# The Fixed Effects Estimator of Technical Efficiency

New Insights and Developments

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Abstract

Firms and organizations, public or private, often operate on markets characterized by non-competitiveness. For example agricultural activities in the western world are heavily subsidized and electricity is supplied by firms with market power. In general it is probably more difficult to find firms that act on highly competitive markets, than firms that are not.

To measure different types of inefficiencies, due to this lack of competitiveness, has been an ongoing issue, since at least the 1950s when several definitions of inefficiency was proposed and since the late 1970s as stochastic frontier analysis. In all three articles presented in this thesis the stochastic frontier analysis approach is considered. Furthermore, in all three articles focus is on technical inefficiency.

The ways to estimate technical inefficiency, based on stochastic frontier models, are numerous. However, focus in this thesis is on fixed effects panel data estimators. This is mainly for two reasons. First, the fixed effects analysis does not demand explicit distributional assumptions of the inefficiency and the random error of the model. Secondly, the analysis does not require the random effects assumption of independence between the firm specific inefficiency and the inputs selected by the very same firm. These two properties are exclusive for fixed effects estimation, compared to other stochastic frontier estimators.

There are of course flaws attached to fixed effects analysis as well, and the contribution of this thesis is to probe some of these flaws, and to propose improvements and tools to identify the worst case scenarios. For example the fixed effects estimator is seriously upward biased in some cases, i.e. inefficiency is overestimated. This could lead to false conclusions, like e.g. that subsidies in agriculture lead to severely inefficient farmers even if these farmers in reality are quite homogenous.

In this thesis estimators to reduce bias as well as mean square error are proposed and statistical diagnostics are designed to identify worst case scenarios for the fixed effects estimator as well as for other estimators. The findings can serve as important tools for the applied researcher, to obtain better approximations of technical inefficiency.

*Keywords:* Stochastic frontier analysis, Technical output (in-) efficiency, Panel data, Fixed effects estimator, Nonparametric kernel estimation, Time-varying firm effects, Bias, Skew, Random error, Hypothesis testing.

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*What are the roots that clutch, what branches grow*
*Out of this stony rubbish? Son of man,*
*You cannot say, or guess, for you know only*
*A heap of broken images, where the sun beats,*
T.S. Eliot

# Contents

## List of Publications

This thesis is based on the work contained in the following papers, referred to by Roman numerals in the text:

I Daniel Wikström. A Finite Sample Improvement of the Fixed Effects Estimator – Applied to Technical Efficiency.

II Daniel Wikström. Estimating Unspecified Time-varying Technical Efficiency.

III Daniel Wikström. The Fixed Effects Estimator of Technical Efficiency – Bias in Relation to Skew.

# 1 Introduction

Almost no markets can be considered to be competitive in the 'perfect market' sense. Western agriculture is heavily subsidized, electricity production is conducted by firms with market power and wage levels for employees in the health care sector, in a country like Sweden, are set under monopsony power. Despite the subsidies, do farmers produce efficiently giving the existing technology? Do managers at electricity companies always push or motivate their co-workers as much as they would have, if the company was acting under fierce competition? How do limited possibilities to negotiate about wages affect individual initiatives and motivation and productivity in the health care sector in Sweden? By the help of frontier analysis, it can be investigated if there are differences in production which can relate to either one of these situations.

The purpose of stochastic frontier analysis is to measure different sorts of inefficiency that might arise due to lack of exposure to fully competitive markets. Usually, these measures are very general and rather reveal a state, than giving an exact explanation to why the observed differences might have arisen. Nevertheless, it is important for stakeholders and policymakers to have indicators of how markets and businesses are functioning, even if the indicators are crude, and do not give any explicit information about possible causes.

The research in this thesis deals with stochastic frontier analysis and estimation of technical output (in-) efficiency. Only panel data models are considered and focus is on fixed effects estimation.

Schmidt and Sickles [1984] proposed the fixed effects estimator of technical inefficiency. The focus on the fixed effects estimator is due to two reasons. Firstly, the fixed effects analysis does not demand explicit distributional assumptions of the inefficiency and the random error of the model. Secondly, the analysis does not require the random effects assumption, of independence between the firm specific inefficiency and the inputs selected by the very same firm. These two properties are exclusive for the fixed effects estimator, compared to other stochastic frontier estimators.

# 2 The Stochastic Frontier Model

In this thesis, I consider the stochastic frontier panel data model, defined as follows:

$$y_{it} = f(x_{it})exp(-u_{it} + v_{it}); \quad i = 1,\dots,N, t = 1,\dots,T, \tag{1}$$

where $y_{it}$ is the observed single output of firm $i$ at time $t$, $x_{it}$ is a $K \times 1$ input vector, and $u_{it}$ ($\geq 0$) is the measure of technical output inefficiency of firm $i$ at time $t$.[1] The frontier production function is represented by $f(\cdot)$; it can be treated parametrically as well as nonparametrically. The error, $v_{it}$, term can be interpreted as a measurement error in $y_{it}$. The actual output is $\tilde{y}_{it}$ and the observed output is

$$y_{it} = \tilde{y}_{it} exp(v_{it}). \tag{2}$$

Alternatively, one can interpret $\tilde{y}_{it}$ as the planned output given the selected inputs, and because of some random occurrence, $y_{it}$ is obtained instead of $\tilde{y}_{it}$.

Figure 1 is a graphical representation of the model in (1) with a single input. The random error makes the observed output to be larger than the output given by the frontier function for input $x_{it}$, i.e. $y_{it} > f(x_{it})$. However, firm $i$ is at the same time inefficient in period $t$, since $\tilde{y}_{it} < f(x_{it})$. The observed output could have been $y_{it}e^{u_{it}}$ in this period with the same amount of input. Therefore, $u_{it} \geq 0$ is a measure of technical output inefficiency of the firm. On the other hand, the firm could also have been efficient if it had used less input, $\tilde{x}_{it}$ instead of $x_{it}$, and still produced the observed output, $y_{it}$. The ratio $\tilde{x}_{it}/x_{it}$ is, therefore, a measure of technical input inefficiency.

Although, I exclusively assume Cobb-Douglas frontier functions in this thesis, and the technical input (in-) efficiency is easily obtained from $u_{it}$ for homogenous frontier functions, I only consider technical output (in-) efficiency henceforth. This is not because technical input efficiency is unimportant. It is merely because I have chosen to focus on other topics in the articles. Actually I would like to encourage the estimation of technical input efficiency as well. If there are diminishing returns to scale, as in Figure 1, the input measure gives a quite different 'picture' of the efficiency of firms. If the frontier function is almost flat the output measure might indicate almost no inefficiency, but at the same time firms with considerable less input may produce almost the same amount of output.

## 3 Estimation of Technical (In-) Efficiency

In the thesis, the production function is treated parametrically and more precisely as a Cobb-Douglas function. In this case the stochastic frontier model simplifies to the following model:

$$y_{it} = \alpha_t + x'_{it}\beta - u_{it} + v_{it} \equiv x'_{it}\beta + \alpha_{it} + v_{it}; \quad i = 1, \ldots, N, t = 1, \ldots, T, \tag{3}$$

[1] In two out of three articles, time-constant inefficiency, $u_i$, is considered.

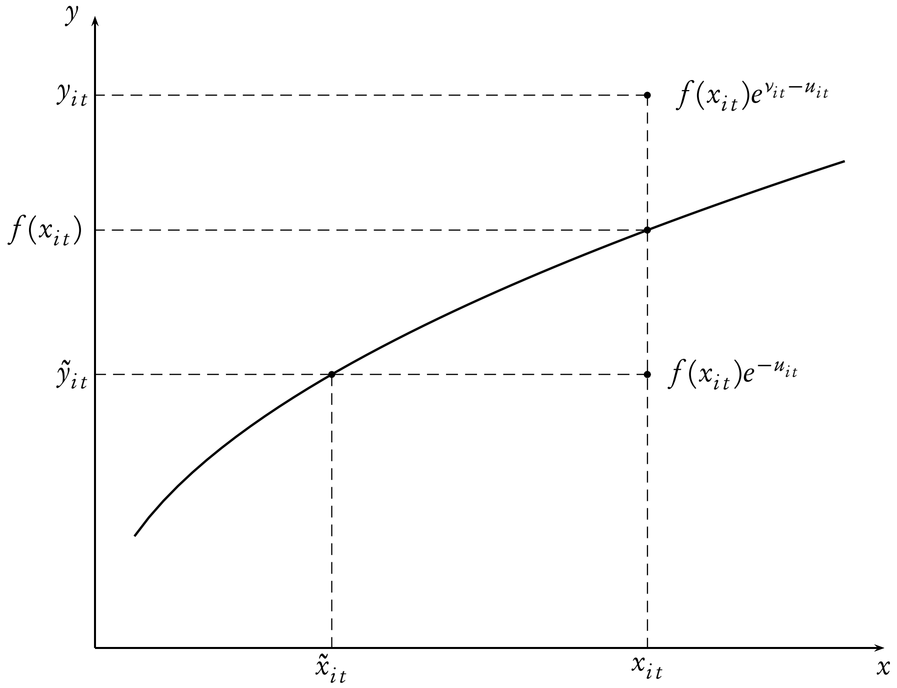


Figure 1: The stochastic frontier model in the case of one input.

where $y_{it}$ is a log-transformed single output, $x_{it}$ is a $K \times 1$ log-transformed vector of inputs, $\beta$ is a $K \times 1$ coefficient vector and $\alpha_{it} = \alpha_t - u_{it}$ is the firm effect of firm $i$, with $\alpha_t$ as the frontier intercept and $u_{it}$ ($\geq 0$) is the measure of technical inefficiency of firm $i$ at time $t$. The error term, $v_{it}$, is assumed to be independent and identically distributed with finite variance, and $E(v_{it}|\alpha_{it}, X_i) = 0$, where $X_i = \left[x_{i1}\, x_{i2} \cdots x_{iT}\right]'$, $i = 1, 2, \ldots, N$.

For Articles I and III focus is only on time-constant inefficiency, which implies $\alpha_i = \alpha - u_i$. However, Article II is on time-varying technical efficiency estimation. Estimation differs a bit between the two cases. For time-constant technical inefficiency, the traditional fixed effects estimator is written as follows:

1

$$\hat{\alpha}_i = \hat{E}(y_{it} - x'_{it}\beta|\alpha_i) = \frac{1}{T}\sum_{t}^{T}(y_{it} - x'_{it}\hat{\beta}) = \bar{y}_i - \bar{x}'_i\hat{\beta}, \tag{4}$$

on the other hand, the kernel FE ('fixed effects') estimator presented in Article I is given by the local constant kernel estimator of the firm effect of firm $i$, written as:

$$\tilde{\alpha}_i = \tilde{E}(y_{it} - x'_{it}\beta|\alpha_i) = \frac{\sum_j^N \sum_t^T \left(y_{jt} - x'_{jt}\hat{\beta}\right) L_j(i,\lambda)}{T \sum_j^N L_j(i,\lambda)} \tag{5}$$

where $L_j(\cdot)$ is a kernel function defined as:

$$L_j(i,\lambda) = \begin{cases} 1, & j = i \\ \lambda \in [0,1], & \text{otherwise.} \end{cases} \tag{6}$$

This local constant kernel estimator actually constitutes a continuum of FE estimators, since $\lambda \in [0,1]$. Which include the traditional FE estimator as a special case when $\lambda = 0$, $\tilde{\alpha}_i = \hat{\alpha}_i$.[2]

The within estimator of $\beta$ can be used for both estimators.

The technical inefficiency estimates are obtained by following Schmidt and Sickles [1984] and use:

$$\hat{\alpha} = \max_j \hat{\alpha}_j, \quad \hat{u}_i = \hat{\alpha} - \hat{\alpha}_i, \; i = 1, \ldots, N, \tag{7}$$

$$\tilde{\alpha} = \max_j \tilde{\alpha}_j, \quad \tilde{u}_i = \tilde{\alpha} - \tilde{\alpha}_i, \; i = 1, \ldots, N. \tag{8}$$

The estimation of the time-varying inefficiencies are slightly more involved. In Article II, I compare the proposed nonparametric estimator of technical inefficiency to a parametric estimator proposed by Cornwell et al. [1990]. The latter estimator is fairly simple and intuitive and is based on a parametric assumption about the function of the time-pattern. Cornwell et al. [1990] assume time-varying firm effect of the following form:

$$\alpha_{it} = \theta_{i1} + \theta_{i2}t + \theta_{i3}t^2. \tag{9}$$

The estimator is very similar to the time-constant case. The objective is still to estimate $E(y_{it} - x'_{it}\beta|\alpha_{it})$. However, instead of averaging the residuals, $y_{it} - x_{it}\hat{\beta}$, $t = 1, 2, \ldots, T$, over the time observations, one regresses the residuals, separate for each firm, on $w'_{it} = \left[1 \; t \; t^2\right]$. Consequently an estimator of the firm effect of firm $i$ at time $t$ is:

$$\hat{\alpha}_{it} = \hat{E}(y_{it} - x'_{it}\beta|\alpha_{it}) = w_{it}\hat{\Theta}_i, \tag{10}$$

[2] See Ouyang et al. [2009] for further details about the local constant kernel estimator in relation to discrete covariates.

where

$$\hat{\Theta}_i = \begin{bmatrix} \hat{\theta}_{i1} \\ \hat{\theta}_{i2} \\ \hat{\theta}_{i3} \end{bmatrix} = \left[ \sum_t^T w_{it} w_{it}' \right]^{-1} \sum_t^T w_{it} (y_{it} - x_{it}' \hat{\beta}). \tag{11}$$

The estimator, $\hat{\beta}$, is the analog to the within estimator. More precisely:

$$\hat{\beta} = \left[ \sum_i^N \sum_t^T \ddot{x}_{it} \ddot{x}_{it}' \right]^{-1} \sum_i^N \sum_t^T \ddot{x}_{it} \ddot{y}_{it}, \tag{12}$$

where $\ddot{y}_{it} = y_{it} - \hat{E}(y_{it}|\alpha_{it})$ and $\ddot{x}_{it} = x_{it} - \hat{E}(x_{it}|\alpha_{it})$. The conditional expected values are estimated analogously to (10).

In Article II, I propose a fixed effects type of kernel estimator. Unlike the estimator proposed by Cornwell et al. [1990], this estimator does not require an explicit time-pattern. Thus, the firm effects are estimated non-parametrically. The estimator of the firm effect for firm $i$ at time $t$ is now the following:

$$\tilde{\alpha}_{it} = \frac{\sum_{j,s} \left( y_{jt} - x_{jt}' \hat{\beta} \right) L\left[ (j,s), (i,t), \lambda \right]}{p(i,t)} \tag{13}$$

where

$$p(i,t) = \sum_{j,s} L\left[ (j,s), (i,t), \lambda \right], \tag{14}$$

$$L\left[ (j,s), (i,t), \lambda \right] = \ell_u \times \ell_o. \tag{15}$$

The so called 'product kernel' in (15) consists of a univariate kernel that handles unordered data, $\ell_u$, and a univariate kernel that handles ordered data, $\ell_o$. The unordered kernel is defined as

$$\ell_u (j, i, \lambda_u) = \begin{cases} 1, & j = i \\ \lambda_u \in [0,1], & \text{otherwise,} \end{cases} \tag{16}$$

while the ordered kernel is defined as

$$\ell_o (s, t, \lambda_o) = \begin{cases} 1, & s = t \\ \lambda_o^{|s-t|} \in [0,1], & s \neq t. \end{cases} \tag{17}$$

The estimator of $\beta$, is estimated like in (12), however, the conditional expectations: $E(y_{it}|\alpha_{it})$ and $E(x_{it}|\alpha_{it})$ are now estimated analogously to (13).

Time-varying inefficiency estimates are obtained as follows for each $t = 1,\ldots,T$:

$$\hat{\alpha}_t = \max_j \hat{\alpha}_{jt}, \quad \hat{u}_{it} = \hat{\alpha}_t - \hat{\alpha}_{it},\ i = 1,\ldots,N, \tag{18}$$

$$\tilde{\alpha}_t = \max_j \tilde{\alpha}_{jt}, \quad \tilde{u}_{it} = \tilde{\alpha}_t - \tilde{\alpha}_{it},\ i = 1,\ldots,N. \tag{19}$$

# 4 Results

The thesis consists of three articles on stochastic frontier analysis and more specifically, technical output inefficiency estimation based on fixed effects estimators. In this section the main results from the three articles are presented.

## 4.1 Article I

Article I is mainly about the poor small sample properties of the traditional fixed effects estimator of technical inefficiency. It is well-known that the fixed effects estimator is upwards biased, when $N$ is large and there is much influence of random error [Kim et al., 2007, Wang and Schmidt, 2009, Satchachai and Schmidt, 2010]. However, in Article III, I show that this bias mainly occurs in the case of right-skewed inefficiencies.[3]

Nevertheless, I propose the kernel fixed effects estimator, defined by (5) and (6), and derive an optimal bandwidth based on a global MSE ('mean square error') criterion. I also prove that there is a continuum of kernel estimators which are efficient to the FE estimator of the firm effects in finite samples, in terms of the MSE criterion. There are also asymptotic results for the kernel estimator provided in the article, e.g. consistency and asymptotic normality. The MSE-efficiency of the kernel estimator is in respect to the firm effects, and not automatically as an estimator of technical inefficiency.

To derive a similar optimal bandwidth for estimation of inefficiency is not trivial and I conjecture it is impossible to find a feasible estimator for such a bandwidth. Nevertheless, I propose two bandwidths which are derived to work as approximations of the optimal bandwidth of a global MSE criterion for the inefficiencies.[4]

Furthermore, kernel estimation has two properties which shows to be very valuable for estimation of inefficiency in cases when the traditional FE estimator works poorly. First, the kernel estimator is biased and is flexible, due to the bias-variance tradeoff, in cases when the traditional FE estimator

[3]This is also shown by Feng and Horrace [2012]

[4]For the inefficiency measure based on comparison to the 'best' firm in the sample.

is influenced by random error. The traditional FE estimator is unbiased but produces very large variance if the influence from the random error is large. It is shown that kernel estimation can decrease the variance quite dramatically in these cases. Thus, the kernel estimator gains in MSE terms from the bias-variance tradeoff. Secondly the bias of the kernel estimator is such that it bias the firm effects towards the mean, which makes it a good candidate when the traditional FE estimator overestimates inefficiency as shown by e.g. Wang and Schmidt [2009].

I show in Monte Carlo simulations that the proposed estimator behaves as expected according to the theoretical results derived in the article. The kernel FE estimator outperforms the traditional FE estimator, in terms of bias and MSE, both when it comes to estimating the firm effects and the inefficiencies.

## 4.2 Article II

The second article is on the estimation of time-varying inefficiency. Analogously to Article I an alternative FE type of estimator is proposed. Again the local constant kernel estimator is used, and in this case an ordered kernel is presented to handle the time-pattern. This estimator is the one presented in Section 3, given by equations (13)-(17). I compare this estimator to the FE estimator proposed by Cornwell et al. [1990]. Unlike in Article I, I do not derive any small sample properties for this estimator. Comparing the proposed estimator, by theoretical small-sample properties, to the Cornwell et al. [1990] estimator is nontrivial. The kernel estimator has the advantage from Article I, across firms, but in the current setting is more general than the parametric FE estimator and therefore will deficit, in statistical terms, if the assumption of the time-pattern of the parametric FE estimator is correct.

I conduct Monte Carlo simulations that indicate that proposed kernel estimator, may outperform the parametric FE estimator, in terms of bias and MSE, even if the time-pattern of the latter estimator is correct. This is entirely due to the efficiency gains recorded in Article I. However, the kernel estimator, suffers from bias in the production function parameters.This is of course a problem, however, it seems to have little impact on the estimated inefficiencies.

I also conduct an empirical analysis on Indonesian rice farmers. The average efficiency levels diverge considerable between the two FE estimators. To investigate if the results of the kernel FE estimator is completely farfetched I also employ a maximum likelihood estimator based on truncated-normal normal composed random error [Battese and Coelli, 1992]. The average efficiency estimates of the MLE is in line with the average of the kernel FE estimator, showing that the proposed estimator is at least not more far-off than a frequently applied ML estimator.[5]

## 4.3 Article III

Article III is about the bias of the FE estimator of technical inefficiency in relation to skew and random error. Feng and Horrace [2012] show that the FE estimator is upward biased, if the population distribution of the inefficiencies is right-skewed. Furthermore, they argue the FE estimator will be less biased if the skew is to the left. This is true, if the measure of interest is the distance to the best firm in the sample.[6] In Article III, on the other hand, focuses is on bias and skew in relation to the population distribution. In this case left-skewed inefficiencies may cause a downward bias, since the estimated maximum firm effect likely is below the population maximum.

Furthermore, I propose a consistent estimator of the population skew, in large $N$ settings. I also propose a $\sqrt{N}$-consistent ratio estimator of a measure of the relative influence of random error. The measure is the ratio between the variance of the distribution of the inefficiencies and the total variance of the composed error term. To perform hypothesis testing and estimation of confidence intervals the bootstrap is used. For FE estimation these two tools are of course useful to get information about what to expect in terms of bias in finite $T$ settings. However, a test of the skew can also be an important diagnostic for researchers using maximum likelihood estimators. If the frequent right-skewed inefficiency assumption is not correct the most common MLEs will be seriously biased.

I conduct Monte Carlo simulations investigating the power and type-I-error for a test of the skew based on normality and bootstrapped standard errors. Also confidence intervals of the ratio estimator are probed. The simulations show that the test of skew is sensitive to random error in relation to the variation in inefficiency. Still, the tests works well for rather small $N$ and $T$ (100 and 5) if the variance of the inefficiencies are around 50 % or more of the total variance of the composed random error. The ratio estimator, on the other hand, is much more robust to random error and can be used as an indicator of what to expect from the skewness test.

[5] I also compare to the half-normal normal model, with similar conclusions.

[6] However, since the maximum firm effect, in this case lies in the thin tail of the distribution, the variance will increase, and the gains in mean square error terms might be small or even negative.

The Monte Carlo experiments also include a comparison of the MSE-efficient FE estimator, derived in Article I, the traditional FE estimator and the frequently applied ML estimator with half-normal normal composed error. The MSE-efficient FE estimator overall, outperforms the two other estimators, in terms of bias and MSE, for right-skewed and symmetrically distributed inefficiencies. If the inefficiencies, on the other hand, are left-skewed, the traditional FE estimator outperforms the two other estimators. However, this estimator is still seriously downward biased in relation to the population. If the MLE instead was based on left-skewed inefficiencies it would probably perform much better than the two FE estimators. Thus, it might be worth investing some time in non-standard ML estimators, i.e. not based on right-skewed inefficiencies.

Finally a small empirical example is conducted, based on the Indonesian rice farmers, also analyzed in Article II. Feng and Horrace [2012] also use this data and conclude that the distribution likely is symmetrical, by heuristic observations of the estimated inefficiencies. I, on the contrary, conclude there is too much random error in the data to make any conclusions about the shape of the distribution. The skewness test does not reject symmetry but the ratio estimator indicates that the variance of the inefficiencies is only about 10 % of the total variance of the composed error term. Thus, there is a lot of random error and the result of the skewness test is inconclusive due to bad power properties.

# 5 Conclusions and Remarks

## 5.1 The Contribution

All three articles are original in their own ways.

The first article introduces a MSE-efficient FE estimator which is characterized by a shrinkage towards the mean firm effect. And because of this it is suited for the standard case, with right-skewed inefficiency, where the traditional FE estimator overestimates inefficiency.

In Article II an extension is made to time-varying inefficiencies. I show that the proposed nonparametric estimator of the inefficiencies may compete with parametric estimation even if the time-pattern of the parametric estimator is correctly specified. This is due to the cross-sectional efficiency gains described in Article I. Thus, Article II gives a new way to estimating unspecified time-varying inefficiency without losing too much efficiency, in terms of bias and MSE, in finite samples.

The work in Article III is different compared with two other articles, since it considers different shapes of the population distribution of the inefficiencies. In the two other articles the frequent assumption of right-skewed inefficiencies is not questioned. Stochastic frontier analysis is about firms which are not under the pressure of fierce competition. Because of this there is no reason why inefficiency always should be right-skewed.

In Article III it is investigated how the traditional FE estimator behaves under different skews. Previous to this study it was well-known that the FE estimator likely is upward biased under the influence of random error. Feng and Horrace [2012] show that this is under right-skewed inefficiencies. I show that the FE estimator may suffer from a much more serious downward bias if the inefficiencies instead are left-skewed. This is in relation to the population distribution and not bias in relation to the distance to the best firm in the sample.

In the article also two diagnostic tools are proposed that can help the applied researcher to select a sensible estimator based on the skew of the inefficiencies and the amount of random error in the sample. These two diagnostics are examples of information which can be obtained from the estimated firm effects in large $N$ settings. I think this is a fairly unexplored source of information.

### 5.2 Important Remark: controlling for heterogeneity

I am not considering heterogeneity in any of the three chapters of the thesis. However this is an important topic and I would like to put forward some thoughts concerning this very delicate issue.

One example of a fixed effects type of estimator for heterogeneity was first presented by Hausman and Taylor [1981]. It is an estimator which can treat observed time-constant heterogeneity in the FE framework.[7] The stochastic frontier model is written as follows:

$$y_{it} = x'_{it}\beta + \alpha_i + \nu_{it} \equiv x'_{it}\beta + z'_i\gamma + \varepsilon_i + \nu_{it}; \quad i = 1, \ldots, N, t = 1, \ldots, T, \tag{20}$$

where $z_i$ is a $Q \times 1$ vector of variables accounting for heterogeneity including a constant, $\gamma$ is a $Q \times 1$ vector of effects due to heterogeneity or inefficiency and $\varepsilon_i$ is an unobserved error term, ideally inefficiency. We need two additional assumptions, compared to (3), to identify $\gamma$. The first assumption is standard full-rank assumption for least-squares estimation: $rank\left[E(z_i z'_i)\right] = Q$. The second assumption is also standard for least squares, but much more debatable: $E(z_i\varepsilon_i) = 0$. This is for sure the strongest assumption. Given $E(\varepsilon_i) = 0$, which is a trivial assumption if $z_i$ includes

[7] See Cornwell et al. [1990] for a similar estimator with time-varying inefficiency.

a constant, $E(z_i\varepsilon_i) = 0$ implies that $\varepsilon_i$ is not correlated with $z_i$. However, note that the relation between the inputs $x_{it}$ and $z_i$ or $\varepsilon_i$ is left unspecified. Thus, both $z_i$ and $\varepsilon_i$ may depend on the inputs. This is the 'beauty' of the fixed effects estimator compared to the random effects estimator and maximum likelihood estimators based on the random effects assumption. No sensible economist really believes that the firm specific inefficiency is independent of the inputs selected by the same firm.

Given the assumptions it is straightforward to estimate $\gamma$ by least squares:

$$\hat{\gamma} = \left[\sum_{i=1}^{N} z_i z_i'\right]^{-1} \sum_{i=1}^{N} z_i \hat{\alpha}_i. \tag{21}$$

It is easy to show the consistency of this estimator as $N$ grows:

$$\begin{aligned}
\hat{\gamma} &= \left[\sum_{i=1}^{N} z_i z_i'\right]^{-1} \sum_{i=1}^{N} z_i \hat{\alpha}_i = \left[\sum_{i=1}^{N} z_i z_i'\right]^{-1} \sum_{i=1}^{N} z_i \left[\alpha_i + \bar{v}_i + \bar{x}_i'(\beta - \hat{\beta})\right] = \\
&= \left[\sum_{i=1}^{N} z_i z_i'\right]^{-1} \left[\sum_{i=1}^{N} z_i \alpha_i + \sum_{i=1}^{N} z_i \bar{v}_i + \sum_{k=1}^{K} (\beta_k - \hat{\beta}_k) \sum_{i=1}^{N} z_i \bar{x}_{ik}\right] \\
&\xrightarrow{p} \left[E(z_i z_i')\right]^{-1} E(z_i \alpha_i) = \left[E(z_i z_i')\right]^{-1} E(z_i z_i')\gamma = \gamma.
\end{aligned}$$

I think this is a realistic alternative to the so called 'true' FE estimator proposed by Greene [2005a,b]. The 'true' FE estimator has several unattractive features. For example, the inefficiency can no longer be correlated with the inputs, the distribution of the inefficiencies is explicitly assumed and there is no way to know if the measure of inefficiency is a measure of inefficiency and/or heterogeneity. How should a mathematical model be able to separate between what is inefficiency and what is heterogeneity? This is a matter of definition, which should be explicitly given by the researcher and not by a 'black box' model.

The estimator $\hat{\gamma}$ is not the product of assumptions of explicit distributions, the variable vector, $z_i$, does not have to include variables which are independent to the inputs and the researcher explicitly defines if the variables are heterogeneity or inefficiency. This for example may give valuable information of the impact of different pre-defined sources of inefficiency.

One drawback is the assumption of no correlation between the observed inefficiency (or heterogeneity) and the unobserved, $E(z_i\varepsilon_i) = 0$. However, compared to complete distributional assumptions of the composed error term and the random effects assumption, this appears as a rather mild assumption. And there is also the possibility of identification through instrumental variables.

A second drawback is that one still cannot be sure that $\varepsilon_i$ solely consists of technical inefficiency. It is not possible to handle unobserved heterogeneity. However, it will be closer to an ideal measure and, at the moment, maybe the best we can do.

## 5.3 Future Research

There are natural extensions of all three articles, e.g. the MSE-efficient FE estimator of Article I could be used to construct the diagnostic tools in Article III. This will likely give finite sample improvements to the skewness test and the confidence intervals of the ratio estimator.

I also think that there are possibilities to improve the estimator in Article II, e.g. I have not investigated the possibility to derive a MSE-efficient bandwidth vector.

Overall though, I think that the third article is the most promising one, in terms of future research. There has been very little done, when it come to retrieving information from the firm effect of the FE estimator. As I show in this article, it is possible to consistently obtain information of the population distribution of the firm effects and therefore, also of the inefficiencies.

There are several possibilities of new FE estimators based on the *Method of Moments*.

- With help of the consistent estimator of the second order central moment (the variance) it is possible to construct standard estimators of inefficiency based on less restrictive assumptions.

  - Method of moments estimators based on a single-parameter distribution of the inefficiencies. For example the Half-normal or the exponential distribution. This is still fixed effects estimation, but with a distributional assumption on the inefficiencies.

- With the help of the second and third order central moments we can estimate non-standard estimators that allow for symmetrical, left or right skewed inefficiencies.

  - Method of moments estimators based on a two-parameter distribution of the inefficiencies. I believe there are distributions much more flexible than the gamma distribution and still fully estimable through the method of moments.

# References

G. Battese and T. Coelli. Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India. *Journal of Productivity Analysis*, 3:153–169, 1992.

C. Cornwell, P. Schmidt, and R. Sickles. Production frontiers with cross-sectional and time-series variation in efficiency levels. *Journal of Econometrics*, 46(1-2):185–200, October 1990.

Q. Feng and W. Horrace. Alternative Technical Efficiency Measures: Skew, Bias and Scale. *Journal of Applied Econometrics*, 2012.

W. Greene. Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics*, 126:269–303, 2005a.

W. Greene. Fixed and Random Effects in Stochastic Frontier Models. *Journal of Productivity Analysis*, 23:7–32, 2005b.

J. Hausman and W. Taylor. Panel Data and Unobservable Individual Effects. *Econometrica*, 49:1377–1398, 1981.

M. Kim, Y. Kim, and P. Schmidt. On the accuracy of bootstrap confidence intervals for efficiency levels in stochastic frontier models with panel data. *Journal of Productivity Analysis*, 28:165–181, 2007.

D. Ouyang, Q Li, and J. Racine. Nonparametric Estimation of Regression Functions with Discrete Regressors. *Econometric Theory*, 25:1–42, 2009.

P. Satchachai and P. Schmidt. Estimates of technical inefficiency in stochastic frontier models with panel data: generalized panel jackknife estimation. *Journal of Productivity Analysis*, 34:83–97, 2010.

P. Schmidt and R. Sickles. Production frontiers and panel data. *Journal of Business & Economic Statistics*, 2:367–374, 1984.

W. Wang and P. Schmidt. On the distribution of estimated technical efficiency in stochastic frontier models. *Journal of Econometrics*, 148:36–45, 2009.

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