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In the working plan for a large area you wish to calculate the annual cut m during the next period of t years. You know the wood capital to be k m^3 with the aid of an assessment, and also the actual percentage of increment, p %. It will now be easy to calculate m if only you can fix the final capital K at the end of the period of t years. In the case of *thinning* you can get the desirable amount of K from a yield table. The *final felling* means that $K = 0$. Other cases may occur when the capital is diminished by successive cuttings during a long period and when you are, e. g., leaving only seed trees standing throughout this period. We do not propose to discuss here how to handle all these different programmes in detail. As soon as the programme has been decided upon you should know the values of K, k, t and p in order to be able to work out m and our present purpose is only to show how this should be done.

First, there is the question of p. Either simple or compound interest has to be used, and the formulae will differ accordingly. Generally speaking, the simple interest method may be applied to old mature stands which grow slowly but the compound interest method is suitable for young stands where thinnings are carried out[1]).

I. Compound Interest

If no felling is done during the period of t years we get $K = k\ 1.\mathrm{op}^t$. But now fellings are carried out t times, on each occasion m m^3 being removed. Hence it follows that the term $t \times m$ must be subtracted. Moreover, we lose the increment of every capital m as soon as it has been cut. Therefore we must also subtract the sum of these losses. If we put Z for the sum of lost increment we arrive at the following formula for the capital at the end of t years.

$$K = k\ 1.\mathrm{op}^t - tm - Z \qquad (1)$$

An expression for Z is obtained as follows:

On the felling at the end of t years we lose no increment
» » » » » » » $t-1$ » » » $m\,(1.\mathrm{op}-1) = m\ 1.\mathrm{op} - m$
» » » » » » » $t-2$ » » » $m\ 1.\mathrm{op}^2 - m$
» » » » » » » 1 » » » $m\ 1.\mathrm{op}^{t-1} - m$

1) See Bulletin No. 14 of the Royal School of Forestry, Stockholm 1953; Sven Petrini: Intérêt simple ou intérêts composés dans le calcul de la croissance.

The sum of the lost increment is $Z = m \dfrac{1.op^t - 1.op}{0.op} - (t-1)\, m$

Instead of Z in formula 1 we insert the last expression, so that

$$K = k\ 1.op^t - tm - m. \frac{1.op^t - 1.op}{0.op} + m\,(t-1) = k\ 1.op^t - m. \frac{1.op^t - 1}{0.op}$$

The formula for m will then be

$$m = 0.op\ \frac{k\ 1.op^t - K}{1.op^t - 1} \quad \ldots\ldots\ldots\ldots \quad (2)$$

II. *Simple interest*

A calculation with a rebate percentage p for t years, when cutting m each year, makes the end capital

$$K = k\,(1 + \frac{tp}{100}) - tm - Z \quad \ldots\ldots\ldots\ldots \quad (3)$$

In formula 3 we presume that the capital k is increased by an equal annual amount, namely $k \times 0.op$. This means that the annual percentage of increment is in reality continually diminishing during the period.

Z can be expressed in the form of a series according to the foregoing reasoning. We have to subtract exactly as much from the increment involved in the term $k\,(1 + \frac{tp}{100})$ as is lost by cutting m m³ on the different occasions. To be able to build up the series we must know the initial capital of the cuttings, because the increment is reckoned to be p % of the capital at the beginning. Taking them in order, these initial capital values are:

The first annual cut at the end of year 1: $\dfrac{m}{1 + \dfrac{p}{100}}$

» second » » » » » » » 2: $\dfrac{m}{1 + \dfrac{2p}{100}}$

» last » » » » » » » t: $\dfrac{m}{1 + \dfrac{tp}{100}}$

On the last cutting we lose no increment, on the preceding one we lose 1 year's increment, and so on.

Hence we get the following series of $(t-1)$ terms

$$Z = m \left[\frac{p\,(t-1)}{100 + p} + \frac{p\,(t-2)}{100 + 2p} + \ldots\ldots + \frac{p}{100 + (t-1)\,p}\right] = mS$$

If we insert mS instead of Z in formula 3 we have

$$K = k\,(1 + \frac{tp}{100}) - tm - m\mathrm{S},\ \text{or}$$

$$m = \frac{k\,(1 + \frac{tp}{100}) - \mathrm{K}}{\mathrm{t} + \mathrm{S}} \qquad \text{(4)}$$

S can be worked out for different values of p and t. Table 1 contains t + S for actual values of the period t and the increment percentage p.

Table 1. **The sum of t + S for different values of t and p.**

Period	Increment percent p		
t years	1 %	2 %	3 %
10	10.434	10.840	11.220
15	15.998	16.905	17.736
20	21.779	23.354	24.769
30	33.96	37.3	40.2

Example: A standing volume of 10.000 m^3, reckoned to increase its volume annually by 2 % simple interest during a period of 20 years, has to be cut entirely during that period. Using formula 4 and table 1, the annual felling is found to be

$$m = \frac{10.000 \times 1.4 - 0}{23.354} = 600\ m^3$$

Note: If the growth had been reckoned at 2 % compound interest, the annual felling would have had to be calculated by formula 2 as follows:

$$m = 0.02\ \frac{10.000 \times 1.02^{20} - 0}{1.02^{20} - 1} = 612\ m^3$$

In the case of simple interest it might be considered inconvenient to be dependent upon table 1. Hence it may be of interest to construct a formula that can be used direct with a fair approximation. The accuray of the approximate formula can be proved by using formula 4 and table 1.

It may be thought reasonable to calculate the lost increment under the assumption that the *percentage* of growth is constant during the period. This will give us the formula 5 below. It is obvious, however, that the result obtained by using this formula will always be too low because the amount of lost increment is over-estimated.

As before, no loss is to be computed on the last annual felling at the end of t years

On the penultimate cut we lose	$m\ 0.0p$
On the preceding cut	$m\ 2 \times 0.0p$
On the first cut we lose	$m\ (t-1)\ 0.0p$
Total lost increment Z =	$\frac{m\ 0.0p \times t\ (t-1)}{2}$

Hence we get from formula 3

$$K = k\ (1 + \frac{tp}{100}) - tm - m\ \frac{0.0p \times t\ (t-1)}{2}, \quad \text{or}$$

$$m = \frac{k\ (1 + \frac{tp}{100}) - K}{t\left[1 + \frac{(t-1)\ p}{200}\right]} \qquad \text{(5)}$$

In the foregoing example the annual felling according to formula 5 will be

$$m = \frac{10.000 \times 1.4 - 0}{23.8} = 588\ m^3$$

The difference between the results from the exact formula 4 and the approximate formula 5 is due to the denominator since the numerator is the same. The error attached to formula 5 reckoned in % of the correct value from formula 4 may be expressed by the formula

$$e = 100\left[\frac{t + S}{t\ (1 + \frac{(t-1)\ p}{200})} - 1\right]$$

In table 2 this error is registered.

Table 2. **Percentile error of the annual cut using formula 5.**

Period t years	Annual percentage of increment p		
	1 %	2 %	3 %
10	— 0.16	— 0.55	— 1.14
15	— 0.38	— 1.14	(— 2.28)
20	— 0.55	(— 1.87)	(— 3.62)
30	— 1.15	(— 3.62)	(— 6.62)

The error is invariably negative. If it is considered expedient always to remain on the safe side, i. e. always to keep slightly on the negative side when estimating the amount of felling, then this will be reached by using formula 5. With a period of not more than 10 years the error is very moderate, and if the growth is slow it will always remain moderate even with such a long period as 30 years. In table 2 those values which are considered to contain too large a systematical error are put within brackets. – It might be mentioned here that such wood capital as has a growth of 3 % or more is practically never doomed to be entirely cut out but should be treated by thinning, in which case the compound interest method should be applied (cfr. formula 2).

It may, however, be possible to find another approximate formula that is better than formula 5. In many old formulae for calculating the possible yield it is assumed that the volume of the fellings, when made continually each year, can be reckoned as having been growing during half the period in question, here for $t/2$ years. Under this assumption the lost increment in formula 3 can be easily determined.

The total volume of the fellings is tm m³. The initial capital of this volume is then found to be

$$\frac{tm}{1 + \frac{tp}{200}} \quad \text{hence} \quad Z = \frac{tm}{1 + \frac{tp}{200}} \times \frac{t}{2} \times 0.0p.$$

From formula 3 we then get the end capital K

$$K = k\,(1 + \frac{tp}{100}) - tm\left[1 + \frac{tp}{200 + tp}\right] \qquad \text{(6)}$$

and the volume of the annual felling

$$m = \frac{k\,(1 + \frac{tp}{100}) - K}{t\,(1 + \frac{tp}{200 + tp})} \qquad \text{(7)}$$

In the example this will mean

$m = \frac{14.000}{23.333} = 600$ m³, wich is the same amount as that obtained by using the exact formula 4. The accuracy of formula 7 is demonstrated in table 3.

Table 3. **The error the formula 7 in % of the result obtained with formula 4.**

Period t pears	Annual percentage of increment p		
	1 %	2 %	3 %
10	— 0.40	— 0.64	— 0.74
15	— 0.30	— 0.31	— 0.11
20	— 0.18	+ 0.09	+ 0.63
30	+ 0.13	+ 1.02	(+ 2.26)

As can be seen in table 3, the approximation with formula 7 is very good even for long periods. Moreover formula 7 normally gives results »on the safe side», so that only with a long period and when the increment percentage has a high value will the result be a positive error.

Summary

The problem is to calculate the volume of the annual felling m when a wood capital k, growing with an increment percentage p, is to be totally or partially cut out during a period of t years. The standing volume at the end of the period is called K.

Using *compound interest* we obtain the correct value as

$$m = 0.op \frac{k\ 1.op^t - K}{1.op^t - 1} \quad \dots\dots\dots\dots \quad (2)$$

When simple interest on the initial capital k is calculated the exact formula is

$$m = \frac{k\,(1 + \frac{tp}{100}) - K}{t + S} \quad \dots\dots\dots\dots \quad (4)$$

Table 1 shows — for different values of t and p — the sum $t + S$ to be written in the denominator of formula 4. The table can easily be completed.

If one desires to be independent of table 1 it is possible to use the approximate formulae 5 and 7. The first is reached by assuming that p remains constant during the period, which, however, does not agree with the definition of simple interest on the initial capital.

$$m = \frac{k\,(1 + \frac{tp}{100}) - K}{t(1 + \frac{(t-1)\,p}{200})} \quad \dots\dots\dots\dots \quad (5)$$

The error with formula 5 is always negative and can be studied in table 2. Formula 5 can be used only if the period t is short or the percentage p is low.

The other approximate formula runs thus

$$m = \frac{k\,(1 + \frac{tp}{100}) - K}{t\,(+ \frac{tp}{200 + tp})} \quad \dots\dots\dots\dots \quad (7)$$

This formula is better than formula 5 and can be used for practically all final fellings, though formula 4 ought, of course, to be preferred. In table 3 are found the percentual errors involved in calculations based on formula 7.

Remark: Formula 2 — recommended for all young and fast-growing forest where it is a question of thinnings — gives exactly the same results as the following formula:

$$K = k \left[1.0p^n - \frac{a}{p}(1.0p^n - 1)\right], \text{ where } m = \frac{ak}{100}$$

Further formula 7 is the same as the following formula

$$m = \left[k\left(1 + \frac{np}{100}\right) - K\right] \frac{\frac{100}{n} + \frac{p}{2}}{100 + np}$$

Both these formulae are published in the book »Sven Petrini: Elements of Forest Economics», 1953 (Oliver & Boyd Ltd, Edinburgh).

Resumé

Beräkning av årsavverkningen.

Formlerna avse årsavverkningens bestämmande under förutsättning att begynnelseförråd, tillväxt, slutförråd och periodens längd är kända storheter. När sammansatt ränta begagnas vid tillväxtberäkningen erhålles en enkel exakt formel. Med enkel ränta bli kalkylerna mera komplicerade, så att antingen en hjälptabell behöver användas eller också approximationer tillgripas.