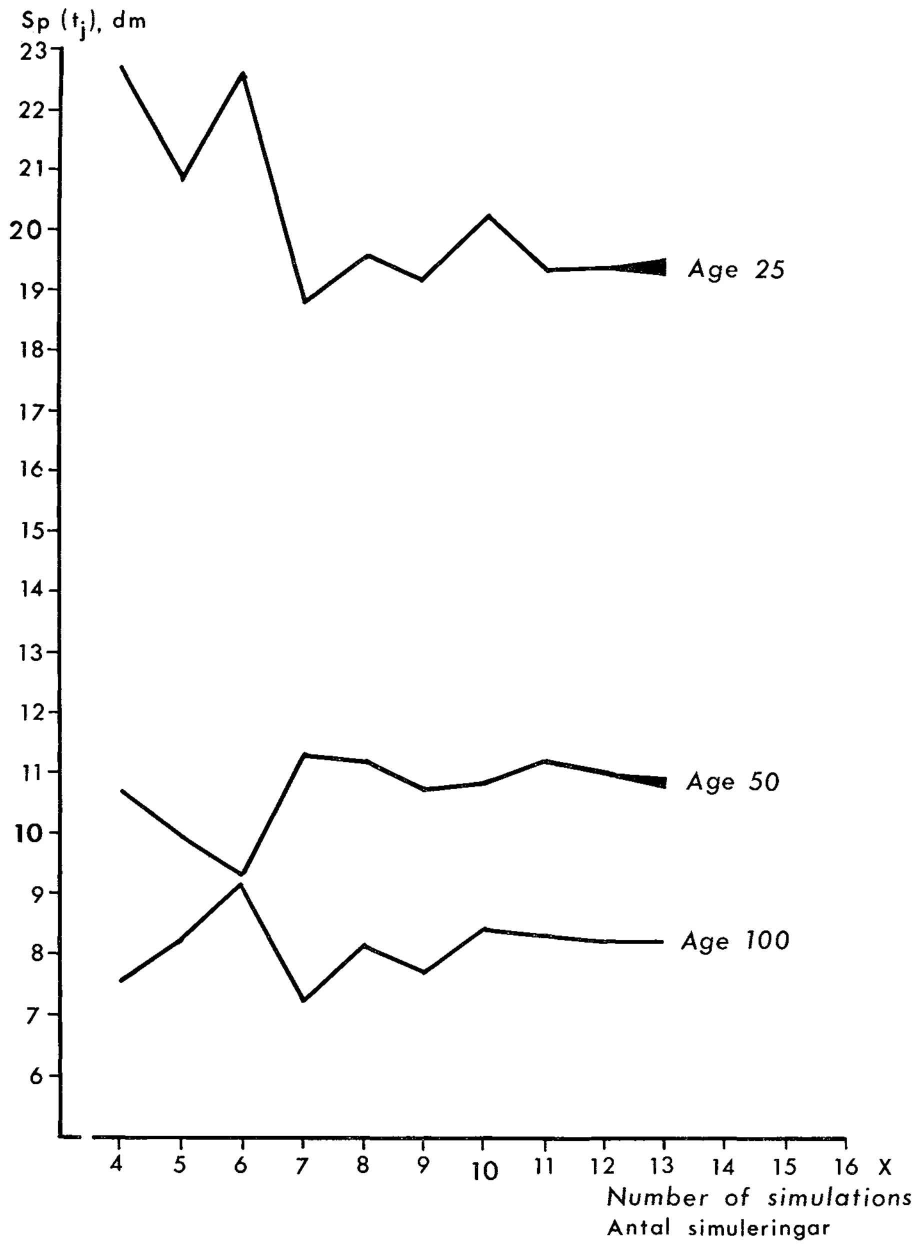


Figure 7. Prediction error $S_p(t_j)$ of h_{100} over number of simulated observation series. $S_p(t_j)$ is not corrected for measurement errors. The number of simulations is expressed as 2^x. When for example $x=4$, the number of simulations is 16, when x is 13 the number of simulations is 8192. For $x=13$, four sets of simulations are illustrated.

Prognosfelet $S_p(t_j)$ i h_{100} över antalet simuleringar. $S_p(t_j)$ är inte korrigerat för mätfel. Antalet simuleringar uttrycks som 2^x. När t ex x är 4 är antalet simuleringar 16. När x är 13 är antalet simuleringar 8192. För $x=13$ visas resultaten av fyra uppsättningar simuleringar.

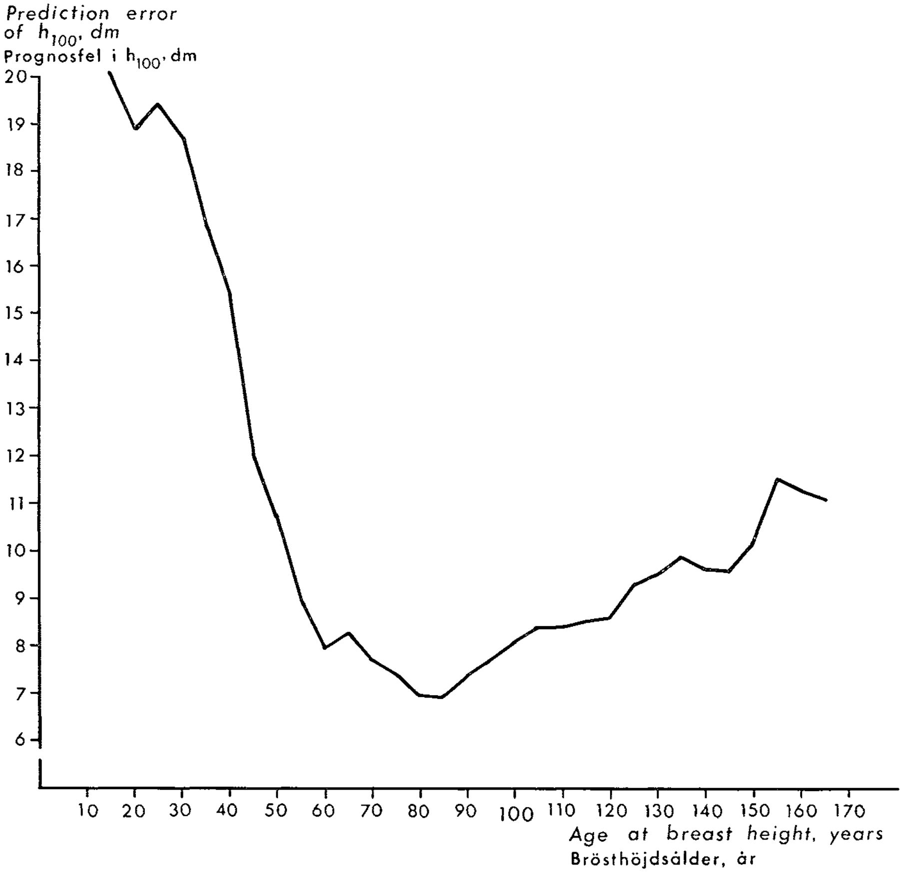


Figure 8. Prediction error of h_{100} over age at breast height. The figures are based on 8192 simulations and corrected for measurement errors.

Prognosfel i h_{100} över brösthöjdsålder. Siffrorna bygger på 8192 simuleringar och är korrigerade för mätfel.

site index estimate for a stand is obtained from the formula

$$S_{si}^2 = S_p^2 + (S_m^2 + S_s^2)/n \qquad (6)$$

where p, m and s are for "prediction", "measurement" and "sampling" respectively. The number of sampling units within the stand is called n. Fries, 1974 reported some results concerning the magnitude of $S_m^2 + S_s^2$. Some of these are found in table 2 below.

The necessary number of plots within a stand to reach some predefined level of accuracy can be calculated by means of these results and figure 8. Let this level be a standard error S_{si} of magnitude E dm. Then the number of plots required (n_E) is obtained from formula (7) below.

$$n_E = (S_m^2 + S_s^2)/(E^2 - S_p^2) \qquad (7)$$

This formula is exemplified in figure 9. The example concerns a stand with "medium" site variation within stand. The plot size chosen is 254 m^2 and some levels of accuracy, E, from 10 to 20 dm are investigated. As can be seen the number of plots required decreases dramatically when E increases from 10 to 14 dm. It would here seem reasonable to accept an E of 14 dm in most cases. In stands younger than 45 years, a higher E must be accepted.

Tabulated values of n_E for various combinations of E, plot size and site variation within stand are found in Appendix 3. These figures might be useful at practical site index estimation. However, their application is limited to even-aged, pure stands of Scots pine in Sweden.

Table 1. Prediction error of h_{100} at various ages at breast height. The figures are based on 8192 simulations and corrected for measurement errors.

Prognosfel i h_{100} vid olika brösthöjdsåldrar. Siffrorna bygger på 8 192 simuleringar och är korrigerade för mätfel.

Age at breast height *Brösthöjdsålder*	Prediction error of h_{100}, dm *Prognosfel i h_{100}, dm*	Age at breast height *Brösthöjdsålder*	Prediction error of h_{100}, dm *Prognosfel i h_{100}, dm*
15	20.1	95	7.7
20	18.8	100	8.1
25	19.4	105	8.4
30	18.7	110	8.4
35	16.8	115	8.5
40	15.4	120	8.6
45	12.0	125	9.3
50	10.8	130	9.5
55	9.0	135	9.9
60	8.0	140	9.6
65	8.3	145	9.6
70	7.7	150	10.2
75	7.4	155	11.5
80	6.9	160	11.3
85	6.9	165	11.1
90	7.3		

Table 2. Some examples of the magnitude of $S_m^2 + S_s^2$ according to Fries, 1974.

Några exempel på storleken av $S_m^2 + S_s^2$ enligt Fries, 1974.

Plot size m² *Ytstorlek m²*	Number of dominant trees per plot used for site estimation *Antal övrehöjdsträd på provytan*	$S_m^2 + S_s^2$, dm — Site variation within stand *Bonitetsvariation inom bestånd*: low *liten*	medium *måttlig*	high *stor*
1 000	10	196	400	676
254	2	324	576	1 024
154	1	400	729	1 225

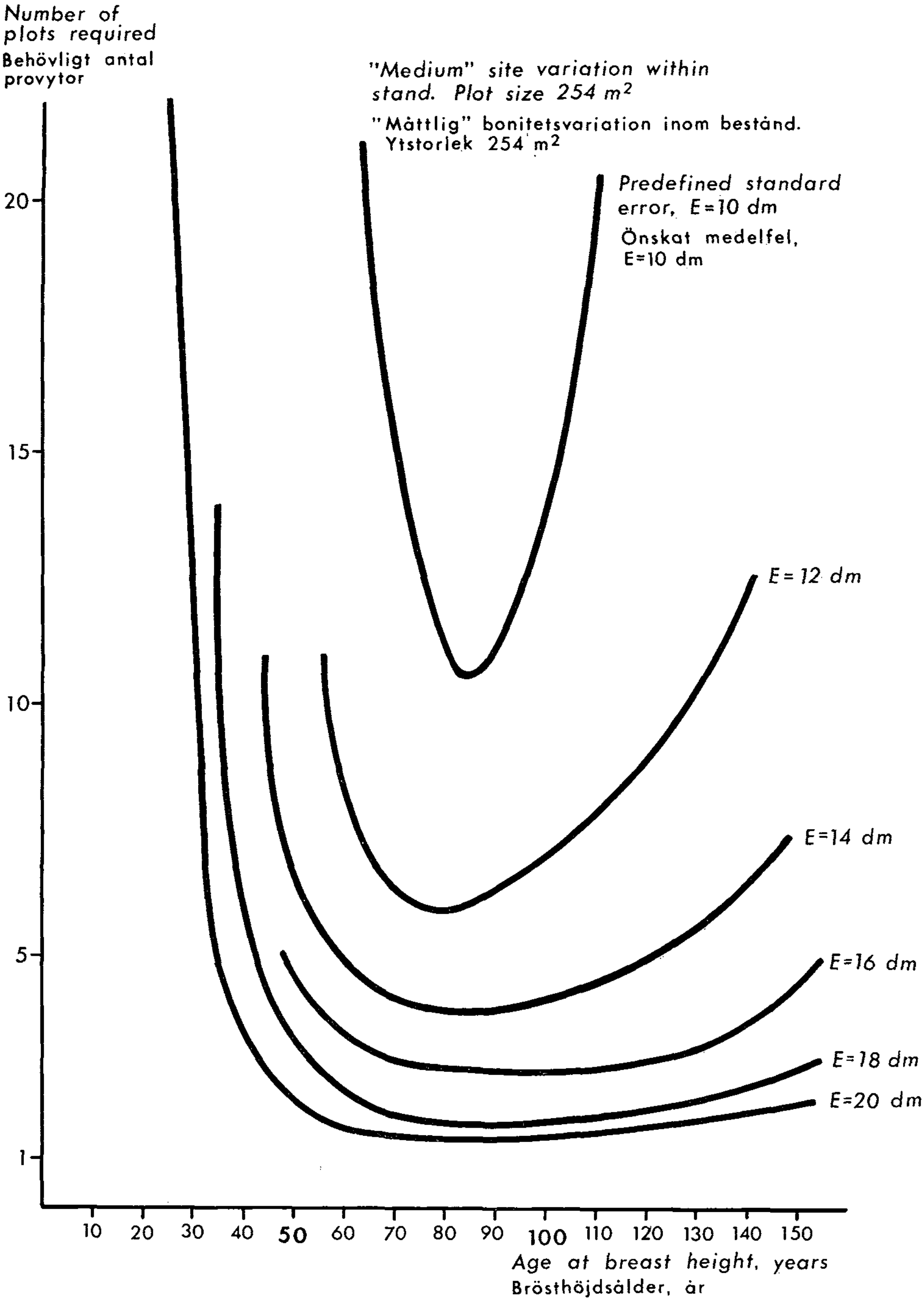


Figure 9. Number of plots within stand required to reach a predefined standard error.

Behövligt antal provytor inom bestånd för att nå ett förutbestämt medelfel.

8 Discussion

Many questions can be discussed in connection with this study. Here however, we limit ourselves to four.

- the simulation model
- division of data by other variables than age
- when should the simulation method be used?
- other applications of simulation in site estimation

For the convenience of the reader, the model (1) is repeated here

$$h_{100}(i, t_j) = \beta_1(t_j) h_{100}(i, t_{j-1}) + \\ + \beta_2(t_j) h_{100}(i, t_{j-2}) + \varepsilon(i, t_j) \tag{1}$$

A more sophisticated model might produce a better description of data. In some cases it might be necessary to introduce a third-order autoregressive term, $\beta_3 h_{100}(i, t_{j-3})$, in the model. However, more data is needed because the observation series must be one age interval (in this case five years) longer than when using (1). The model can be made more flexible by bringing terms of higher order than 1 into the model, for instance $[h_{100}(i, t_{j-1})]^2$.

It is obvious that no constant term should be brought into the model. If such a constant is not 0, the functions obtained at regression analysis cannot have the following desirable feature for more than two values of h_{100}: "If $h_{100}(i, t_{j-1}) = h_{100}(i, t_{j-2})$ then the conditional expectation of $h_{100}(i, t_j) = h_{100}(i, t_{j-1})$." An attempt to perform simulation with regression functions involving constant terms has been made. As expected, the simulated observation series got a preposterous shape when approaching the limits of the data set.

In this investigation data has been divided by age. The aim of this division was to form groups (age intervals) with as constant an interior variance (S_p^2) as possible. An alternative way of division is to use the dominant height. However, when prediction errors are searched for, it seems more reasonable to use age than height for division. The prediction error is hardly the same in a 12 m stand of age 18 ($h_{100} = 30$ m) as in a 12 m stand of age 81 ($h_{100} = 12$ m). This holds for prediction errors only. Measurement errors are probably more constant at constant height than at constant age.

Besides the division by age, also some other divisions, for instance by h_{100}, might be done. Such a division should be made if there are reasons to believe that S_p varies with h_{100} and the data set is large enough. This division by h_{100} is probably especially valuable if the simulation model will be used for prediction purposes.

The simulation technique used for estimating prediction errors should of course only be used when more straightforward methods are not applicable. For instance when site index curves are constructed as functions

site index = f(height, age)

they can often be investigated for prediction errors by means of standard error of estimate, obtained at regression analysis. The situation is not quite as simple (Heger, 1971) when site index curves are constructed as functions

height = f(site index, age)

For a discussion of these questions, see Curtis, DeMars & Herman, 1974. Whatever model used, one thing must be pointed out. When calculating S_p from the data used at the construction of the site index curves, there is a risk of underestimating S_p. This is because "overfitting" often occurs. See Gardner, 1972 for a discussion of this question. In the investigation carried out here, the data used for the calculation of S_p is not the same as that used in the construction of the site index curves.

An alternative way of calculating the prediction errors is to omit the simulation and use a more traditional statistical approach by calculating expectations for S_p. This way of solving the present problem seems possible if the regression coefficients and standard errors in Appendix 1 are known. However, the simulation method is more illustrative and much simpler.

The simulation technique described here has been used for estimating prediction errors. It might also be useful for some other purposes. Studying effects of "sampling in time" is closely related to estimating prediction errors. This means using observations from more than one occasion to estimate h_{100}. In practice this can be done by measuring of whorls. The most proper weighting of such observations in order to form an estimate of site index with maximal accuracy can be studied by simulated observation series. Simulation might also be used to develop aids for prediction of h_{100}. Assume that in some data set a trend of site index over age, which can be related to some variable x, exists. For simplicity, let us say that x is altitude. It is then possible to include a term b_3x in model (1). The magnitude of b_3 for different ages can be used to develop different sets of site index curves for different altitudes.

9 Sammanfattning

Syftet med den här genomförda undersökningen är att utveckla en metod för att skatta noggrannheten vid höjdbonitering med höjdutvecklingskurvor, som är oberoende av det sätt på vilket kurvorna konstruerats. Metoden illustreras med hjälp av höjdutvecklingskurvor för tall i Sverige (Hägglund, 1974).

Det totala felet i en skattning av höjdboniteten för ett bestånd kan skrivas som

$$S_{si}^2 = S_p^2 + (S_m^2 + S_s^2)/n$$

där S_p är prognosfelet, S_m mätfelet och S_s samplingfelet. Antalet provytor inom bestånd betecknas n. Med prognosfel menas det fel som uppstår på grund av att beståndets faktiska höjdutveckling mera sällan exakt följer höjdutvecklingskurvorna. Detta kan bero antingen på att kurvorna ej är förväntningsriktiga eller på onormal väderlek, skador, vertikala bördighetsvariationer i markprofilen m fl faktorer. Undersökningen inriktas främst på skattning av prognosfelen, eftersom mät- och samplingfelen studerats i tidigare undersökningar (Eriksson, 1970, Fries, 1974).

Materialet till undersökningen är hämtat från 203 permanenta försöksytor och består av vid olika åldrar upprepade observationer av brösthöjdsålder och övre höjd. Med hjälp av höjdutvecklingskurvorna beräknades h_{100}, övre höjden vid 100 års total ålder, för varje yta och brösthöjdsålder. Erhållna h_{100} omräknades med lineär interpolering till att avse vart femte år under den period respektive yta observerats. Till de sålunda erhållna ”observationsserierna” av successiva skattningar av h_{100} anpassades en andra ordningens autoregressiv modell med följande utseende

$$h_{100}(i, t_j) = \beta_1(t_j) h_{100}(i, t_{j-1}) + \\ + \beta_2(t_j) h_{100}(i, t_{j-2}) + \varepsilon(i, t_j)$$

Beteckningen t_j står för brösthöjdsålder medan index i avser provyta. β_1 och β_2 är konstanter medan ε är ”felet” — den stokastiska komponenten. Modellen anpassades till materialet med regressionsanalys på så sätt att separata funktioner utarbetades för varje ålder t_j. Härigenom erhölls ett antal funktioner där den beroende variabeln $h_{100}(i, t_j)$ svarar mot åldrarna 15, 20, 25, ..., 165 år. Funktionerna användes för att simulera observationsserier som sträcker sig från 15 till 165 års brösthöjdsålder. Härvid antogs att $\varepsilon(i, t_j)$ är oberoende av $\varepsilon(i, t_{j+l})$ där l är ett heltal. Vidare antogs att $\varepsilon(i, t_j)$ har medeltalet 0, är normalfördelat och har en konstant varians inom femårsintervall. Flertalet av dessa antaganden kontrollerades i undersökningen.

För varje simulerad observationsserie skattades det ”sanna” värdet på h_{100} som det aritmetiska medeltalet av de 31 värden på h_{100} som erhölls vid simuleringen. Prognosfelet beräknades för varje ålder som

$$S_p(t_j) = \sqrt{(1/k) \sum_{i=1}^{k} (h_{100}(i, t_j) - \hat{h}_{100}(i))^2}$$

där k är antalet simuleringar och $\hat{h}_{100}(i)$ skattningen av det ”sanna” h_{100} för den i:te simulerade observationsserien. De slutliga beräkningarna av S_p grundades på 8 192 simulerade observationsserier. Resultatet av beräkningarna är att prognosfelet är av storleksordningen 2 m vid brösthöjdsåldern 15—20 år. Därefter sjunker felet snabbt med stigande ålder och når ett minimum på ca 7 dm vid 80—85 år. När åldern ökas ytterligare stiger felet långsamt och är ca 11 dm vid 155—165 år.

Undersökningen avslutas med beräkningar avseende det antal provytor som krävs för att skatta h_{100} för ett bestånd med viss förutbestämd noggrannhet. Härvid användes dels uppgifter rörande storleken av mät- och samplingfel hämtade från Fries, 1974, dels de här gjorda skattningarna av prognosfelet.

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Stockholm, April 1975

Björn Hägglund

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Appendix 1

Some information about the regression functions

$h_{100}(i, t_j) = b_1(t_j)h_{100}(i, t_{j-1}) + b_2(t_j)h_{100}(i, t_{j-2}) + e(i, t_j)$
h_{100}: site index, dm
i: observation series no
t_j: age at breast height
b_1, b_2: estimated coefficients
e: estimated error

Dependent variable h_{100} at age	No of observations	Regression coefficients b_1	b_2	$b_1 + b_2$	Standard error of estimate, dm	Residuals deviation from normal distribution χ^2
20	5	1.00589	—	1.00589	6.7	0.90
25	5	0.94013	0.07116	1.01129	4.8	1.17
30	7	−0.48922	1.49818	1.00896	4.3	2.24
35	23	0.77101	0.24554	1.01655	7.5	1.49
40	25	1.04847	−0.04476	1.00371	5.0	1.83
45	24	1.44649	−0.44001	1.00648	8.1	3.29
50	25	1.29390	−0.29095	1.00295	4.6	6.57
55	30	1.24068	−0.23872	1.00196	4.8	6.45
60	36	1.47007	−0.46845	1.00112	3.7	4.92
65	36	1.57425	−0.56612	1.00813	2.9	2.59
70	39	1.04606	−0.04456	1.00150	3.4	6.14
75	41	1.33604	−0.32697	1.00907	3.4	7.88*
80	38	1.25789	−0.25065	1.00724	3.4	1.40
85	43	1.41382	−0.41154	1.00228	2.8	2.63
90	47	1.58841	−0.58718	1.00123	2.8	2.50
95	44	1.22279	−0.21754	1.00525	2.4	2.92
100	41	1.31390	−0.31076	1.00314	2.4	1.89
105	28	1.13941	−0.13709	1.00232	2.0	3.45
110	31	0.81972	0.18197	1.00169	2.2	4.18
115	24	1.29629	−0.29914	0.99715	2.6	1.34
120	20	1.10471	−0.10166	1.00305	2.0	3.00
125	11	1.55710	−0.55152	1.00558	2.4	5.46
130	9	1.05536	−0.05616	0.99920	2.5	0.98
135	6	1.33444	−0.33349	1.00095	2.3	1.59
140	7	0.61485	0.38056	0.99541	2.1	0.56
145	8	1.26072	−0.26331	0.99741	1.7	3.56
150	10	1.23046	−0.22642	1.00404	3.4	2.21
155	7	1.75400	−0.75305	1.00095	2.5	0.56
160	5	0.94351	0.05001	0.99352	1.4	5.87
165	4	0.87931	0.11572	0.99503	0.6	4.45
All	679	1.01140	−0.00704	1.00436	3.9	11.1*

Appendix 2

Simulated series of successive site index estimates

128 simulated series of successive h_{100}

Age at breast height

15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165
h_{100} in dm																														
140	139	139	141	161	160	158	163	168	172	176	178	180	185	187	187	186	188	189	193	193	195	198	197	194	192	192	194	194	193	192
160	158	156	158	157	161	164	161	156	153	147	147	154	158	160	161	163	161	160	159	160	160	159	156	158	156	153	151	150	147	146
180	178	178	176	185	183	179	183	179	181	189	186	187	182	181	186	187	187	186	187	183	183	187	184	184	188	187	184	182	179	178
200	197	205	197	179	177	181	180	184	190	192	189	189	192	190	197	202	198	195	194	189	188	186	184	181	181	180	176	173	170	169
220	217	220	216	213	202	177	177	180	185	193	195	193	203	207	210	213	220	221	223	221	221	224	224	223	223	220	217	216	214	213
240	239	238	236	250	255	272	281	281	281	286	291	296	305	311	317	322	324	328	328	326	331	335	336	339	337	337	343	344	343	341
260	266	272	262	270	257	247	250	250	251	254	255	266	267	269	266	264	264	268	269	271	273	277	274	275	277	276	277	280	280	278
280	279	293	275	274	272	273	272	265	268	268	266	266	275	277	278	276	280	281	285	284	281	284	286	288	285	285	283	282	279	279
140	130	143	116	127	130	147	151	149	152	155	155	157	166	172	180	184	182	181	176	174	176	178	180	186	181	181	181	181	179	178
160	159	156	161	167	168	163	161	151	150	155	164	167	166	163	162	165	165	164	165	164	166	165	165	168	165	163	160	157	154	153
180	187	188	190	188	185	182	179	190	192	199	193	187	183	183	188	194	195	196	194	194	192	193	194	191	190	191	187	186	183	183
200	203	198	211	211	216	224	220	220	217	219	220	215	216	216	221	223	223	224	229	231	237	239	241	240	239	238	236	239	238	236
220	213	216	206	212	214	205	205	205	205	210	208	206	205	211	215	220	222	224	222	220	220	218	218	217	217	216	214	212	213	211
240	245	249	244	239	239	243	240	237	238	241	237	237	236	233	239	241	247	247	246	245	244	245	249	252	249	250	255	262	259	258
260	265	269	266	273	284	281	279	272	272	272	275	277	279	275	270	270	270	270	268	270	270	273	272	268	263	262	268	272	271	270
280	279	285	283	280	283	284	282	279	276	276	275	273	273	273	281	285	285	287	284	284	283	287	289	289	286	286	290	291	290	288
140	144	144	153	157	160	150	144	140	134	133	133	137	136	140	138	137	138	140	141	142	140	137	135	140	139	139	139	140	139	139
160	166	164	171	177	180	182	185	190	193	195	192	192	195	197	198	198	197	194	190	190	187	191	190	192	191	190	191	195	193	192
180	168	162	176	180	190	196	198	187	180	179	182	181	183	182	181	182	188	193	194	191	189	189	186	186	180	178	178	178	174	174
200	200	196	206	212	225	235	240	234	232	231	240	239	240	240	237	242	246	247	245	246	251	255	253	254	251	250	249	249	246	246
220	225	220	225	227	222	227	228	234	239	240	243	246	248	249	256	261	259	258	258	256	259	264	264	265	262	259	255	251	248	248
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260	262	266	264	279	286	302	311	317	316	316	316	325	329	330	331	339	349	349	350	348	349	351	349	352	352	352	353	355	354	352
280	277	281	278	280	278	267	264	274	275	277	274	274	271	273	274	278	277	278	275	276	277	277	273	271	271	265	269	269	269	268
140	154	161	150	155	154	157	156	149	141	140	144	140	141	138	141	143	142	140	140	138	140	141	139	138	136	137	140	143	141	140
160	171	177	174	183	192	199	199	199	199	202	202	203	206	210	210	208	210	210	209	206	205	205	204	201	203	201	204	207	205	204
180	189	199	198	212	202	213	224	223	220	216	219	222	219	224	227	227	227	230	229	228	228	229	231	230	225	225	227	232	231	230
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280	280	291	268	273	266	260	256	262	264	270	266	273	271	272	272	269	269	272	269	267	268	266	263	264	261	258	259	262	261	260

140	133	137	131	125	129	145	153	159	160	160	159	161	161	156	153	151	153	156	154	149	150	151	152	153	154	153	156	151	148	148
160	160	172	155	160	170	173	178	177	179	175	176	177	182	180	181	182	186	189	187	186	187	189	191	195	191	189	190	188	186	186
180	174	173	176	190	189	192	186	186	183	178	178	180	184	185	185	188	188	191	193	195	194	193	192	188	189	187	185	184	183	182
200	209	218	200	203	203	196	194	195	199	203	208	205	208	208	217	221	225	223	220	223	228	232	230	231	232	231	236	240	238	238
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260	260	272	246	252	254	256	258	251	244	239	235	233	230	226	225	226	225	227	228	222	220	220	219	217	216	218	220	220	216	215
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140	143	146	148	145	147	125	120	115	118	125	123	126	129	132	138	140	138	141	140	136	134	133	133	135	132	131	133	138	133	134
160	176	171	182	176	183	192	200	201	203	204	200	200	198	199	193	196	197	194	194	192	192	197	200	200	199	196	196	196	196	195
180	185	191	183	196	203	201	197	199	198	202	212	217	217	214	213	211	211	210	212	212	209	208	209	209	208	205	204	199	200	198
200	203	209	208	209	206	197	194	190	192	201	201	205	208	207	204	204	208	210	209	205	204	203	204	207	209	212	217	220	217	217
220	217	226	211	216	217	231	233	241	246	248	251	252	255	259	266	269	269	271	275	274	273	275	272	277	274	275	279	279	278	276
240	237	240	239	233	239	249	251	247	245	243	243	246	247	247	247	245	245	247	247	246	245	245	243	242	241	243	243	242	243	241
260	257	267	259	268	274	264	266	268	272	276	276	278	277	281	281	284	285	286	289	290	293	295	290	293	290	287	288	284	281	280
280	267	274	270	285	290	285	286	295	293	291	292	292	298	302	306	307	305	306	307	307	309	313	314	316	312	308	314	324	321	320

Cont. 128 simulated series of successive h_{100}

Age at breast height

15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165
h_{100} in dm																														
140	145	153	140	139	136	128	122	114	117	122	124	125	126	126	126	124	122	122	118	118	119	120	118	114	115	116	115	114	114	113
160	159	162	155	148	146	145	151	151	152	157	163	166	164	164	164	166	166	166	168	170	170	169	168	169	167	168	168	171	169	168
180	174	171	173	172	173	172	179	186	189	193	187	182	186	195	202	203	206	209	208	205	205	211	211	211	209	207	209	207	206	205
200	209	205	208	218	222	238	253	259	262	270	269	276	286	286	287	289	289	288	289	287	287	288	289	288	286	284	285	288	286	286
220	236	241	230	230	223	228	232	239	244	254	250	247	250	251	252	249	251	251	253	251	249	247	245	244	242	242	247	251	248	246
240	235	226	245	242	233	226	218	213	211	214	209	207	207	210	213	216	211	214	210	211	210	206	209	212	208	205	205	210	208	207
260	274	281	273	283	283	280	283	290	298	304	305	308	311	314	311	313	315	316	312	313	312	310	312	314	313	311	307	306	305	303
280	286	292	284	285	284	303	302	301	304	309	308	302	299	296	298	301	304	305	304	301	300	303	300	298	302	300	299	302	299	298
140	141	146	143	146	156	163	158	155	154	152	155	154	159	162	171	172	174	174	175	175	176	181	186	187	188	186	185	186	185	186
160	157	155	151	158	161	164	159	156	152	152	156	156	151	150	152	155	155	153	154	156	157	156	154	158	155	152	153	158	157	156
180	193	195	194	205	204	203	201	208	210	210	213	216	222	226	224	222	221	221	219	216	217	219	217	217	217	219	218	220	217	216
200	211	221	211	207	210	217	214	220	228	236	237	239	235	236	233	234	235	234	235	237	234	238	240	247	245	243	242	240	240	238
220	215	219	213	225	223	222	218	214	208	209	206	208	210	217	217	218	221	222	222	225	226	230	232	229	226	223	221	222	220	220
240	250	249	254	247	250	260	269	268	265	265	267	269	270	272	276	276	279	281	282	283	283	282	280	277	282	282	289	295	294	293
260	254	258	261	272	272	281	290	287	283	276	273	278	283	283	284	289	290	290	286	283	281	282	281	282	279	277	276	275	273	273
280	287	292	287	285	281	276	274	264	266	272	271	270	268	269	268	269	271	272	272	273	275	271	269	271	268	268	270	274	273	272
140	136	146	135	144	141	151	148	142	138	136	134	139	136	132	133	139	143	144	144	142	143	142	142	141	138	137	137	140	138	137
160	164	155	166	165	166	169	169	159	145	139	139	137	134	137	140	142	145	147	148	143	142	140	138	135	135	134	136	136	133	133
180	187	192	189	204	197	191	187	186	190	191	197	202	206	210	210	212	212	212	215	214	215	214	213	216	212	212	213	216	214	214
200	214	216	209	219	214	220	215	211	209	208	209	211	211	212	209	210	206	209	208	206	212	215	215	211	211	212	214	217	214	214
220	226	232	225	232	237	243	246	247	251	256	256	260	263	267	272	280	280	281	284	288	290	290	292	291	293	293	294	298	293	292
240	246	241	258	250	242	246	246	241	234	234	229	229	229	223	221	223	225	221	224	221	221	221	224	223	225	221	222	228	225	225
260	264	262	272	283	279	282	281	273	266	264	262	261	259	255	250	249	251	252	255	257	259	264	259	256	258	256	260	265	265	264
280	289	292	290	291	287	305	312	313	318	326	324	332	341	344	351	354	353	352	352	350	352	358	360	360	360	358	357	351	349	348
140	143	139	146	147	148	159	160	162	160	164	166	167	171	177	175	175	172	171	172	168	167	170	163	160	162	163	160	157	153	153
160	165	169	158	169	170	175	170	169	169	174	173	178	181	185	185	185	186	187	185	183	181	184	182	183	182	181	173	164	165	164
180	178	185	168	183	184	194	205	201	201	204	206	214	220	223	224	226	226	226	227	227	231	234	233	229	229	229	227	224	223	222
200	199	194	201	202	212	209	212	213	212	219	216	217	216	220	225	228	232	231	231	228	233	238	236	233	233	234	243	251	250	249
220	229	227	227	227	224	224	229	224	222	224	224	231	228	227	229	232	239	238	237	236	236	240	237	238	240	239	236	238	238	237
240	236	235	249	249	255	249	252	252	250	253	258	267	270	275	284	286	290	293	292	290	292	298	297	293	292	293	296	301	300	299
260	268	273	269	267	268	270	264	258	259	255	256	260	262	265	268	266	267	269	268	266	265	269	264	268	267	268	268	271	269	269
280	293	296	290	302	310	329	335	340	340	343	341	339	342	342	343	344	345	346	345	349	349	352	352	356	357	356	358	363	361	360

140	129	129	132	135	138	133	135	132	135	139	141	146	145	147	146	144	144	145	142	138	141	142	141	142	144	145	148	151	148	146
160	155	156	152	150	152	141	138	133	128	128	130	135	136	136	135	133	133	133	133	132	137	142	143	139	139	138	134	130	130	130
180	177	182	180	181	179	189	190	186	177	174	174	176	185	191	197	202	204	200	201	203	200	199	202	204	201	201	198	194	191	190
200	205	207	210	208	209	206	205	212	218	220	223	226	228	231	236	235	237	235	235	234	234	237	236	237	237	239	234	230	230	228
220	218	217	227	229	234	239	238	232	235	239	233	227	224	223	224	224	220	223	223	220	222	224	226	226	223	220	219	219	222	220
240	241	244	240	253	263	271	277	276	270	271	269	266	270	274	278	280	284	284	284	282	284	288	289	288	287	287	291	296	295	294
260	255	255	263	251	258	260	256	255	254	257	260	263	266	270	274	276	277	279	279	279	281	281	277	275	275	275	276	280	278	276
280	278	289	273	286	292	304	306	296	299	305	304	307	308	308	308	305	304	304	300	299	300	300	301	301	299	296	297	298	299	297
140	146	140	152	136	129	116	115	110	105	102	103	109	107	106	106	106	105	108	105	102	100	97	92	93	92	90	91	93	91	90
160	151	144	161	153	143	141	141	147	149	152	154	157	163	170	174	171	172	173	176	179	181	182	184	180	182	183	178	179	178	178
180	190	183	192	194	196	196	200	204	206	210	209	211	211	210	207	211	214	212	212	209	207	205	201	201	201	202	197	194	192	190
200	198	197	203	212	222	234	234	233	223	218	223	220	217	212	215	214	215	216	215	216	215	211	209	208	207	207	211	215	213	212
220	217	226	210	224	225	232	235	225	222	220	217	216	218	212	216	218	219	217	215	210	209	204	206	207	206	202	195	187	187	186
240	239	244	239	236	231	238	246	249	252	254	255	257	258	258	260	258	255	253	254	253	252	254	245	240	240	241	245	250	248	246
260	249	257	243	239	237	241	251	255	254	258	256	257	259	262	261	262	263	264	267	269	271	276	278	276	274	273	274	272	270	270
280	276	277	275	289	286	304	304	309	309	310	313	314	321	323	326	327	329	331	327	323	327	327	325	330	325	322	326	328	325	324
140	135	130	149	126	130	142	149	147	142	141	140	140	139	135	133	135	136	135	135	132	132	130	128	129	128	128	128	132	130	130
160	155	161	160	166	167	149	138	136	132	130	128	127	124	128	132	134	134	135	134	135	137	140	143	145	144	144	139	136	135	134
180	178	189	177	185	191	192	197	202	207	213	212	222	229	229	230	233	232	234	232	232	235	239	236	235	232	231	227	224	221	220
200	198	199	198	206	205	199	192	186	187	193	186	184	185	186	187	188	189	191	190	190	192	197	195	200	199	196	195	200	200	198
220	221	222	224	232	236	232	242	245	244	246	249	254	261	260	258	257	258	263	264	268	271	273	272	276	272	267	271	276	274	273
240	231	228	239	238	244	240	235	240	245	254	249	253	253	249	245	246	248	248	248	248	248	251	250	252	250	249	247	247	247	245
260	264	270	266	258	262	273	281	275	273	271	273	272	271	274	273	274	276	275	277	276	277	278	276	272	270	270	267	267	265	264
280	274	283	271	272	274	287	298	293	299	301	303	307	306	305	307	309	314	313	313	312	316	319	316	317	314	312	318	327	324	322
140	134	135	133	137	146	152	151	147	146	149	146	151	153	154	153	153	150	151	151	155	153	153	156	158	155	156	158	156	155	154
160	152	156	142	154	152	149	153	153	156	158	158	162	161	157	156	156	155	158	156	156	155	152	154	155	155	155	149	139	139	138
180	194	196	195	200	198	208	211	202	197	198	196	195	192	193	197	196	196	194	195	199	202	202	202	197	199	199	199	200	198	198
200	197	195	200	197	201	207	210	208	205	207	207	215	217	213	207	208	210	214	215	216	221	224	226	226	226	226	225	221	221	220
220	216	228	219	235	234	217	217	213	206	206	209	212	217	219	223	228	227	225	225	221	222	221	225	220	220	220	224	228	228	226
240	250	257	251	257	252	260	264	269	273	278	278	281	285	289	289	288	289	294	295	297	299	302	302	301	297	294	297	296	294	293
260	253	261	257	249	241	227	222	227	228	231	235	240	242	242	242	243	239	242	247	245	247	250	251	255	254	253	252	255	252	251
280	278	281	284	286	285	283	275	271	268	264	264	264	270	271	273	281	285	286	286	287	287	289	288	288	289	287	288	292	291	289

Appendix 3

Number of plots required to estimate site index within a stand with predefined standard error. A "0" notation means either that the predefined standard error cannot be reached, or that the necessary number of plots exceeds 99.

Site variation within stand	Plot size m^2	Age at breast height														
		15	25	35	45	55	65	75	85	95	105	115	125	135	145	155
		Necessary number of plots														
Predefined standard error 10 dm																
Low	1 000	0	0	0	0	10	6	4	4	5	7	7	15	98	25	0
Low	254	0	0	0	0	17	10	7	6	8	11	12	24	0	41	0
Low	154	0	0	0	0	21	13	9	8	10	14	14	30	0	51	0
Medium	1 000	0	0	0	0	21	13	9	8	10	14	14	30	0	51	0
Medium	254	0	0	0	0	30	19	13	11	14	20	21	43	0	73	0
Medium	154	0	0	0	0	38	23	16	14	18	25	26	54	0	93	0
High	1 000	0	0	0	0	36	22	15	13	17	23	24	50	0	86	0
High	254	0	0	0	0	54	33	23	20	25	35	37	76	0	0	0
High	154	0	0	0	0	64	39	27	23	30	42	44	91	0	0	0
Predefined standard error 12 dm																
Low	1 000	0	0	0	0	3	3	2	2	2	3	3	3	4	4	17
Low	254	0	0	0	0	5	4	4	3	4	4	5	6	7	6	28
Low	154	0	0	0	0	6	5	4	4	5	5	6	7	9	8	34
Medium	1 000	0	0	0	0	6	5	4	4	5	5	6	7	9	8	34
Medium	254	0	0	0	0	9	8	6	6	7	8	8	10	13	11	49
Medium	154	0	0	0	0	12	10	8	8	9	10	10	13	16	14	62
High	1 000	0	0	0	0	11	9	8	7	8	9	9	12	15	13	58
High	254	0	0	0	0	16	14	11	11	12	14	14	18	22	20	87
High	154	0	0	0	0	19	16	14	13	14	17	17	21	27	24	0

Predefined standard error 14 dm																
Low	1 000	0	0	0	4	2	2	1	1	1	2	2	2	2	2	3
Low	254	0	0	0	6	3	3	2	2	2	3	3	3	3	3	5
Low	154	0	0	0	8	3	3	3	3	3	3	3	4	4	4	6
Medium	1 000	0	0	0	8	3	3	3	3	3	3	3	4	4	4	6
Medium	254	0	0	0	11	5	5	4	4	4	5	5	5	6	6	9
Medium	154	0	0	0	14	6	6	5	5	5	6	6	7	7	7	11
High	1 000	0	0	0	13	6	5	5	5	5	5	5	6	7	7	11
High	254	0	0	0	20	9	8	7	7	7	8	8	9	10	10	16
High	154	0	0	0	24	11	10	9	8	9	10	10	11	13	12	19
Predefined standard error 16 dm																
Low	1 000	0	0	0	2	1	1	1	1	1	1	1	1	1	1	2
Low	254	0	0	0	3	2	2	2	2	2	2	2	2	2	2	3
Low	154	0	0	0	4	2	2	2	2	2	2	2	2	3	2	3
Medium	1 000	0	0	0	4	2	2	2	2	2	2	2	2	3	2	3
Medium	254	0	0	0	5	3	3	3	3	3	3	3	3	4	4	5
Medium	154	0	0	0	7	4	4	4	3	4	4	4	4	5	4	6
High	1 000	0	0	0	6	4	4	3	3	3	4	4	4	4	4	5
High	254	0	0	0	9	6	5	5	5	5	6	6	6	6	6	8
High	154	0	0	0	11	7	7	6	6	6	7	7	7	8	7	10
Predefined standard error 18 dm																
Low	1 000	0	0	5	1	1	1	1	1	1	1	1	1	1	1	1
Low	254	0	0	8	2	1	1	1	1	1	1	1	1	1	1	2
Low	154	0	0	10	2	2	2	1	1	2	2	2	2	2	2	2
Medium	1 000	0	0	10	2	2	2	1	1	2	2	2	2	2	2	2
Medium	254	0	0	14	3	2	2	2	2	2	2	2	2	3	2	3
Medium	154	0	0	17	4	3	3	3	3	3	3	3	3	3	3	4
High	1 000	0	0	16	4	3	3	3	2	3	3	3	3	3	3	4
High	254	0	0	25	6	4	4	4	4	4	4	4	4	5	4	5
High	154	0	0	29	7	5	5	5	4	5	5	5	5	5	5	6

Cont. Appendix 3

Site variation within stand	Plot size m²	Age at breast height														
		15	25	35	45	55	65	75	85	95	105	115	125	135	145	155
		Necessary number of plots														
Predefined standard error 20 dm																
Low	1 000	0	8	2	1	1	1	1	1	1	1	1	1	1	1	1
Low	254	0	14	3	1	1	1	1	1	1	1	1	1	1	1	1
Low	154	0	17	3	2	1	1	1	1	1	1	1	1	1	1	1
Medium	1 000	0	17	3	2	1	1	1	1	1	1	1	1	1	1	1
Medium	254	0	24	5	2	2	2	2	2	2	2	2	2	2	2	2
Medium	154	0	31	6	3	2	2	2	2	2	2	2	2	2	2	3
High	1 000	0	29	6	3	2	2	2	2	2	2	2	2	2	2	3
High	254	0	43	9	4	3	3	3	3	3	3	3	3	3	3	4
High	154	0	52	10	5	4	4	4	3	4	4	4	4	4	4	5