STUDIA FORESTALIA SUECICA

Nr 18 1964

Studies of Cross-Country Transport Distances and Road Net Extension

Studier över terrängtransportens längd och vägnätets utbyggnad

by

GUSTAF VON SEGEBADEN

Abridged version of Research Note no. 23 from the Department of Operational Efficiency at the Royal College of Forestry

Sammandrag av rapport nr 23 från institutionen för skogsteknik vid Skogshögskolan

SKOGSHÖGSKOLAN

STOCKHOLM

CONTENTS

PREFACE

The first part of this paper deals with the relationship between cross-country transport and road net density while the second part is devoted to determinations of total length of roads. Although the parts are independent of each other, they belong to the same subject matter, and it has been considered feasible to publish them together under a common title.

In a concentrated form the paper reports on the content of a licentiate thesis for degree in forest work techniques presented in the spring of 1962 and later published in a slightly revised version as report no. 23 from the Department of Operational Efficiency at the Royal College of Forestry.

Stockholm, April, 1964.

Gustaf von Segebaden

1*—412713

INTRODUCTION

"King Anund ordered roads to be built throughout Svithiod, through forests, over bogs and mountains, and he was named Anund the Road Builder" Snorre Sturlasson (1179—1241) wrote in the Norwegian "Tales of the Kings" (26).

The extension of a rational road net currently being the most profitable measure of rationalization in forestry, road building is an urgent undertaking in our times as well.

Since forestry is carried out over large areas, the product, timber, must be terminally collected into bulk quantities for rational manufacturing. Moreover, labour and appliances must be distributed from centres of population to the individual places of operation in the forests. The extraction of timber is one of the most expensive links in the chain of transport operations, be it manual, by horse, or by machines. It is therefore advantageous by road construction to shorten the distances of extraction and to forward the timber expediently to transport ways with great capacity and, hence, low direct costs of transport. To labour road construction also means shorter cross-country walking distances from the road to the place of operation. Amongst other utility aspects of an extended road net that may be mentioned here are cheaper transport of appliances for silvicultural work, fire control and, in some cases, facilitated use of machines.

To solve the problem of transport economy by computing the optimum net density and standard of the forest roads *Sundberg* (28), *Larsson* (17), and *Larsson & Rydstern* (18) have recently designed various models. The main principle has been to establish a function for the total of those costs within the area influenced by the road which are affected by the construction of the road. The optimum design of the road net is obtained when the total cost per volume unit of timber is at a minimum. The factors most significant at calculations of this kind are generally the direct costs of transporting timber and carrying labour cross-country and on the road, the cost of road construction and the volume of timber to be felled and, hence, the suitable time of road extension with respect to age and condition of the forests (20, 23 and 1).

Optimum calculations of this kind and other computations concerning the extension of the forest road net must comprise a relationship between the degree of road extension (road net density) and the distance of carrying the timber and labour cross-country (cross-country transport distance).

Moreover, the original status of the road net extension in the area concerned must be known. Knowledge of the distance to road not only for the forest land but also for the quantities of timber, if possible distributed between felling classes, species, sizes, etc., is of great value at a priority rating of various projects of road construction and at other computations of management economy.

The following presentation reports on some methods of collecting data for a determination of the relationship between the cross-country transport distance and the road net density, and for a determination of length of roads. The methods, which are primarily intended for summary calculations for large areas, might be used for inventory purposes and for the analysis of a certain status of extension as well as prognoses and evaluations of continued extension.

A. Cross-country transport distance and road net density

1. General review of the relationship between cross-country transport distance and road net density

No mathematically accurate relationship that is generally applicable exists between the cross-country distance and the road net density since the design of the road net and topography, both of which influence on this relationship, vary irregularly.

Provided that all transport occurs on a plane ground, that the roads are straight lines and parallel, that the perpendicular distances between the roads are equal, and that the cross-country transport is straight-lined and perpendicular to the road, the following relationships nevertheless apply (cf. figure 1)

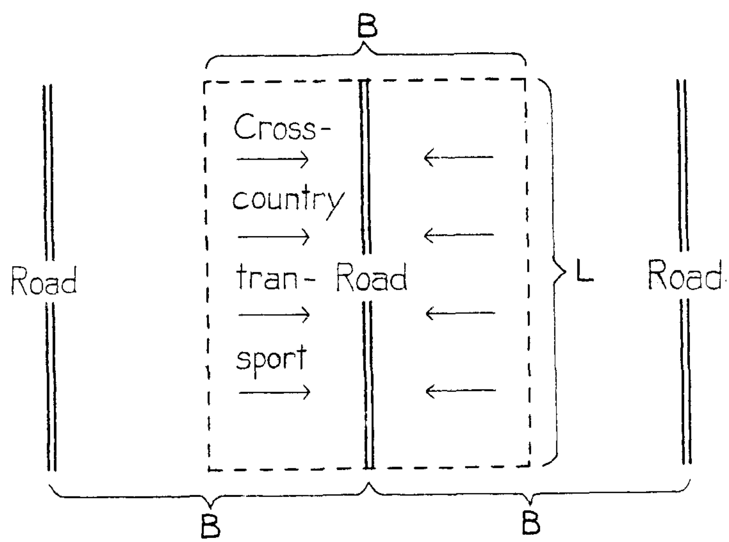


Fig. 1. Model of transport with straight and parallel roads situated at equal distances from each other.
The cross-country transport is carried out perpendicularly to and straight toward the roads.

$$\begin{cases} Mg = \dfrac{B}{4} \\ V = \dfrac{10 \cdot L}{B \cdot L} \end{cases}$$

$$Mg = \frac{2.5}{V} \tag{1}$$

where Mg = the geometrical mean cross-country transport distance, km
V = the road net density, metres of road per hectare[1]
B = width of the area influenced, km (= distance between the roads)
L = length of the area influenced, km (= road-length within the area)

The geometrical cross-country transport distance is the shortest straight-line distance from a given point to the nearest road. The geometrical mean cross-country transport distance of an area is an arithmetic mean of the geometrical cross-country transport distances from an infinite number of points evenly distributed over the area, each point representing an infinitely small area.

The relationship expressed in formula (1) is currently used for summary calculations of the road net density. Since the cross-country transport to an access road in practice seldom moves perpendicularly nor straight toward the road, the geometrical mean cross-country transport distance is given a percentage allowance in order to obtain the value of the actual transport distance.

Occasionally, this allowance is made by means of an adjustment for the increase in the geometrical cross-country transport distance caused by hauling the timber to special landings instead of leaving the timber evenly distributed along the entire length of the road, and by means of an adjustment due to the fact that the course of haulage between the stump and the landing does not follow a straight line, an allowance for winding course. The value of the first of these adjustments varies with the geometrical mean cross-country transport distance and the distances between the landings (cf. table 10, p. 32). The allowance for the winding course of horse logging is usually estimated at 10—30 % (28). In other cases both these adjustments are lumped into one amounting to 30—40 % (2).

Any change in the conditions concerning the roads in formula (1) effects an increase in the geometrical mean cross-country transport distance. Just the difference that the road haulage winds symmetrically along the course of the postulated straight lines means that formula (1) results in too short

[1] 1 metre/hectare (m/ha) = 0.1 km/sq. km = 0.16 miles/sq. mile = 1.33 feet/acre.

geometrical mean cross-country transport distances. Computing empirically the values of Mg and V, both of which are affected in this case by the changes in the course of the haulage, and introducing these values in the formula

$$Mg = \frac{2.5 \cdot k}{V} \tag{2}$$

we obtain by a solution of k an adjustment for the deviations from the "ideal" conditions of formula (1). Values of adjustment can be similarly obtained for other deviations and combinations of deviations with respect to the courses of roads according to formula (1); hence, adjustment can be obtained for real, irregular road nets. This adjustment is here called road net adjustment (V-corr).

Values of adjustment can be obtained in a similar way for deviations in the course of cross-country transports from the conditions of formula (1):

$$Mp = Mg \cdot T\text{-corr} \tag{3}$$

where Mp is "the practical mean cross-country transport distance" (= actual mean haul).

This adjustment is called the cross-country transport adjustment (T-corr). If the relationships according to the formulas (2) and (3) are known, formula (4) can be established for the practical mean cross-country transport distance

$$Mp = \frac{2.5 \cdot \text{V-corr} \cdot T\text{-corr}}{V} \tag{4}$$

An adjustment, *the road net adjustment, V-corr, is* thus explored in the following presentation on the basis of formula (2) *to compensate for the fact that the roads are not straight nor parallel and that the distances between the roads are not equal,* while another adjustment, *the cross-country transport adjustment, T-corr, is* obtained on the basis of formula (3) *for the conversion of the geometric cross-country transport distance to the practical cross-country transport distance.* (Thus, the adjustments do not comprise adaptations caused by variations in the volume of timber felled in various parts of the road net system etc.)

2. Road net adjustment

a) Models of road nets

A study of the change in the road net adjustment at certain given alterations in the models of road nets may serve as a basis for the judgement of the road net adjustments obtained in practice.

Various models of road nets are shown in the series of figures 2—5. Series

Fig. 2

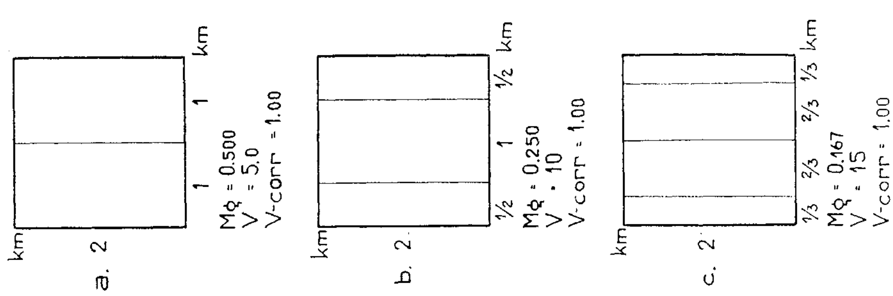


Fig. 3

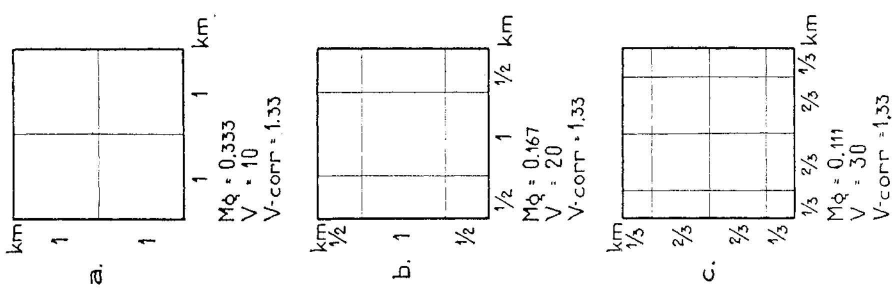


Fig. 4

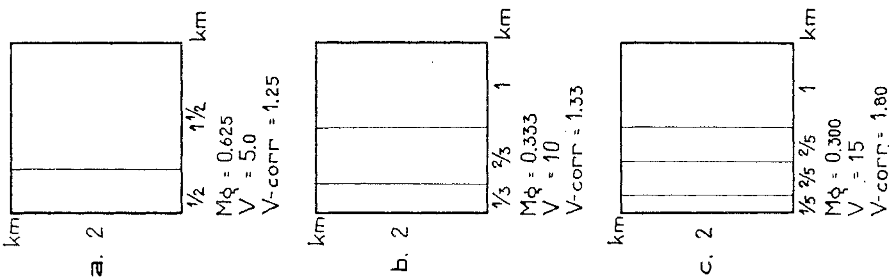


Fig. 5

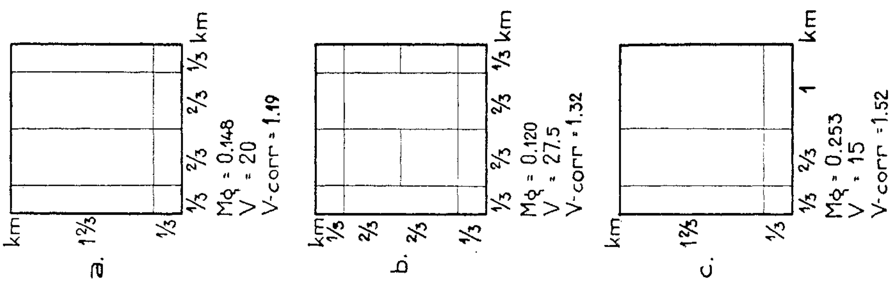


Fig. 2—5, a—c. Road net adjustments for various models of road nets.
Mg = geometric mean cross-country transport distance, km
V = density of the road net, m/hectare
V-corr = road net adjustment

no. 2 shows the type of road net that entirely meets the requirements of formula (1) by having straight, parallel roads situated with equal distances apart. The road net adjustment in this case naturally becomes 1.00. Series no. 3 shows a type of road net where the roads form a net of equally large squares. The *V*-corr in all three cases a—c has the same value, 1.33. Series no. 4 is a road net with parallel roads situated at different distances apart. Rising with increasing unevenness in the distribution of roads, *V*-corr varies between 1.25 and 1.80 in the area.

Series no. 5 of the figures shows different road nets of squares and rectangles where *V*-corr assumes the values 1.19, 1.32, and 1.52.

A comparison between the figures 2c, 5a, and 5c shows how an extension of the lengthwise roads in order to achieve improved evenness in the road system provides an essentially greater effect by reducing the cross-country transport distance than does an extension of the cross roads. It is to be kept in mind, however, that the cross roads may have an influence on i.a. the distance of lorry haul, a matter beyond the scope of this work.

Series no. 2 and no. 3 of the figures have shown that the road net adjustment is constant and independent of the net density in road systems of equal design. These facts are shown in a more general way for nets where the roads constitute sides of equally large equilateral triangles, equally large squares, and equally large, regular hexagons; these figures being the only equally large, regular polygons that can cover a surface entirely (figure 6):

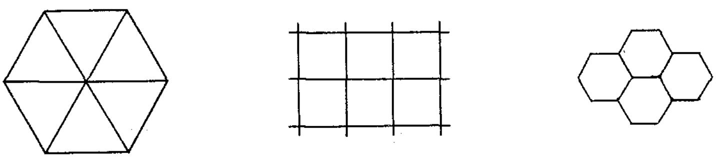


Fig. 6. Approximate outlines of road nets forming equally large equilateral triangles, equally large squares, and equally large regular hexagons.
The outlines are drawn to a scale giving equal road net density in the three cases.

	Equilateral triangle	Square	Regular hexagon
Length of side	a_1	a_2	a_3
Area served per side	$\frac{a_1^2\sqrt{3}}{6}$	$\frac{a_2^2}{2}$	$\frac{a_3^2\sqrt{3}}{2}$
Road-length per unit area, V	$\frac{2\sqrt{3}}{a_1}$	$\frac{2}{a_2}$	$\frac{2\sqrt{3}}{3a_3}$
Geometric mean cross-country transport distance, Mg	$\frac{a_1\sqrt{3}}{18}$	$\frac{a_2}{6}$	$\frac{a_3\sqrt{3}}{6}$
V-corr $\left(=\frac{V\cdot Mg}{2.5}\right)$	1.33	1.33	1.33

Evidently, the value of the road net adjustment is constant and independent of the length of the side a, and, hence, also independent of the road net density.

The road net adjustment being equal in the three cases depends on the fact that the area influenced by each side is bordered by two isosceles triangles with common base. The side thus serves an area only half that served by the corresponding road-length in a parallel system, and the mean cross-country transport distance is 2/3 of the corresponding value in the "ideal case":

$$\begin{cases} \dfrac{2}{3}\,Mg = \dfrac{2.5 \cdot V\text{-corr}}{2 \cdot V} \\[2ex] Mg = \dfrac{2.5}{V} \end{cases}$$

$$V\text{-corr} = \frac{4}{3} = 1.33$$

This statement also applies to other road net models with triangular areas of influence such as certain types of road nets with zig-zag roads. (The quadratic road net may be considered a special case of such a road net).

Since the road net adjustment is equally large in the road nets, the types are equal from the point of geometric cross-country transport; at equal road distance per unit area an equally large geometric mean cross-country transport distance being obtained.

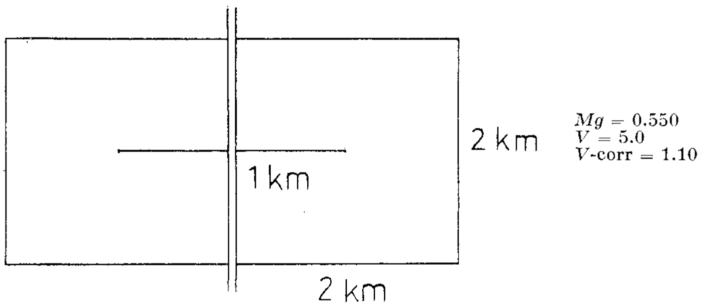


Fig. 7. Road net model with branch roads from a throughfare.

A model with roads branching out from a throughfare is shown in figure 7. The terminal points of the branch roads have been chosen at a distance from the far boundary of the area corresponding to the range of effect, i.e. equal to half the distance between the branch roads. The spacing between the throughfares is twice the spacing of the branch roads. The length of the branch roads constitutes 50 per cent of the total road distance, and the road net adjustment is 1.10. If the spacing of the throughfares is redoubled the road net adjustment will be 1.05.

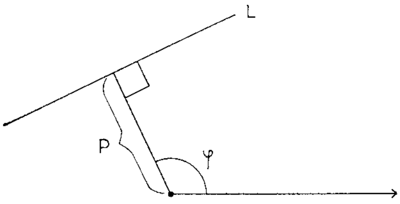


Fig. 8. Design of a random road net.

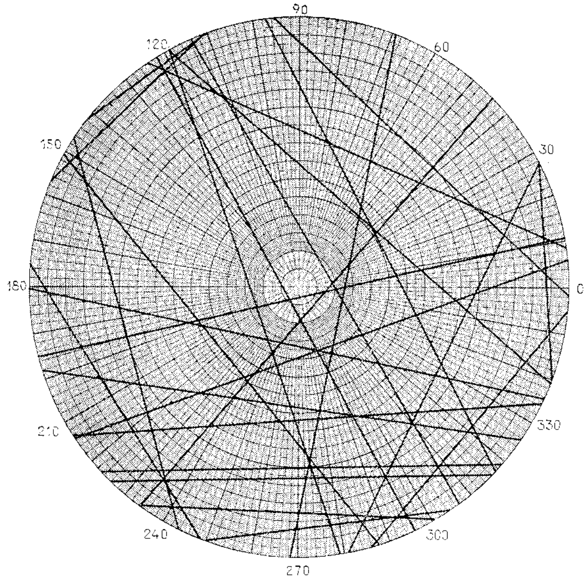


Fig. 9. Example showing the design of a random road net where φ has been chosen with so-called rectangular distribution between 0 and 2π.

Dr. *B. Matérn* has computed the V-corr of randomly designed road nets. Consisting of infinitely long lines, such a road net is constructed in the following way.

A straight line L is determined by its normal coordinates P and φ (cf. figure 8). Values of φ are alotted between 0 and 2π according to some distribution which is symmetric around π, but requires no further definition. Values of P are chosen between 0 and ∞, on an λ average values in each interval of length *1* (more precisely expressed: according to a Poisson process with intensity λ).—Figure 9 shows an example of a road net designed in this way where φ was chosen with rectangular distribution. In the road net thus obtained the average distance ($\bar{a}$) from an arbitrarily chosen point to the nearest point on a road (mathematical expectation) is

$$\bar{a} = \frac{1}{\lambda}$$

The mean road-length per unit of area is

$$\bar{v} = \frac{\lambda}{2}$$

Thus, we get

$$\bar{a} \cdot \bar{v} = \frac{1}{2}$$

which, introduced into formula (2) gives V-corr $= 2.00$.

The distance to the nearest road from an arbitrarily chosen point has a distribution expressed by the following density (or frequency) function:

$$\lambda \cdot e^{-\lambda x}$$

the mean value of which is $\frac{1}{\lambda}$ ($= \bar{a}$, above) with a dispersion of $\frac{1}{\lambda}$

The value of road net adjustment, 2.00, obtained here is probably the highest one that can be derived for model nets with an even distribution of roads when it is entirely designed without consideration being taken to the rational viewpoints with respect to the cross-country transports.

Figure 4 shows that the road net adjustment rises as unevenness of the distribution of roads over the area increases. This matter has also been discussed in more general terms for an area consisting of two parts with road nets of equal design but with different road-length per unit of area (table 1). When the two parts of the area are equally large and the road-length per unit of area in one part is twice that in the other part, the table shows that the road net adjustment for the entire area is 12 per cent higher than for the parts. Larger differences in road net density cause V-corr to increase strongly.

Table 1. Road net adjustment for an area comprising two parts with road nets of equal design but different road-length per unit area.

V-corr of the entire area is expressed in per cent of V-corr of the parts. Each part may consist of one area or of several part areas.

Area of the small part in per cent of the large part	Road-length of the small part per unit area in per cent of that of the large part									
	25	50	75	100	125	150	200	300	400	500
25	136	108	101	100	101	103	108	121	136	151
50	150	111	102	100	101	104	111	130	150	171
75	155	112	102	100	101	104	112	133	155	178
100	156	112	102	100	101	104	112	133	156	180

At calculations including road net adjustment for an area that consists of, or should consist of, parts with differences in the density and form of the road net, each part should be treated separately, if possible. The road net adjustment for the entire area means little information and may eventually be directly misleading with respect to the "effect" in the part areas.

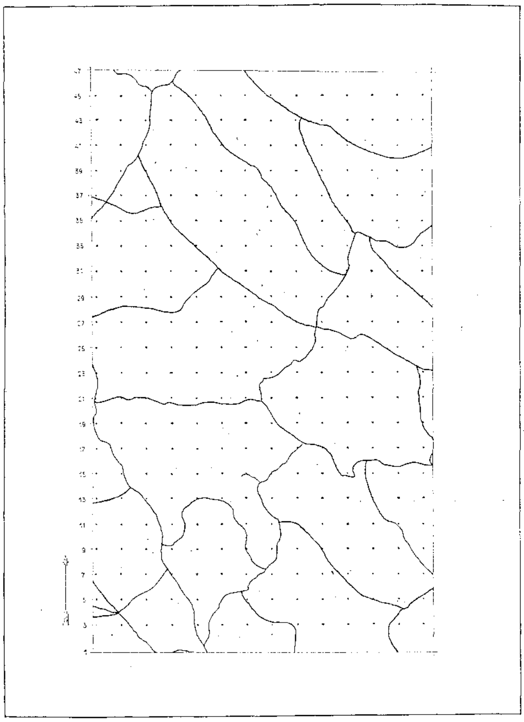


Fig. 10. Part of map sheet "Stensele".
Existing and proposed roads.

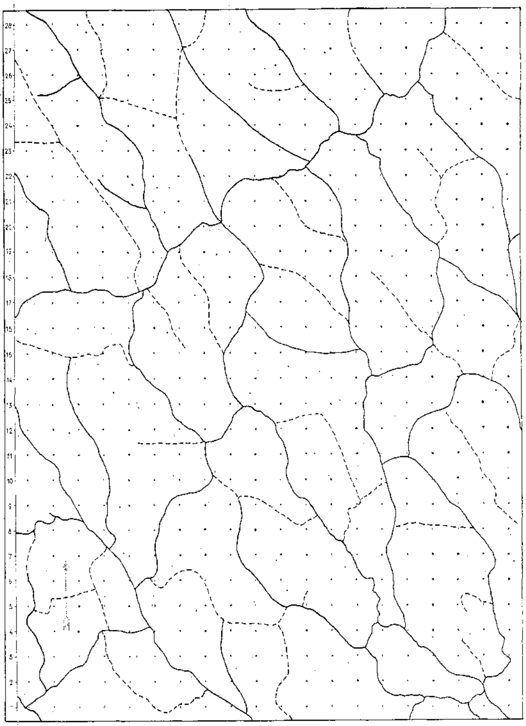


Fig. 11. Part of map sheet "Harads".
Existing roads: full lines.
Proposed roads: dashed lines.

b) Real road nets

After exploratory investigations the geometrical mean cross-country transport distance has been computed from maps of areas situated in northern and middle Sweden.

Methods

Regular systems of points, mostly in a square spacing (cf. figures 10 and 11), have been overlaid on maps showing roads. The shortest straight-line distance (= the geometric cross-country transport distance) to the nearest road has then been measured from each point in the system. The sum of all these "shortest straight-line distances" divided by the number of points gives an estimate of the geometric mean cross-country transport distance.

The accuracy of the determination of the geometric mean cross-country transport distance will depend on the accuracy of the measurements and on the number of points as well as on the size of the area and the value of the geometric mean cross-country transport distance.

Accuracy

The measurements of the straight-line distances were made with compass permitting an accuracy of 0.5 mm, which corresponds to 50 metres at a scale of 1:100 000. The rounding-off error of the mean value thus becomes $\pm \frac{50}{\sqrt{12\,n}}$ metres, where n is the number of individual measurements. Already at a point number of 10 the rounding-off error of the mean value is less than $\pm$ 5 metres, which can be considered entirely acceptable in this instance. However, the difficulty in reading correctly half millimetres on a scale graduated with 1-mm units contributes to give an error of measurement of the mean value that is slightly greater than that cited above.

In order to study how the precision of the mean value depends on the number of points the variation (expressed as variance per point) has been investigated for square nets of points of various densities. Corresponding studies have also been carried out for point nets where the points were the corners of the survey tracts used by the National Forest Survey.[1] It is significant in the latter case that the distance between the corners of a tract (1.8 km in the area concerned) is small in relation to the distance between the tracts (cf. table 12 and figures 19 and 20, p. 43—45). Although the points will be situated in a regular spacing, they will not be evenly distributed but occur in clusters. Reference is made to the theoretical background of these calculations as discussed by *Matérn* (19).

Based on the precision investigations outlined above at various densities of road nets, an approximate formula has been constructed for the computation of the standard error of the estimate of the mean distance of an area at different conditions in respect of the size of the area (A), mean distance (Mg), and the number of sample points (n) in a regular spacing.

The formula is based on experiences gained concerning the relative standard deviation (σ) per point in the spacing at various densities of point pattern:

Area	Road net	n	$\frac{A}{n \cdot Mg^2}$	σ per cent
Harads	Present	48	18.2	52
	Planned	48	39.9	78
	Present	108	8.1	44
	Planned	108	17.7	60
Province of Jämtland				
Part I	Present	38 (1)	49.8 (1 882)	98.5 (109.8)
	Planned	38 (1)	86.0 (3 256)	101.8 (117.0)
Part II	Present	34 (1)	256.6 (8 770)	82.0 (88.1)
	Planned	34 (1)	375.6 (12 836)	100.8 (89.6)
Part III	Present	20 (1)	81.9 (1 647)	99.2 (105.3)
	Planned	20 (1)	141.4 (2 842)	90.1 (100.8)

[1] The survey is a combined line-plot survey along the periphery of systematically spaced squares, so-called "tracts". In north Sweden the sidelength of the tracts is 1.8 km, in southernmost Sweden 1.2 km.

Figures within parentheses refer to the case when only one sample point is chosen at random in the area. The n-values 38, 34, and 20 are number of tracts, the other n-values are number of points. The density of the point pattern has been expressed by means of the ratio $\frac{A}{n \cdot Mg^2}$, which is approximately proportionate to the number of road net meshes per sample point. Provided the road net is composed of squares, the distance between the roads is six times the mean distance and the area of the road net mesh is $36Mg^2$. The number of road net meshes within the area then becomes $A/36Mg^2$ and the number of road net meshes per point $A/(n \cdot 36Mg^2)$.

The formula designed has the following appearance

$$(5) \qquad {}^{10}\log \sigma = 1.42 + 0.30\, x \cdot \frac{20}{20 + x^2}$$

where $x = {}^{10}\log \left(\frac{A}{n \cdot Mg^2}\right)$ from which the relative standard error ε of the mean distance is obtained from the equation

$$(6) \qquad \varepsilon = \frac{\sigma}{\sqrt{n}}$$

The validity of the formula is based on the condition that the points are spaced regularly in squares. The formula is less accurate for negative values of x. For values of x exceeding 4.5 the value 4.5 shall be used in the formula. The formula may also be applicable to regular triangle spacings.

In spite of the condition concerning the design of the point net, formula (5) can also be applied for the precision of mean distances based on determinations for the four corners of the tracts used by the National Forest Survey. By this method, however, only limits are obtained which encompass the standard error. These limits are computed by using for n alternatively the number of tracts, and the number of determinations from the corners; the standard error should then be closer to the upper limit than to the lower one.

The number of points required in a regular square spacing has been presented in table 2 for the standard errors 2.5, 5, and 10 %, and for various areas and geometric mean distances.

Results and discussion

A study of the road net models in figures 2—5 shows that the road net adjustment varies strongly with the geometric design of the road net. The

Table 2. Number of points required in a regular square spacing to obtain a certain standard error of estimate of the geometric mean distance at various sizes of areas and mean distances. Parentheses in the table pertain to cases where the area of the road net mesh (36 Mg^2) is larger than the size of the actual area concerned. Number of the points given in these cases can be considered valid on an average for alternative locations of the area in relation to the road net.

Area sq. km	Standard error, per cent														
	2.5					5					10				
	Geometric mean distance to road, km														
	0.5	1.0	1.5	2.0	2.5	0.5	1.0	1.5	2.0	2.5	0.5	1.0	1.5	2.0	2.5
25	110	(65)	(50)	(40)	(30)	45	(25)	(20)	(15)	(15)	20	(10)	(10)	(5)	(5)
50	140	80	(60)	(50)	(40)	60	35	(25)	(20)	(15)	25	15	(10)	(10)	(10)
75	160	95	(70)	(60)	(50)	70	40	(30)	(25)	(20)	30	15	(10)	(10)	(10)
100	180	110	80	(65)	(55)	75	45	35	(25)	(20)	30	20	15	(10)	(10)
150	210	120	90	75	(60)	85	50	40	30	(25)	35	20	15	15	(10)
200	230	140	100	80	(70)	95	60	45	35	(30)	40	25	20	15	(15)
250	250	150	110	90	75	100	65	45	35	30	40	25	20	15	15
300	270	160	120	95	80	110	65	50	40	35	40	30	20	15	15
400	300	180	130	110	90	120	75	55	45	40	45	30	25	20	15
500	320	190	140	120	100	130	80	60	50	40	50	35	25	20	15
750	370	230	170	130	110	150	95	70	55	50	55	35	30	25	20
1 000	410	250	190	150	130	160	100	75	60	55	60	40	30	25	20
1 250	440	270	200	160	140	170	110	85	70	60	65	40	35	30	25
1 500	470	290	220	170	150	180	120	90	75	60	65	45	35	30	25
2 500	550	350	260	210	180	210	140	110	90	75	75	50	40	35	30
5 000	680	440	330	270	230	250	170	130	110	95	90	65	50	40	35
10 000	820	540	430	350	300	300	210	160	140	120	100	75	60	50	45
20 000	1 000	680	520	440	380	350	250	200	170	150	110	90	75	65	55

Table 3. Calculation of the road net adjustment for the area Stenträsk.

	Entire area	Part S	Part N
Total area, hectares	127 218	63 609	63 609
Total length of roads, km	316.6	169.6	142.0
Road net density, *V*, m/hectare	2.45	2.67	2.23
Geometric mean cross-country transport distance, *Mg*, km	1.37	1.26	1.46
V-corr	1.34	1.35	1.30

Table 4. Calculation of the road net adjustment for the area Harads.

Degree of extension: 1 = present, 2 = planned road net.

	Entire area		Part S		Part N	
Total area, hectares	229 900		114 950		114 950	
Degree of extension	1	2	1	2	1	2
Total length of roads, km	497.4	757.5	263.5	379.9	233.9	377.6
V, m/hectare	2.16	3.27	2.29	3.30	2.03	3.28
Mg, km	1.46	0.97	1.39	0.96	1.53	0.99
V-corr	1.26	1.28	1.27	1.27	1.24	1.30

Table 5. Calculations of road net adjustments for the areas Malå, Jörn, and Åsele.

	Malå		Jörn		Åsele	
Degree of extension	1	2	1	2	1	2
Total length of roads, km	922.5	1 417.7	994.0	1 485.7	759.7	1 148.8
Total area, hectares	390 110		389 958		407 822	
V, m/hectare	2.36	3.63	2.55	3.81	1.88	2.84
Mg, km	1.56	0.94	1.46	0.99	1.98	1.31
V-corr	1.47	1.36	1.49	1.51	1.49	1.49
Land area, hectares	374 110		363 840		379 022	
V, m/hectare	2.47	3.79	2.73	4.08	2.00	3.03
Mg, km	1.57	0.94	1.48	0.99	2.05	1.34
V-corr	1.55	1.43	1.62	1.62	1.64	1.62

Table 6. Calculation of the road net adjustment on the basis of the geometric mean cross-country transport distance of the forest land and of the total land area, respectively, and the road net density of the land area within parts of the province of Jämtland.

"Difference in mean distance, per cent" is expressed in per cent of the mean distance of the forest land.

Part of province	Small area no.	Road net	Road net density m/hectare of land area	Forest land		Land area excl. mount.		Difference in mean distance per cent	No. tract corners	
				Mean distance km	*V*-corr	Mean distance km	*V*-corr		On forest land	On land excl. mount.
I	J:1-14	Present	1.83	2.53	1.85	2.55	1.87	1	611	877
		Planned	2.42	1.91	1.85	2.00	1.94	5		
II	J:15-24	Present	3.18	1.38	1.76	1.68	2.14	22	648	766
		Planned	3.83	1.10	1.69	1.26	1.93	15		
I-II	J:1-24	Present	2.49	1.92	1.91	2.09	2.08	9	1 259	1 643
		Planned	3.10	1.49	1.85	1.60	1.98	7		
III	H:1-8	Present	2.16	1.55	1.34	2.02	1.75	30	334	490
		Planned	2.79	1.22	1.36	1.51	1.69	24		
Whole province		Present	2.40	1.84	1.77	2.07	1.99	13	1 593	2 133
		Planned	3.02	1.43	1.73	1.58	1.91	10		

models considered most representative of actual road nets are the square rectangular net in figure 5 and the nets of equally large regular polygons in figure 6. For these nets the road net adjustment has been computed at 1.32 and 1.33 respectively.

For real road nets with even distribution over the area, the road net adjustment has been calculated at 1.24—1.35 when the measurements are based on the total size of the areas (tables 3 and 4). For larger areas with rather more uneven road nets (Malå, Jörn and Åsele) the adjustment has been estimated at 1.36—1.51 when the total area has been taken into account (table 5). When the land area only is considered within the same region, the values 1.43—1.64 have been obtained. The corresponding values for the

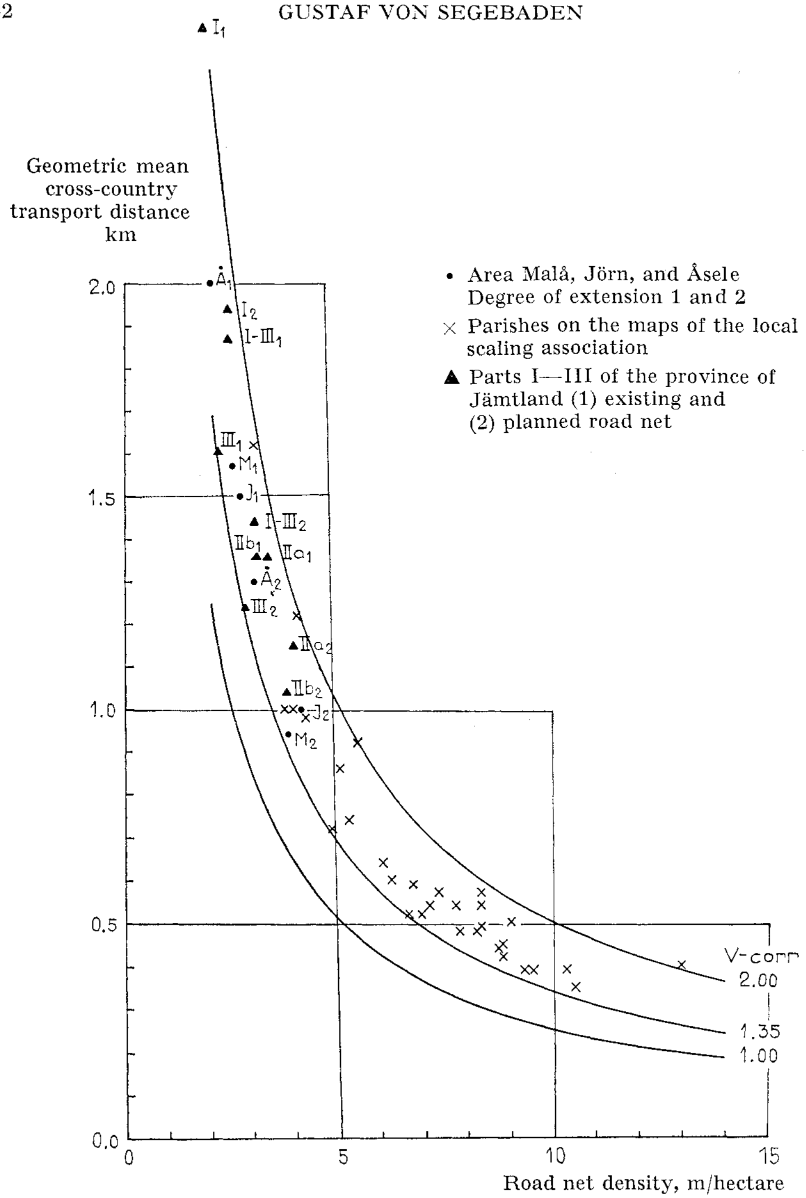


Fig. 12. Relationships obtained between road net density and geometric mean cross-country transport distance.

province of Jämtland amount to 1.69—2.14 (table 6). These values of road net adjustment for the land area closely agree with the road net adjustments computed in a similar way for 31 parishes in middle Sweden (table 7) despite the road net type and density are essentially different in the various areas (cf. figure 12).

Table 7. Calculation of road net adjustment on maps with roads approved for timber storage by a local scaling association in middle Sweden.

Parish	Total land area hectare	Approved roads km	Road net density m/hectare	Mean cross-country transport distance km	*V*-corr
Nora	28 942	206	7.11	0.54	1.53
By	30 970	273	8.82	0.45	1.59
Folkärna	18 959	166	8.74	0.44	1.56
Grytnäs	10 765	101	9.38	0.39	1.46
Hedemora	46 040	405	8.80	0.42	1.48
Husby	37 090	253	6.82	0.59	1.62
St. Skedvi	20 552	212	10.34	0.44	1.84
Vika	17 790	160	8.97	0.50	1.81
Sundborn	19 330	140	7.26	0.57	1.65
Svärdsjö	85 321	445	5.21	0.74	1.54
Enviken	34 170	185	5.40	0.92	1.98
St. Kopparberg	33 500	258	7.70	0.54	1.66
Gustafs	25 550	211	8.26	0.57	1.88
Floda	37 520	157	4.18	0.98	1.65
Ludvika	19 219	157	8.18	0.48	1.46
Grangärde	75 882	497	6.55	0.52	1.36
Säfsnäs	56 990	225	3.94	1.03	1.62
Nås	48 810	186	3.81	1.00	1.53
Gagnef	40 110	122	3.04	1.62	1.98
Ål	13 620	84	6.19	0.60	1.47
Leksand	81 903	389	4.75	0.72	1.37
Siljansnäs	26 871	133	4.95	0.86	1.71
Ockelbo	*100 162*	*694*	*6.93*	*0.52*	*1.44*
Järbo	14 061	85	6.02	0.64	1.54
Ovansjö	39 976	378	9.45	0.39	1.47
Torsåker	30 440	396	13.00	0.40	2.07
Årsunda	19 774	208	10.53	0.35	1.48
Österfärnebo	31 130	242	7.76	0.48	1.50
Hedesunda	43 134	357	8.27	0.54	1.78
Valbo	58 778	486	8.27	0.49	1.62
Bollnäs (part)	abt. 32 890	133	4.05	1.22	1.98

In the northern regions the road nets mainly consist of throughfares at a density of about 2—4 metres of road per hectare. The road nets in parishes in middle Sweden consist of both throughfares and secondary roads, the latter ones often being branch roads. The road net density varies between 3 and 13 metres of road per hectare, the average being 6.7 metres per hectare.

The road net adjustment for forest land in certain parts of the province of Jämtland has been calculated at values ranging between 1.34 and 1.91. The values are generally lower than those for the total land in the same area (table 6).

In the cases where it has been possible to compute the adjustment for two alternative degrees of road net extension in the area, the road net adjustment has usually been slightly less for the alternative with denser road net. The difference, however, has generally not exceeded 8 per cent of the value for the more open road net although the increase in road net density in a couple of areas has been even larger than 50 per cent. The planned road nets have then been consisted of branch roads only to a small extent. However, when extension caused a marked change in the form of the road net, the road net adjustment has changed rather more clearly.

According to the investigations a value of road net adjustment about 1.60—1.70 can be recommended for use at summary calculations pertaining to large areas of normal Swedish country, when the computations are meant for forest land or for the total land area. In cases where the road net is very evenly distributed, a slightly lower road net adjustment may be chosen. Concerning calculations of alternative degrees of extension in an area, the road net adjustment should be computed for outset on the basis of direct measurements of mean distances and road-length. The value of road net adjustment thus obtained can then after reduction (if any) be used for calculations at the further extension of the road net. In the cases when the design of the road net becomes more comprehensively changed by the road net extension, the road net adjustment, however, should be directly computed not only for outset but also for other degrees of extension. Sometimes, it may then be suitable to limit the studies to selected model areas.

3. Distribution of area by cross-country transport distances

It is often of value to know for an area not only the mean cross-country transport distance but also how the area is distributed around this mean distance.

In the road net models formed by the equally large, regular polygons in figure 6, the area influenced by each road (each side) is composed of two isosceles, congruent triangles with common base. For road nets of this type the following distribution of area applies.

Geometric cross-country transport distance	Percentage of the area
$\leq Mg$	55.6
$\leq 2\ Mg$	88.9
$\leq 3\ Mg$	100.0

The corresponding areal distributions have been computed for certain areas in north Sweden and for the transport investigation of the province of Jämtland. The results are presented in table 8. The figures in the table

Table 8. Percentage distribution of area by geometric cross-country transport distance (*mg*) in relation to the geometric mean cross-country transport distance (*Mg*).

The points supporting the areal distribution in the areas Malå, Jörn, and Åsele are situated on land and on various categories of estates. In the Jämtland material the points in the two degrees of road net extension are points located on forest land.

Area	*Mg* km	Percentage area within various geometric cross-country transport distances			$\frac{mg \text{ max}}{Mg}$	No. measurements of *mg*
		$\leq Mg$	$\leq 2\ Mg$	$\leq 3\ Mg$		
Malå						
degree of ext. 1 ..	1.57	57	88	96	4	198
» » » 2 ..	0.94	56	89	97	5	198
Jörn						
1.............	1.48	56	89	97	4	209
2.............	0.99	58	89	98	6	209
Åsele						
1.............	2.05	57	87	97	4	203
2.............	1.34	58	86	98	5	203
Province of Jämtland						
Area A						
1.............	2.9	67	86	95	8	about 460
2.............	2.1	68	87	93	8	» 460
Area B						
1.............	1.5	62	90	98	5	» 800
2.............	1.1	57	91	98	6	» 800
Area C						
1.............	1.6	63	89	96	8	» 330
2.............	1.3	62	91	97	7	» 330
Whole province						
1.............	1.9	67	89	96	12	» 1 590
2.............	1.4	66	91	96	12	» 1 590
Average (arithm.)		61	89	97	7	
Range of variation.............	0.94—2.9	56—68	86—91	93—98	4—12	

indicate that there is a rather high stability with regard to the distribution of the area on distance classes. The maximum distance seems seldom to be less than four times the mean distance.

The mean distance weighted with the costs is the same as the area weighted mean distance only in cases when the cost of moving is changed rectilinearly on distance. In the other cases occurring at summary computations, the values reported above for the percentage distribution of area between geometric cross-country transport distances may give guidance at the determination of the cost weighted mean distance.

4. Distance adjustment for cross-country transport

The cross-country transport adjustment has previously been defined as *the adjustment, T-corr, which is required to convert the geometric cross-country transport distance to the practical one,* or according to the formula (3):

$$Mp = Mg \cdot T\text{-corr} \tag{3}$$

Expressed in other words, the cross-country transport adjustment may be said to be the relationship between the distance of cross-country haulage to a landing and the shortest straight-line distance from corresponding stump to the nearest transport road:

$$T\text{-corr} = \frac{Mp}{Mg}$$

The road used for computing the geometric cross-country transport distance can be 'but need not be' the same as the one used for computing the practical cross-country transport distance on account of adverse slopes to the nearest road, or due to other obstacles.

a) Accuracy

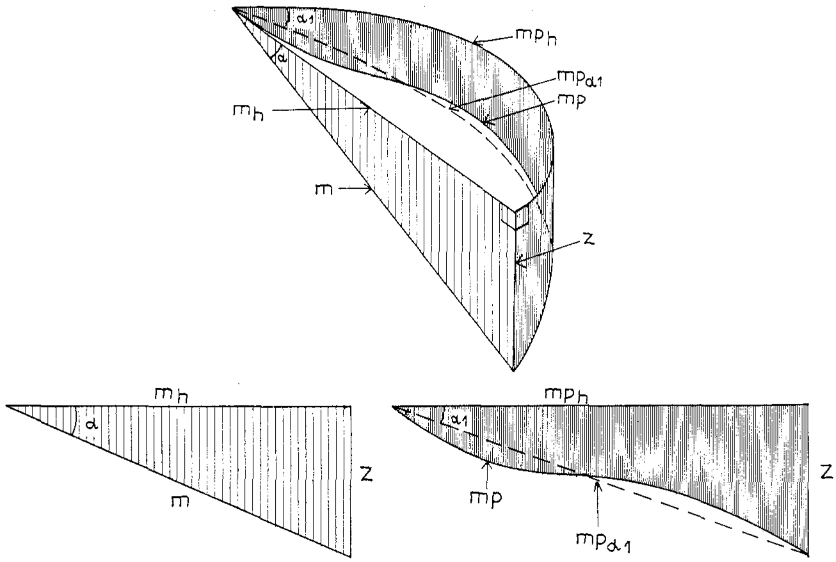
This section deals with matters pertaining to the fact that measurements of distances on the maps constitute the horizontal projections of the distances. When dealing with the problem, an expression is sought for the ratio between the practical distance of cross-country transport (mp) and the distance of its orthogonal projection in the horizontal plane (mp_h).

The ratio sought, mp/mp_h, has been presented in table 9 for various values of α, St_h, and St_v (cf. figure 13 with denotations) by means of the formulas (7) and (8):

Sought: The ratio mp/mp_h

$$\begin{cases} \dfrac{mp}{mp_h} = \dfrac{mp_{\alpha_1} \cdot St_v}{mp_h} \\[2ex] \dfrac{mp_{\alpha_1}}{mp_h} = \dfrac{1}{\cos \alpha_1} \end{cases}$$

$$\frac{mp}{mp_h} = \frac{St_v}{\cos \alpha_1} \tag{7}$$



Vertical plane through the terminal points of the cross-country road

The unfolded plane of the vertical section through the practical cross-country transport road

Fig. 13. Outline of a practical cross-country transport road (*mp*) and its orthogonal projection in the horizontal plane (*mph*), and the geometric elements used for the computation of the ratio *mp*/*mph*.

mp = practical cross-country transport distance
mph = orthogonal projection of the practical cross-country transport road in the horizontal plane (= distance measured on the map)
mp_{α_1} = straight line between the terminal points of the practical cross-country transport road in the unfolded plane of the vertical section through the practical cross-country transport road
α_1 = angle between mph and mp_{α_1}
z = height difference between the terminal points of the practical cross-country transport road
m = the straight line between the terminal points of the practical cross-country transport road in the vertical plane through the points
mh = the horizontal projection of m (= straight line distance of the practical cross-country transport road measured on the map)
α = angle between mh and m
Sth = allowance for winding course in the horizontal plane: mph/mh (computed from measurements on the map)
Stv = allowance for winding course in the vertical plane: mp/mp_{α_1}

The value of the angel α_1 is obtained from the following equations:

$$\begin{cases} tg\,\alpha = \dfrac{z}{m_h} \\[2ex] tg\,\alpha_1 = \dfrac{z}{mp_h} \\[2ex] mp_h = m_h \cdot St_h \end{cases}$$

(8) $$tg\,\alpha_1 = \frac{tg\,\alpha}{St_h}$$

Table 9. The ratio mp/mp_h at various values of α, St_h and St_v.

$\alpha°$	St_h	$\alpha_1°$	mp/mp_h				
			St_v				
			1.00	1.10	1.20	1.30	1.40
0	1.00	0	1.000	1.100	1.200	1.300	1.400
	1.10	0	1.000	1.100	1.200	1.300	1.400
	1.20	0	1.000	1.100	1.200	1.300	1.400
	1.30	0	1.000	1.100	1.200	1.300	1.400
	1.40	0	1.000	1.100	1.200	1.300	1.400
10	1.00	10.00	1.015	1.116	1.218	1.320	1.421
	1.10	9.11	1.013	1.114	1.216	1.317	1.418
	1.20	8.36	1.011	1.112	1.213	1.314	1.415
	1.30	7.72	1.009	1.110	1.211	1.312	1.413
	1.40	7.18	1.008	1.109	1.210	1.310	1.411
20	1.00	20.00	1.064	1.170	1.277	1.383	1.490
	1.10	18.31	1.053	1.158	1.264	1.369	1.474
	1.20	16.87	1.045	1.150	1.254	1.358	1.463
	1.30	15.64	1.038	1.142	1.246	1.349	1.453
	1.40	14.57	1.033	1.136	1.240	1.343	1.446
30	1.00	30.00	1.155	1.270	1.386	1.501	1.617
	1.10	27.70	1.129	1.242	1.355	1.468	1.581
	1.20	25.70	1.110	1.221	1.332	1.443	1.554
	1.30	23.95	1.094	1.203	1.313	1.422	1.532
	1.40	22.41	1.082	1.190	1.298	1.407	1.515
40	1.00	40.00	1.305	1.436	1.566	1.696	1.827
	1.10	37.34	1.258	1.384	1.510	1.635	1.761
	1.20	34.96	1.220	1.342	1.464	1.586	1.708
	1.30	32.84	1.190	1.309	1.428	1.547	1.666
	1.40	30.94	1.166	1.283	1.399	1.516	1.632

On the basis of determinations of the factors α, St_h, and St_v, it may be concluded that the values obtained by means of measurements in the horizontal plane at the applications concerned are rather good approximations of the true distances. A reservation, however, must be made in respect of such errors that might occur at measurements on maps that are simplifications of reality.

b) Determination of the cross-country transport adjustment at the Royal College of Forestry exercises in road planning at Malingsbo

The students at the Royal College of Forestry obtain practical tasks in the subject of road planning. One task has included an investigation of how the road net should be designed within a specified area to meet the requirements of the area in the best way regarding transport lines for forest products, for residents and for labour etc.

This task has been fullfilled according to a method reported by *Janlöv* (12). It includs studies of the cross-country transport within the area as an important element.

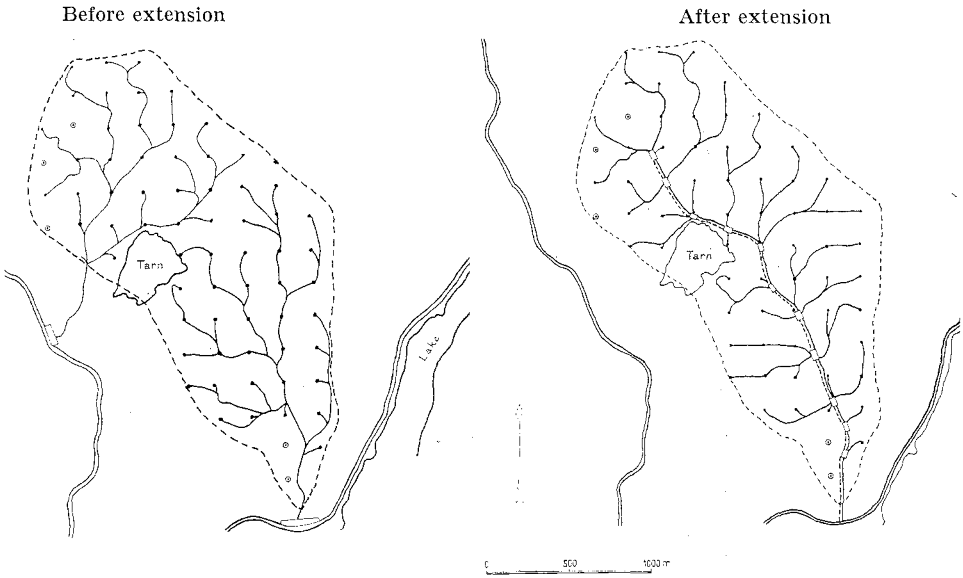


Fig. 14. Cross-country transport outline of a natural transport area on the range of Kloten. Before and after the extension of a road.

•	Point on productive land	——	Cross-country transport road
⊙	Point on impediment	- - - -	Boundary of natural transport area
═══	Permanent road	☐	Landing
╤╤╤	Planned forest road		

Quadratic patterns of points have been introduced on maps of the natural transport areas (scale 1: 10 000). Each point may be said to represent a fixed quantity of timber that is to be transported along the natural line of transport to a landing on the road side or on a floatway. The cross-country transport distance may be determined with rather good accuracy after a careful study of the country before the establishment of the "transport area maps". This method of studying the design of the cross-country transport in a certain area by means of an overlay with a systematic point pattern has proved advantageously applicable both at the practical management planning, especially when the country could be studied in a stereo-model, and at more theoretical investigations, examples of which are given by *Arvidson* (3), *Hall* (9), and *Hjelmström* (10).

Figure 14 shows an example of the transport conditions in an area before and after the extension of a road.

The cross-country transport distance from each point on the map has been compared with the straight-line distance measured from the same point to the nearest transport line, in this case a lorry road.

The ratios between the values obtained from the maps with respect to the actual cross-country transport distance (*mp*) and the geometric cross-

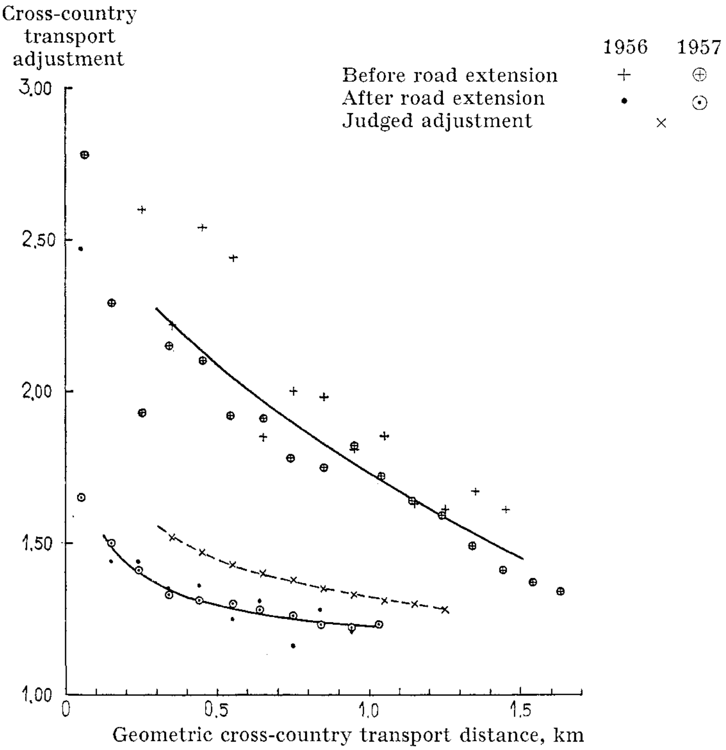


Fig. 15. Cross-country transport adjustment applied at the exercises in road planning at the Royal College of Forestry at Malingsbo.

country transport distance (*mg*) have been grouped into 100-metres classes for the latter distance. Since the objects chosen as tracts of exercises displayed factual needs for more roads, the values obtained from the maps with respect to "before road extension" are meant to represent poor cross-country transport conditions which will mean a high cross-country transport adjustment. It should be noticed that these values consequently do not represent average conditions at lower road net densities but conditions prevailing in areas that are poorly planned with respect to cross-country transports. "After road extension" may represent good or "ideal" cross-country transport conditions with a presumably rather low cross-country transport adjustment achieved by correct placing of the roads in their natural location.

Figure 15 shows the values of cross-country transport adjustment graphically presented and fitted with curves. The interval of geometric cross-country transport distance of the greatest interest currently ranges between 0.3 km and 1.5 km. The optimum mean cross-country transport distance can be expected somewhere between these values.

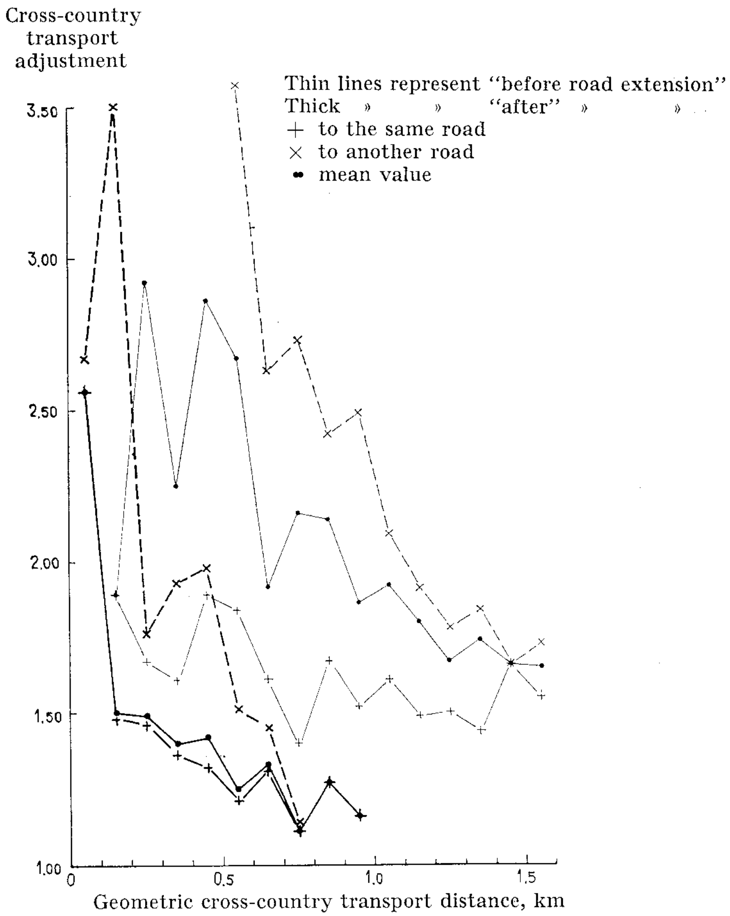


Fig. 16. Comparison between the cross-country transport adjustment for points from which timber is hauled to the same road as that from which *mg* has been measured and for points from which timber is hauled to another road—before and after road extension; Royal College of Forestry exercises at Malingsbo.

Distinction has been made in one part of the material between points from which the timber has been transported to the same road as that to which the geometric cross-country transport distance has been measured, and points from which the timber has been transported to some other road.

Figure 16 displays this distribution graphically. The figure shows that the cross-country transport adjustment is essentially higher for the points from which the timber has been hauled to some road other than that which is nearest, than for the points from which the timber has been hauled to the nearest road. The position of the curve of the weighted mean value of "*T*-corr same road" and "*T*-corr other road" in relation to the positions of the curves of these two adjustments provides a concept of the relative distribution of "points toward same road" and "points toward some other road".

It appears before the extension of the road that the timber from about half of the points was hauled to some road other than the nearest one, while after the road extension the timber from only a few points were hauled to some other road. From the point of cross-country transport this may be said to constitute a measure of the efficiency of the road systems in these transport areas. This statement, however, does not infer anything from the point of cross-country transport about the better or worse placing of the roads in the current road net. These roads may be placed quite logically in the large natural transport areas which may be said to be composed of a number of natural transport areas of lower order, i.e. those areas which are situated most distant from the current roads, naturally become less favoured from the point of cross-country transport than the others and they are therefore the prime objects of road construction at an extension of the road net. These areas are just the kind apportioned to the forestry students as tasks of exercise.

When the cross-country transport is carried out to special landings, the transport distance is higher than that obtained when the timber is unloaded evenly scattered along the roads. Compiled according to *Sundberg* (28), table 10 shows the percentage increase in the geometric mean cross-country transport distance at various distances between the landings.

Table 10. Percentage increase in the geometric mean cross-country transport distance when timber is hauled to special landings.
Timber is evenly scattered in the forest.

Geometric mean cross-country transport distance, (*Mg*) km	Percentage increase in *Mg* at various distances (metres) between the landings			
	150	250	350	450
0.05	40	78	118	154
0.15	6	13	28	39
0.25	3	6	8	13
0.35	1	4	6	8
0.45	1	2	4	6
0.55	0	2	3	4
0.65	0	1	3	3

The table shows that no essential increase in the cross-country transport distance is incurred when the timber is hauled to special landings, as long as the distance between the landings is less than the geometric mean cross-country transport distance.

The review above shows that the distance adjustment for cross-country transport is caused by the following conditions.

— From some parts of a transport area the cross-country transport must frequently be directed to some road other than the nearest one.
— The timber is usually hauled to special landings by the roads.
— The practical course of haulage deviates from the straight line between the point of loading to the landing on the road side.

Somewhere between the curves "before" and "after" the road extension there should be a value of cross-country transport adjustment that is useful at summary calculations for large areas of a certain type of country. Roads that cross over watershed divides and other boundaries between transport areas as well as roads in steep country that are mainly fed with timber from one side only have a raising effect on this value, which must be judged with respect to the place of existing and planned roads.

The curve showing "judged adjustment" in figure 15 is an example of a compromise between the values "before" and "after" road extension when these values have been given the weights 1 and 4, respectively.

Since the precision of the performance of the task certainly varied between the students, the values obtained may be conceived as examples showing the application of the method.

While the road net adjustment is a purely geometric coefficient it is evident that the distance adjustment for cross-country transport is bound to logging technique and its application to topography.

c) Determination of the cross-country transport adjustment at a special study conducted at Malingsbo

In the summer and autumn of 1959 a special investigation of the cross-country transport adjustment was carried out in the districts of Malingsbo and Kloten.

A forestry map drawn to the scale 1: 10 000 in 1955, partly on the basis of aerial photography, was overlaid a quadratic point pattern with a distance of 1 km between the points, the position of which was subsequently ascertained in the country by means of forestry maps and aerial pictures. The distance of the judged course of haulage was measured in the country from each point (numbering 255). Both the shortest distance to the nearest road, i.e. the geometric cross-country transport distance, and the straight-line distance to the landing by permanent road were measured from each point on the map.

The result of this investigation of the cross-country transport adjustment is shown in figure 17.

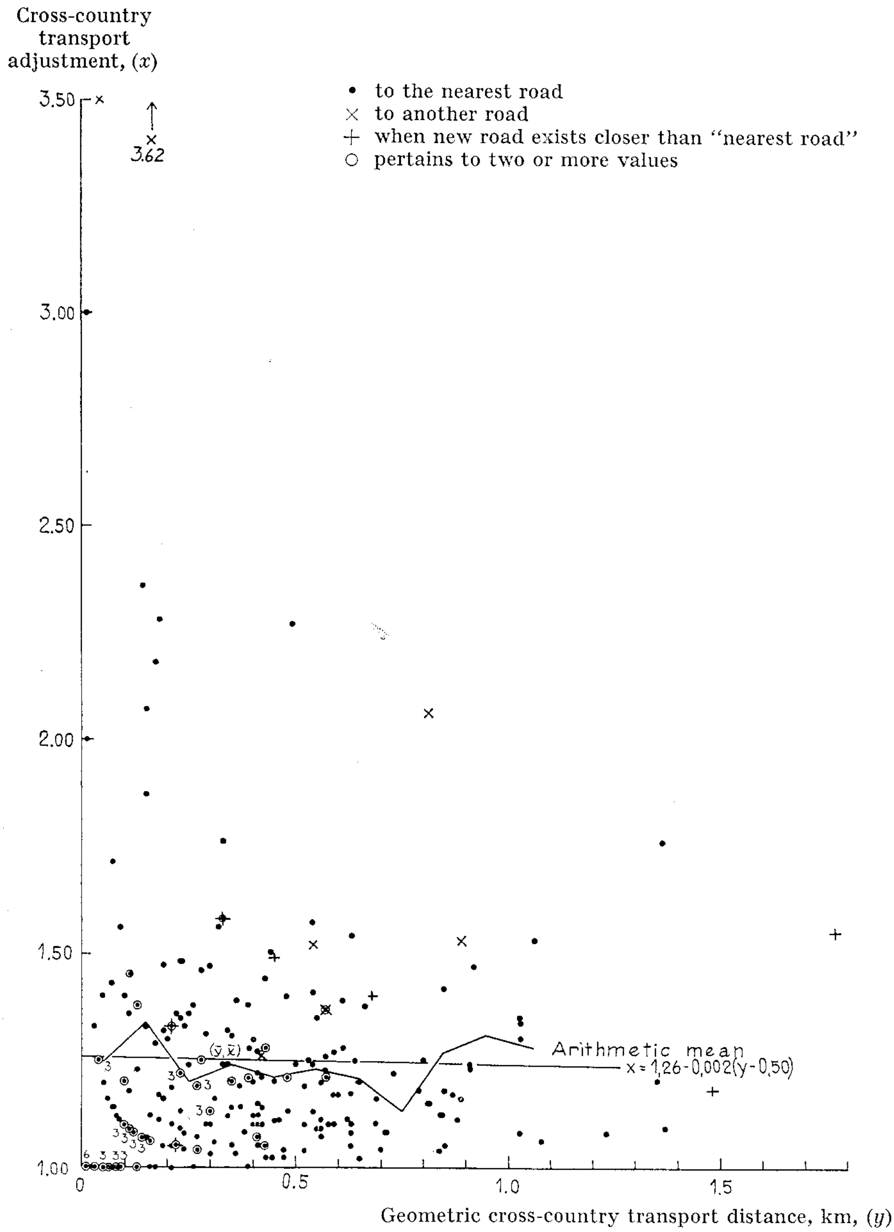


Fig. 17. Cross-country transport adjustment at a study specially carried out at Malingsbo.

The data have been investigated concerning the correlation between the cross-country transport adjustment (x) and the geometric cross-country transport distance (y). This investigation produced a correlation coefficient of -0.017 for the formula $x = 1.26 - 0.002\ (y - 0.50)$.[1] On account of this weak correlation, the coefficient can definitely not be stated significantly different from 0. The material should be about 100 times larger to produce significance at this correlation.

The cross-country transport adjustment for the area concerned may be computed by means of relatively few points and by making the arithmetic mean valid for various geometric cross-country transport distances. In the present case the mean value of the cross-country transport adjustment amounted to 1.25.

An essential increase in adjustment is only obtained at decreasing geometric cross-country transport distance when the timber is hauled to special landings and when the geometric cross-country transport distance is short in relation to the distance between the landings.

The allowance for winding in this investigation was also determined at an average of +11 per cent of the straight-line distance between the stump and the landing on a road side.

d) Determination of the cross-country transport adjustment at a transport investigation in the province of Jämtland

An investigation of the accessibility of forests in the province of Jämtland also included a determination of the cross-country transport adjustment (21). The material was entirely based on studies of the public maps drawn to the scale 1: 100 000.

The determination was carried out for the northern corner of the "tracts" of the third National Forest Survey in the area, provided the corner was situated on forest land. Two alternatives of road net extension were then considered, viz. the present net of permanent main roads and a planned extension of same. Furthermore, the cross-country transport adjustment for timber haulage to floatways was determined for some of the survey tracts. The number of determinations of the cross-country transport adjustment factors amounted to a total of about 1 900. The result is shown in figure 18. The following mean values were obtained for the cross-country transport adjustment:

to the present road net 1.36,
to the planned road net 1.34,
and to the floatways 1.37;
the mean value of all the determinations being 1.35.

[1] In the formula y is expressed in $^1/_{10}$-km units.

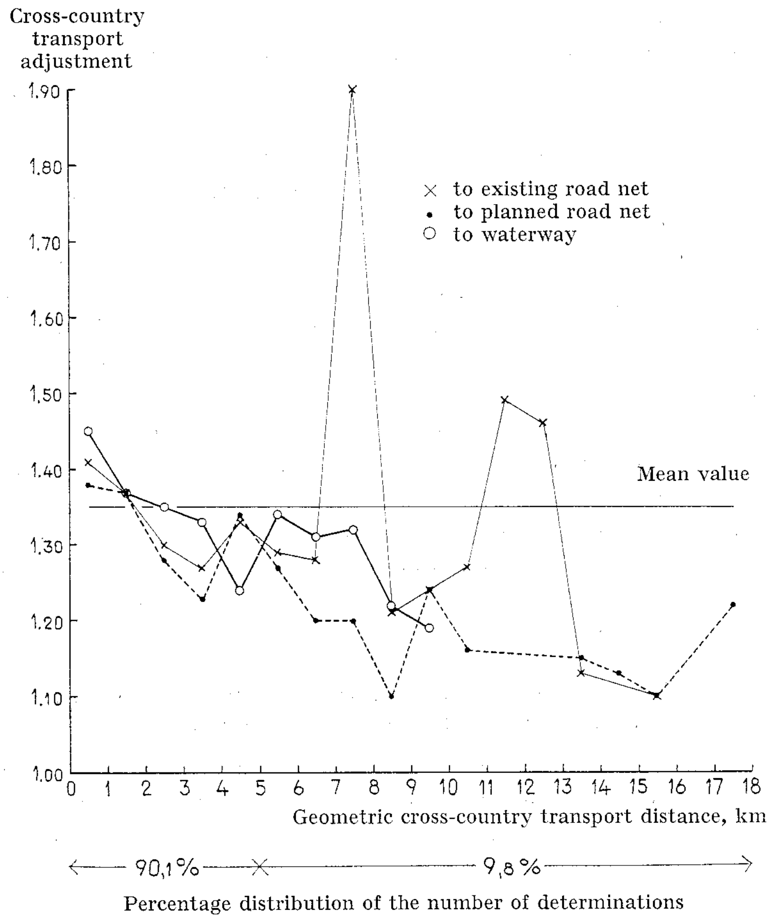


Fig. 18. Cross-country transport adjustment at a transport investigation in the province of Jämtland.

5. Introduction of the adjustments of road net and cross-country transport into the calculations of the road net density

Based on the principal discussion of the coordination of two given transport operations carried on by *Sundberg* in "Studies of Transportation in Forestry" (28), calculations of the optimum road net density were aimed at "the meeting point between the two transport methods where the sum of the direct and the indirect distance costs at haulage of timber from the points of origin to the terminal point is at the lowest".

The task is thus to find the minimum point of the function

$K = DUK_1 + IUK_1 + DUK_2 + IUK_2$

where K = the total cost of transport

DUK = the direct distance cost of a certain transport

IUK = the indirect distance cost of a certain transport

index 1 = primary transport

» 2 = secondary transport

» u = outside the area

The following additional symbols are introduced (cf. figure 1, p. 9):

B = width of the area concerned (= the distance between the roads of the secondary transport)

L = length of the area concerned

Q = volume of timber per unit area

duk = specific direct distance cost for a certain transport

iuk = specific indirect distance cost for a certain transport

S = road-length of the secondary transport

k_2 = cost per unit of length of the secondary road

Mean cross-country distance (Mp) of the primary transport:

$$Mp = \frac{B}{4} \cdot T\text{-corr}$$

Relationship between the mean cross-country transport distance (Mp) and the road net density (V):

$$Mp = \frac{2.5 \cdot V\text{-corr} \cdot T\text{-corr}}{10 \cdot V}$$

(where the value 10 is entered into the denominator to convert Mp and V to the same system of units).

$$\frac{B}{4} \cdot T\text{-corr} = \frac{V\text{-corr} \cdot T\text{-corr}}{4 \cdot V}$$

$$V = \frac{V\text{-corr}}{B}$$

The road net density V corresponds to the road-length S within the area:

$$S = V \cdot B \cdot L = \frac{V\text{-corr}}{B} \cdot B \cdot L = L \cdot V\text{-corr}$$

The following costs per volume unit of timber are obtained provided the mean haulage of the secondary transport within the area equals half the road distance $\left(\frac{S}{2}\right)$:

$$\begin{cases} DUK_1 = duk_1 \cdot \dfrac{B}{4} \cdot T\text{-corr} \\ IUK_1 \ = iuk_1 \ \cdot \dfrac{B}{4} \cdot T\text{-corr} \\ DUK_2 = duk_2 \cdot \dfrac{L}{2} \cdot V\text{-corr} + duk_{2u} \cdot S_u \\ IUK_2 \ = iuk_2 \ \cdot \dfrac{L}{2} \cdot V\text{-corr} + iuk_{2u} \cdot S_u \end{cases}$$

$$\begin{cases} iuk_2 \ = \dfrac{k_2 \cdot L \cdot V\text{-corr}}{B \cdot L \cdot Q \cdot \dfrac{L}{2} \cdot V\text{-corr}} = \dfrac{2\ k_2}{B \cdot L \cdot Q} \\ IUK_2 = \dfrac{2\ k_2}{B \cdot L \cdot Q} \cdot \dfrac{L}{2} \cdot V\text{-corr} + iuk_{2u} \cdot S_u \end{cases}$$

$$IUK_2 = \frac{k_2 \cdot V\text{-corr}}{B \cdot Q} + iuk_{2u} \cdot S_u$$

Subsequently is obtained the total cost of transport:

$$K = \frac{B}{4} \cdot T\text{-corr} \cdot (duk_1 + iuk_1) + duk_2 \cdot \frac{L}{2} \cdot V\text{-corr} +$$

$$+ duk_{2u} \cdot S_u + \frac{k_2 \cdot V\text{-corr}}{B \cdot Q} + iuk_{2u} \cdot S_u$$

The point of minimum (if any) of this function is obtained by derivation with respect to B:

$$K' = \frac{T\text{-corr}}{4} \cdot (duk_1 + iuk_1) - \frac{k_2 \cdot V\text{-corr}}{B^2 \cdot Q}$$

$$\begin{cases} K' = 0 \\ T\text{-corr} \cdot (duk + iuk) = \dfrac{k_2 \cdot V\text{-corr}}{B^2 \cdot Q} \end{cases}$$

Since the second derivate of the function is positive, B at $K' = 0$ represents a minimum value of the function

$$B_{min} = \sqrt{\frac{4 \cdot k_2 \cdot V\text{-corr}}{T\text{-corr} \cdot (duk_1 + iuk_1) \cdot Q}} \tag{9}$$

An area the B_{min} of which is computed according to the formula above (9), thus has a real mean cross-country transport distance of $\frac{B_{min}}{4} \cdot T$-corr and a road net density of $\frac{V\text{-corr}}{B_{min}}$

Since the "optimum road distance" becomes a slightly abstract concept for non-parallel road nets, it may be suitable instead to use the term "optimum (practical) mean cross-country transport distance" (Mp_{min}):

$$Mp_{min} = \sqrt{\frac{k_2 \cdot V\text{-corr} \cdot T\text{-corr}}{4 \cdot (duk_1 + iuk_1) \cdot Q}} \tag{10}$$

B. Determinations of length of roads

1. Statistics on the extent of the Swedish road net

As previously mentioned, it is important at calculations of the optimum extension of the road net to be able to determine the actual extent of the road net. Our knowledge of the extent of the national road net at various times will be reviewed in an introduction in order to provide a background to the following presentation.

Data on the extent of the Swedish net of roads and streets are annually furnished in the official statistics on the basis of information from the Board of Roads and Waterways. In the statistics dependable information is given on public roads in rural areas and on roads and streets in towns and communities which are independent with respect to road maintenance, and on private roads entitled to subsidies for maintenance. On private roads which are not entitled to such subsidies, however, there are at present no figures, not even estimates.

Public roads are constructed by decision of public authorities or they are considered of old as public. They are maintained by the civil service and they are open for public communications. All roads not public are considered private.

Our earliest official statistics on the public road system in the entire country comprise five-year periods from 1856—1860 to 1901—1905 (14). In the "Statistical Abstract of Sweden" (27) there are data on the total length of the public roads annually from 1906. Data have been compiled below for certain years concerning the extent of the road net in the rural areas.

Year	Length of roads, km
1860	53 858
1870	56 994
1880	59 929
1890	59 937
1900	56 395
1910	62 097
1920	65 276
1930	76 203
1941	88 595
1950	90 409
1960	93 481

Data for the years 1860—1900 are rather independable because of i.a. "difficulties at the classification of roads, outdated information on length of roads, necessitity of remeasurements etc.". The reduction in length from 1890 to 1900 is caused by the fact that certain roads according to the Law of 1891 (16) then lost their status of being public.

Knowledge of the private road net is scanty, the reason being the changes in legislation concerning the private road maintenance. These circumstances are described in the Forestry Investigation of 1936 (29) as follows:

"Legislation concerning private roads is based i.a. on the principle that private road maintenance is of private concern. Hence, the parts interested should have option to negotiate road matters and then to determine whether and how the roads are to be built as well as how the burden of costs of maintenance are to be distributed. In consequence of this attitude of the State, the civil service authorities have contributed very little to the private roads and their ability to serve the communications. State subsidies to these roads were not possible until late [1918] and they have then been awarded with relatively small amounts".

Regarding private road projects of forestry importance, however, there are certain data from 1870 and later reported by the Swedish Forest Service (15, 13 and 5) and from 1942 by the National Board of Private Forestry (4 and 24). In "Lorry Transports in Forestry", the most detailed outline of the structure and extent of lorry transports in forestry, *Fredén* (6) also reported on the result of an inventory comprising roads owned by Forest Service and the companies by the end of 1954. The forest land areas included in the report represent 44 per cent of the total forest land area.

The latest official information on the total length of roads and streets (7) is finally presented in table 11.

2. Determination of length of roads at the road inventory carried out by the National Forest Survey

a) Methods

When information on the road-length in a certain area is required and when such data are lacking, there is near at hand to measure the length of the roads on maps. However, no differentiation of the length can then be made regarding such factors as surface material, road width, carrying capacity and speed standards etc. When such a division is required the collection of data must be made in field. *Matérn* at the Royal College of Forestry has advised on a collection of data by a method of random sampling to determine the length of roads. The method may be used for work on maps as well as in field.

Table 11. Total length (km) of the Swedish roads and streets as of January 1st, 1963, distributed by the types of the roads and the surface material.

National road: road which is of great importance for the through long-distance traffic.
Provincial road: road other than national road.
. . Value not available.

Road type	Total length of roads	Length of surfaced roads	Surfaced roads per cent	Total length of gravelled roads	Of which	
					Oiled	Other
A. Public roads in rural areas	96 027	18 288	19	77 739	12 195	65 544
1. National roads . .	12 102	8 295	69	3 807	1 965	1 842
2. Provincial » . .	83 925	9 993	12	73 932	10 230	63 702
B. Roads and streets in towns and communities with independent road maintenance	10 879	6 925	64	3 954	. .	. .
C. Private roads entitled to financial support by subsidies	55 223			55 223	. .	. .
Total	162 129	25 213	16	136 916	. .	. .

Concerning the theoretical background of the method and its description reference is made to the appendix "A method of estimating the total length of roads by means of a line survey" by *Matérn.*

b) Collection of data

During the current National Forest Survey, which started in 1953, the entire country is annually covered with a low percentage survey (8). Cf. figures 19 and 20. Special problems with respect to biology, geology or technology can then be considered and data on the problems can be gathered throughout the country in one single year. Precision desired in the total result is obtained by repeated collection for a number of years. An exploratory collection of data on the roads called "road inventory" was made in 1956. On the basis of experiences gained at this inventory, certain changes were made in the instruction for the field work after which the inventory has been repeated annually during the period 1957—1963. The recording of roads has then proceeded according to the following excerpt from the instruction (11):

» — — — special description shall be made according to the following instruction for permanent roads and seasonal lorry roads the centre line of which is intersected by the survey line.

Permanent roads are trafficable all year (occasionally excluding the period of frost lift) with heavy duty lorries; where necessary trenched and gravelled or surfaced by some other material.

Seasonal lorry roads are planed main roads trafficable with heavy duty lorries or tractors in winter.

Main horse roads, roads under construction, and old roads no longer used for traffic due to new roads being built are not reported.

Recording of permanent roads

The following characteristics are reported:
Ownership at intersection
Nature of road from a forestry point of view
Three different groups are reported:
S — roads which at the intersection pass forest land or so near forest land that timber can be hauled to and piled on the road side. If the road passes land of some category other than forest land, it is to be considered as S-road provided the distance of the cross-country transport is shortened if the timber is piled on the road side. If the distance to the nearest stand margin exceeds 200 metres, however, T-road is recorded as follows.
T — roads on which timber is transported and to which timber is not extracted directly. It is understood that S-roads according to above are also used for transport from the forest.
O — other roads where transport of forest products does not occur.
Public or private roads
Ownership group
Width of the road

Recording of the seasonal lorry roads

When there is doubt whether the main road is used for horse or lorry transport, the road is recorded within brackets.»

By "owner" is here meant the owner of the land (forest land or other land) served by the road and not the legal owner of the road.

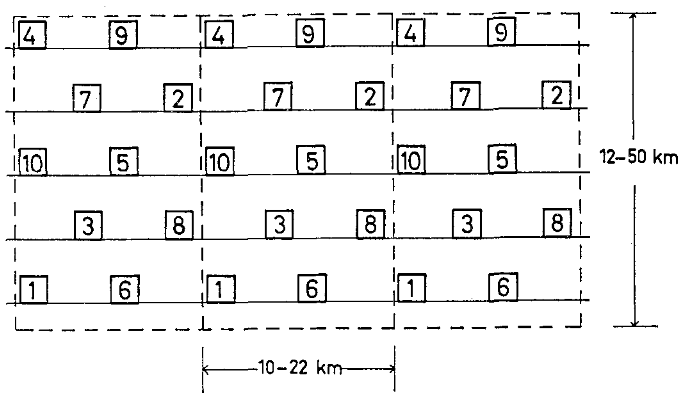


Fig. 19. Approximate outline of the layout of the survey tracts.
Thin full line represents "survey line" at the second National Forest Survey (1938—1952), now replaced by "tracts" located along the former line
Thick » » » "tract"
Dashed » » boundary of "survey area" with one tract surveyed per year
$\boxed{1}$ Tract surveyed in the first year
$\boxed{2}$ » » » » second year
a. s. o.

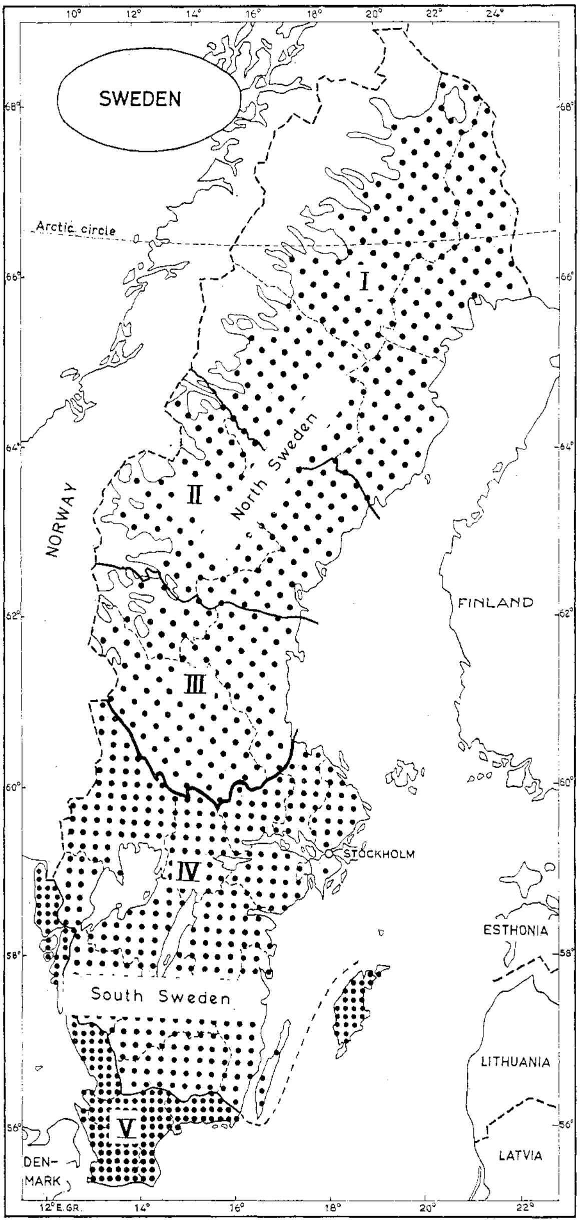


Fig. 20. Division of the country into "regions" (I-V), the report areas of the road inventory and the tracts of one year of survey.

c) Computation

At computation the material collected in the field has first been checked with respect to the formal correctness of the records. The material has also been supplemented with the public roads not included in the field work. According to the instruction, distances over water and over large farm fields and over impediment may be obtained from the maps by means of ruler. Moreover, roads crossing over farm fields are to be reported only when the registration can be made without great time consumption. In this case intersections between roads and the tract side may be lacking.

After this check the data have been punched on cards and sorted with respect to various factors. After sorting the number of intersections between roads and tract sides is obtained in each group of sorting. The road-length (Y) corresponding to the number of intersections (n) is obtained according to formula (A 3)[1]:

$$Y = \frac{\pi}{2} \cdot \frac{n}{t}$$

Table 12. Theoretical land area factor at the third National Forest Survey and the actual land area factor at the surveys of 1956, 1957, and 1958.

Region	Distance between lines and tracts at the survey of one year km		Length of tract side km	Area per km of line surveyed, sq. km			
				Theoretical at the survey of one year*	Actual at the survey in		
					1956	1957	1958
I	25	22.4	2.2**	63.6364**	64.2980	63.9376	77.8668
II	25	18.4	1.8	63.8889	65.6507	63.3957	63.2781
III	50	10.8	1.6	84.3750	85.0264	83.5310	84.9596
IV	25	12.0	1.4	53.5714	54.2340	54.0630	53.7534
V	12.5	10.0	1.2	26.0417	26.4169	25.6748	25.8805

* After two years of survey the theoretical area per km of line surveyed is half that presented above etc.

** From the 1958 survey the tract side in region I is 1.8 km and the area per km of line surveyed is theoretically 77.7777 km².

The length of the line system per unit area (t) depends on the length of the tract side and on the distances between lines and between tracts. These factors vary between the regions of the National Forest Survey (figure 19) according to table 12 where the corresponding area of land per unit of length on land surveyed in one year is presented for both the theoretical case (22) and the actual one. It is assumed in the theoretical case that the areas covered with water are evenly distributed over the total area and that they

[1] Formula (A 3) = formula (3) in appendix (p. 68).

are thus equally represented at each annual survey and that the line length surveyed is equally long each year. Actually, the representation of areas covered with water differs between years and hence the actual land area per unit of length surveyed on land. Moreover, the total line length per year varies since the number of survey tracts (or parts thereof on the area boundaries) per year is not constant. At repeated survey the actual land area factor approaches the theoretical one, nearly to concur at the end of the survey cycle. In the road inventory the theoretical factor has been chosen for use.

At a division of the material by parts of regions (e.g. provinces) it should be noted that neither the nominal land area factor nor the actual one definitely applies to the separate parts.

If the theoretical land area factor is used, an intersection between road and tract side corresponds to the following road-length in kilometres.

Region	Road inventory 1957—1960
I	29.1550
II	25.0892
III	33.1340
IV	21.0375
V	10.2266

d) Results and accuracy

The results of 1957—1960 road inventories (denoted "road inventory 1957—60") are reported in tables 13, 14 and 15. The country has been divided into two areas (cf. figure 20):

a) North Sweden comprising the regions I, II, and III.
b) South Sweden comprising the regions IV and V.

The registration of the roads is made in four ownership groups:
State forests
Other public forests
Company forests
Other private forests

Some data, however, are distributed by individual regions without division into ownership groups.

By means of the statistics on the area of forest land according to table 16, the road net density expressed in metres of road per hectare of forest land for various ownership groups has been computed in table 17.

Table 13. Public roads according to the road inventory.

Ownership group Total	Length of roads, km		
	North Sweden	South Sweden	Whole of Sweden
State	3 880	1 160	5 040
Other public	830	2 210	3 040
Companies	7 950	3 430	11 380
Other private	17 520	27 620	45 140
Total, road character S 1957—60	30 180	34 420	64 600
Total, road characters T and O 1957—60	6 230	24 730	30 960
Grand total 1957—60	36 410	59 150	95 560

Table 14. Private permanent roads with character *S* according to the road inventory.

Ownership group Total	Length of roads, km		
	North Sweden	South Sweden	Whole of Sweden
State	7 990	8 200	16 190
Other public	2 260	7 300	9 560
Companies	16 290	12 820	29 110
Other private	25 180	70 710	95 890
Total, road character S 1957—60	51 720	99 030	150 750

Table 15. Seasonal lorry roads according to the road inventory.

Ownership group Total	Length of roads, km
	North Sweden
State	9 850
Other public	1 750
Companies	13 010
Other private	27 030
Total 1957—60	51 640

Table 18 gives a summary of the road-length and the road net density of public and private permanent *S*-roads distributed by regions.

To check the dependability of the method, adjustments have been made in accordance with statistics on the length of public roads from the Board of Roads and Waterways of January 1st, 1960 (27).

Table 16. Forest land according to the National Forest Survey 1953—57.
All land except mountains, farm fields and roads, is considered forest land.

Ownership group Total	Forest land area, sq. km		
	North Sweden	South Sweden	Whole of Sweden
State	53 575	5 843	59 418
Other public	11 616	5 763	17 379
Companies	56 053	13 976	70 029
Other private	82 638	67 029	149 667
Total Region I 103 950, » II 48 621, » III 51 311	203 882	Region IV 79 007, » V 13 604 — 92 611	296 493

Table 17. Road net density expressed in metres of road per hectare of forest land according to the road inventory pertaining to public and private permanent roads with character *S*, and seasonal lorry roads.
The standard errors of the determinations are reported in table 20.

Ownership group	North Sweden					South Sweden		
	Public roads	Private roads	Seasonal lorry roads	Total perm. roads	Total all roads	Public roads	Private roads	Total perm. roads
State	0.7	1.5	1.8	2.2	4.0	2.0	14.0	16.0
Other public	0.7	1.9	1.5	2.6	4.1	3.8	12.7	16.5
Companies	1.4	2.9	2.3	4.3	6.6	2.5	9.2	11.7
Other private	2.1	3.0	3.3	5.1	8.4	4.1	10.5	14.6
Average 1957—60	1.5	2.5	2.5	4.0	6.5	3.7	10.7	14.4

Table 18. Permanent roads with character *S* distributed by regions according to the road inventory.

	Region					
	I	II	III	IV	V	I-V
Length of roads, km						
Public roads	13 730	9 450	7 000	29 770	4 650	64 600
Private roads	17 930	10 860	22 930	81 900	17 130	150 750
Total, S-roads	31 660	20 310	29 930	111 670	21 780	215 350
Forest land area, sq. km	103 950	48 621	51 311	79 007	13 604	296 493
Road net density, m/hectare						
Public roads	1.3	2.0	1.3	3.8	3.4	2.2
Private roads	1.7	2.2	4.5	10.3	12.6	5.1
Total, S-roads	3.0	4.2	5.8	14.1	16.0	7.3

The length of the roads according to the road inventories of 1956 and 1960 for four province groups and for the entire country, and the mean value of the five inventories 1956—1960 put in relation to the values of the official statistics have been compiled below.

	Length of roads according to			
	Official statistics	Road inventory		
		1956	1960	1956—1960
Province groups............ (range of variation)	100.0	95.6—109.4	93.6—120.0	100.2—106.5
Whole of Sweden...........	100.0	103.8	105.7	102.2

A comparison between the values of separate provinces naturally gives considerably greater deviations, especially when only one year survey is used, e.g. province of Östergötland 1956 (65 intersections) + 32.9 %; 1956—1960 (65 + 59 + 45 + 42 + 39 intersections) + 1.5 %.

This adjustment of the length of public roads obtained at the road inventories to the length according to official statistics shows that the material from the road inventory on the survey tracts of one year provides usable values of road-length for large areas.

The precision of the estimate of the length of permanent roads with the character S has been investigated by computations of the relative standard errors of the estimates. At the computations it has been assumed that the length of a road net within a region estimated by means of x intersections has a relative standard error that amounts to $130/\sqrt{x}$ per cent. The formula has been obtained in a numerical way by means of tract systems placed on maps.

A system of tracts from the area concerned in one year has then been placed in various positions on the map by parallel translations corresponding to a certain number of replications of one year survey.

At the computation of the relative standard error ε (= standard error expressed in per cent) of the length of roads estimated of one year, the following computations have been carried out.

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$$

$$s^2 = \frac{1}{n-1}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2\right]$$

$$\varepsilon = 100 \cdot \frac{s}{\bar{x}}$$

where n = no. positions (= no. surveys) and $x_1, x_2, \ldots, x_n$ = no. intersections between tract sides and roads in the n positions.

Table 19. Compilation for the computation of an approximate formula for the relative standard errors of the road-length estimates.

Area	Region	n	Road type	x	s	ε	$\frac{100}{\sqrt{x}}$
Part of the province of Norrbotten	I	20	Public	30.65	8.0	26.0	18.1
			Private	6.25	3.0	47.2	40.0
			All	36.90	8.7	23.7	16.5
Parts of the units of Dalarna, Jämtland and Hälsingland	III	20	Public	10.70	4.2	38.8	30.6
			Private	9.55	3.9	41.3	32.4
			All	20.25	5.4	26.7	22.2
Unit of Ångermanland	III	28	Public	37.96	8.9	23.4	24.7
Unit of Medelpad	III	28	Public	16.39	4.6	28.3	16.2
Province of Östergötland	IV	23	Public	51.35	8.3	16.1	14.0
Province of Kronoberg	IV	23	Public	42.00	10.5	25.0	15.4
Total						296.5	230.1

Table 19 shows a compilation for determination of ε. The following approximate formula for ε is obtained.

$$\varepsilon = \frac{100 \cdot \frac{296.5}{230.1}}{\sqrt{x}} = \frac{128.9}{\sqrt{x}} \approx \frac{130}{\sqrt{x}} \tag{11}$$

Table 20 presents the computed relative standard errors for the length of the permanent roads with character S, which are reported in tables 13 and 14.

Table 20. Calculated relative standard errors for permanent roads with character S according to the road inventory.

Ownership group Total	North Sweden			South Sweden		
	Public roads	Private roads	Total perm. roads	Public roads	Private roads	Total perm. roads
State	11	8	7	17	6	6
Other public	25	15	13	12	7	6
Companies	8	6	5	10	5	5
Other private	5	4	3	3	2	2
Total, S-roads	4	3	2	3	2	2

It appears at a division of the material into small areas or ownership groups of small extent such as "other public forests" in North Sweden that the separate values become rather inaccurate.

The relative standard errors computed for the length of *S*-roads presented in table 18 are reported in table 21.

Table 21. Calculated relative standard errors for permanent roads with character *S* distributed by regions according to the road inventory.

Road type Total	Region					
	I	II	III	IV	V	I—V
Public roads	6	7	9	3	6	2
Private roads	5	6	5	2	3	2
Total, S-roads	4	5	4	2	3	1

The road-length obtained for each of the two report areas North Sweden and South Sweden shows rather good agreement between the various inventories as regards the total length of public roads while the length of private permanent roads with the character *S* displays a considerably greater variation. It has therefore been considered of value to analyze further these variations.

The relative standard error of the mean value has therefore been primarily computed for the length of public, respectively, private permanent roads with character *S* at the four inventories 1957—1960. The standard error has been computed on the basis of the actual results from the inventories of the separate years. Below this "standard error obtained" has been compared with the standard error computed by means of formula (11) reported in table 20.

	North Sweden	South Sweden
Public S-roads		
Standard error obtained	4.4 %	1.6 %
» » computed	4.0 %	3.1 %
Private S-roads		
Standard error obtained	7.4 %	7.4 %
» » computed	3.1 %	1.8 %

At tests of significance of the standard errors no significant difference is obtained between the errors obtained and those calculated for the public roads, but a significant difference on the 0.001-level for the private roads.

The differences between the results of the various years for the private permanent *S*-roads may not be randomly conditioned only. The extension carried out during the period for such roads, which for the years 1957—1960 is reported at between 2 400 and 3 300 km per annum (24), cannot have any influence of importance in this context. The variations are likely caused by the difficulties at the field work to make a correct judgement of the carrying capacity of the roads, i.e. to make a correct distinction between permanent roads and other types of roads (e.g. cart roads and main seasonal roads). It may be a matter of differences in the judgement between various team leaders, as well as between judgements made by the same team leader at different occasions. This situation is to be expected since the judgement is subjective. The members of the survey teams generally lack the local information that would enable a more correct classification of the carrying capacity. The difficulty of making a correct classification of the carrying capacity has been elucidated by *Hjelmström* (10).

Differences in the classification of roads with respect to the road-character should not have had any decisive influence on the variations between the inventory results of the various years for the private permanent *S*-roads. This statement is vindicated by the good agreement between the obtained and the computed precision of the inventory results for the public roads with character *S*. Since the public roads have been classified as permanent with extremely few exceptions, the judgement of the carrying capacity in this case has no influence on the variations between the various yearly inventory results.

Uniformity in the judgements made at the road inventory, as in the other registrations in field made by the National Forest Survey, is aimed at by excursions with the team leaders before the beginning of the field work season and by inspections of the field work.

The length of private permanent roads with character *S* presented in table 14 also includes such roads which have been subject to doubts whether they are trafficable with heavy lorries even in rainy periods (autumn). The share of "doubtful" roads reported in table 14 is 14—15 per cent of the total length.

The standard of the private permanent roads may also be elucidated to

Table 22. Hard surfaced width of private roads with character *S* according to the road inventory.

Part of the country	Total length of roads percentage distributed by various widths (metres) of hard surfaced roads									
	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	Total
North Sweden...........	7	30	31	17	10	2	2	1	0	100
South Sweden...........	12	48	28	9	3	0	0	0	0	100

some extent by the surfaced width of the roads which is reported in table 22. The table shows for the private permanent roads with character *S* that about 70 per cent of such roads in North Sweden and about 90 per cent in South Sweden have a surfaced width less than 3.25 metres.

According to the instruction, permanent roads with character *S* are roads "which at the intersection pass over forest land or so near forest land that timber can be hauled to and piled by the road". These roads thus affect directly the transport distance from the stump to the road.

By means of the road net densities reported in table 17 summary computations of the mean cross-country transport distance can thus be made for the various areas of registration. More accurate results are then obtained if the position of the roads in relation to the forest land they serve is taken into consideration. The land category on which the intersection between road and tract side is situated can then give some information.

The registration of the land category at the field work is shown below.

Location of the intersection	(Symbol in table 23)
On forest land	(S)
farm fields	(I)
other land	(O)
In the border line between	
forest land and farm fields	(S/I)
forest land and other land	(S/O)
farm fields and other land	(I/O)

The division by land categories between the public and the private roads with character *S* is presented in table 23.

Table 23. Public and private permanent roads with character *S* distributed by land categories according to the road inventory.

Area Road type Total	Length of roads, km						
	Land category						Total
	S	I	O	S/I	S/O	I/O	
North Sweden							
Public roads	20 730	3 560	800	3 100	1 740	250	30 180
Private roads	40 910	4 200	910	4 100	1 580	20	51 720
Total	61 640	7 760	1 710	7 200	3 320	270	81 900
Per cent	75.3	9.5	2.1	8.8	4.0	0.3	100.0
South Sweden							
Public roads	17 100	9 000	340	6 990	690	300	34 420
Private roads	63 140	17 510	540	16 340	1 160	340	99 030
Total	80 240	26 510	880	23 330	1 850	640	133 450
Per cent	60.1	19.9	0.6	17.5	1.4	0.5	100.0

At the continued computations it has been assumed that roads on forest land are fed with timber from both sides while roads in the boundary between, or on, other land categories are fed with timber from one side only; of the latter roads is therefore counted half the length. Furthermore, it has been assumed that the mean cross-country transport distance to roads on farm land and in the boundary between farm land and other kinds of land is extended by 100 metres in relation to that on forest land.

At the computation the road net adjustment has been given the limit values 1.35 and 2.00. The cross-country transport distance adjustment has been assumed to amount to 1.20.

The computation of the mean cross-country transport distance is:

	North Sweden		South Sweden	
S-roads, total length, km	81 900		133 450	
deduction for one-sided cross-country transport, km	$\frac{0.5 \cdot 24.7 \cdot 81\ 900}{100} = 10\ 110$		$\frac{0.5 \cdot 39.9 \cdot 133\ 450}{100} = 26\ 620$	
"Adjusted length", km	71 790		106 830	
Forest land area, sq.km	203 882		92 611	
"Adjusted road net density", m/ha	3.52		11.54	
Cross-country transport adjustment	1.20		1.20	
Road net adjustment	1.35	2.00	1.35	2.00
Mean distance on forest land, km	1.15	1.70	0.35	0.52
allowance 100 m for the land categories I and I/0, km	$\frac{9.8}{100} \cdot 0.1 = 0.01$		$\frac{20.4}{100} \cdot 0.1 = 0.02$	
Mean cross-country transport distance, km	1.16	1.71	0.37	0.54

Under the conditions given above the mean cross-country transport distance from the stump to the permanent road in North Sweden is cited at about 1.5 km and for South Sweden at about 0.5 km.

Based on i.a. information in table 18 an analogous computation (however, only for V-corr = 1.60, T-corr = 1.20) of the mean cross-country transport distance has been made for the various regions. The result of this computation is:

	Region					
	I	II	III	IV	V	I—V
Mean cross-country transport distance, km	1.8	1.4	0.9	0.4	0.4	0.8

In the northern area the permanent road net is supplemented by the seasonal lorry roads trafficable in winter. If the computation of the mean cross-country haulage is made with consideration taken to the 2.5 metres of seasonal lorry road per hectare of forest land reported in table 17, a mean cross-country haulage to this combined road net of 0.7—1.0 km is obtained under conditions equal otherwise.

Primarily in the northern area some transport occurs in the form of cross-country transport direct to waterways. When the mean cross-country transport distance is sought for such a combined net of permanent roads and waterways, it should be noted when a permanent road is built within a short distance from the waterway and along the same that the length of one of these transport ways only should be included in the computations.

The registration of seasonal lorry roads has also been made in the southern parts of the country. The results show a very great variation between the years, the variation being so great as to make the inventory a failure in this respect. Objective standards of classification for the seasonal lorry roads as well as the private permanent roads might be required to obtain greater accuracy in the estimates.

3. Determination of length of main roads within small areas in the province of Jämtland

The following presentation is primarily intended to provide an example showing how the method of estimating the length of a road net by "line survey" may be used at opportunities other than a national forest survey. To support similar applications in other cases, the report has been made rather elaborate.

a) Methods

The maps in scale 1: 100 000 on which the main roads concerned had been drawn were overlaid with transparent millimeter-graded paper. The line survey was carried out along lines in both directions, thus along two line systems at right angle.

The distance between the lines was chosen at 5.0 cm. A line surveyed for 5 km then corresponds to an area of 25/2 sq. km and the length of the line system per unit of area became 0.400 units of length. When this value was entered into formula (A 3), each intersection between the road nets and the line systems is found to represent a length of 3.9270 km.

By means of knowledge of the length of the existing and the planned road nets being about 9 000 km and 11 300 km respectively in the province, and on the basis of the approximate formula (11) for the relative standard error of the road inventory of the National Forest Survey, the line density chosen was judged to be of precision required.

At the "survey" the number of intersections was recorded with distinction made between the two systems of lines. Differentiation was then also made between four types of main roads, viz. existing public roads respectively private roads, proposed private roads, and proposed reconstruction of existing private roads.

b) Results

The total length (km) obtained for the various road types within the province has been compiled below. The compilation also includes data on the roads reported by the provincial forestry committee (25).

	Existing roads		Proposed roads	
	Public	Private	Private	Reconstruction of existing private roads
Survey........	4 795	4 210	2 156	157
Committee.....	5 100	4 000	2 100	160

The agreement between the data from the two sources is apparently good. Information on the distribution of roads by small areas (cf. figure 21) according to the survey is given in table 24.

Table 24. Length of main roads (km) within small areas in the province of Jämtland.

Small area	Roads				Road net		Small area	Roads				Road net	
	Existing		Proposed					Existing		Proposed			
	Public	Private	Private	Reconstruction	Existing	Planned		Public	Private	Private	Reconstruction	Existing	Planned
J: 1	125.7		3.9		125.7	129.6	J:18	216.0	153.1	110.0		369.1	479.1
2	137.4	43.2	35.4	3.9	180.6	219.9	19	286.7	133.5	121.7		420.2	541.9
3	125.7	23.5	55.0		149.2	204.2	20	176.7	192.5	58.9	3.9	369.2	432.0
4	23.6	58.9	74.6		82.5	157.1	21	125.7	219.9	70.7		345.6	416.3
5	62.8	15.7	51.1	15.7	78.5	145.3	22	290.6	235.6	82.5		526.2	608.7
6	94.3	43.2	31.4		137.5	168.9	23	235.6	310.3	145.3		545.9	691.2
7	66.7	11.8	39.3		78.5	117.8	24	184.6	310.2	47.1	4.0	494.8	545.9
8	90.3	27.5	31.4		117.8	149.2	H: 1	200.3	90.3	47.1		290.6	337.7
9	184.6	70.7	106.0		255.3	361.3	2	78.5	145.3	66.8	43.2	223.8	333.8
10	247.4	90.3	70.7		337.7	408.4	3	23.6	133.5	66.7	39.3	157.1	263.1
11	200.3	94.2	66.8		294.5	361.3	4	117.8	188.5	58.9	7.9	306.3	373.1
12	188.5	58.9	137.4		247.4	384.8	5	78.6	192.4	74.6		271.0	345.6
13	141.4	109.9	58.9		251.3	310.2	6	54.9	302.4	47.1	23.6	357.3	428.0
14	168.9	106.0	58.9		274.9	333.8	7	62.8	212.1	43.2	11.8	274.9	329.9
15	47.1	110.0	90.3		157.1	247.4	8	106.0	102.1	70.7		208.1	278.8
16	196.4	145.3	78.5		341.7	420.2	Whole province						
17	455.6	278.8	55.0	3.9	734.4	793.3		4 795.1	4 209.6	2 155.9	157.2	9 004.7	11 317.8

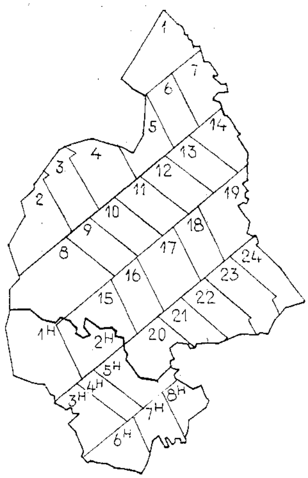


Fig. 21. Map outline of the province of Jämtland showing division into 32 small areas.

c) Accuracy

The distribution of the number of intersections on each of the two systems of survey lines for the various types of roads in each small area has provided a basis for the computation of an approximate formula for the relative standard error of the road-length determinations. The surveys along the two nets of lines are considered as two double determinations independent of each other. The number of intersections in the various cases is reported in table 25.

The variance of the sum of the two determinations x_1 and x_2 of the same type of roads in a small area may be estimated at

$$(x_1 - x_2)^2$$

Within the area J:1 the estimate of the variance of (20 + 12) intersections with existing public roads is $(20 - 12)^2 = 64$.

For the total number of intersections with a certain type of roads, the estimate of the variance equals the total of the variances in each small area:

$$\sigma_T^2 = \sum (x_1 - x_2)^2$$

For the 1 221 intersections with existing public roads the estimate is:

$$\sigma_T^2 = (20 - 12)^2 + (17 - 18)^2 + \ldots + (15 - 12)^2 = 661.00$$

Table 25. Number of intersections with horizontal (H) and vertical (V) line system recorded at line survey for different types of main roads within small areas in the province of Jämtland.

Small area	Line system	No. intersections: Roads, Existing, Public	Roads, Existing, Private	Roads, Proposed	Road net, Existing	Road net, Planned
J: 1	H	20			20	20
	V	12		1	12	13
2	H	17	7	5	24	29
	V	18	4	5	22	27
3	H	13	4	6	17	23
	V	19	2	8	21	29
4	H	3	7	11	10	21
	V	3	8	8	11	19
5	H	8	2	10	10	20
	V	8	2	7	10	17
6	H	13	5	3	18	21
	V	11	6	5	17	22
7	H	9	1	5	10	15
	V	8	2	5	10	15
8	H	12	5	4	17	21
	V	11	2	4	13	17
9	H	24	6	14	30	44
	V	23	12	13	35	48
10	H	29	12	10	41	51
	V	34	11	8	45	53
11	H	23	7	8	30	38
	V	28	17	9	45	54
12	H	23	7	18	30	48
	V	25	8	17	33	50
13	H	15	16	9	31	40
	V	21	12	6	33	39
14	H	26	16	10	42	52
	V	17	11	5	28	33
15	H	3	15	9	18	27
	V	9	13	14	22	36
16	H	32	22	13	54	67
	V	18	15	7	33	40
17	H	59	39	8	98	106
	V	57	32	7	89	96
J:18	H	26	16	14	42	56
	V	29	23	14	52	66
19	H	37	18	16	55	71
	V	36	16	15	52	67
20	H	25	24	11	49	60
	V	20	25	5	45	50
21	H	14	32	9	46	55
	V	18	24	9	42	51
22	H	38	30	12	68	80
	V	36	30	9	66	75
23	H	29	42	20	71	91
	V	31	37	17	68	85
24	H	25	42	9	67	76
	V	22	37	4	59	63
H: 1	H	25	9	4	34	38
	V	26	14	8	40	48
2	H	11	17	13	28	41
	V	9	20	15	29	44
3	H	1	16	13	17	30
	V	5	18	14	23	37
4	H	12	23	10	35	45
	V	18	25	7	43	50
5	H	11	23	9	34	43
	V	9	26	10	35	45
6	H	8	38	8	46	54
	V	6	39	10	45	55
7	H	7	30	8	37	45
	V	9	24	6	33	39
8	H	15	16	9	31	40
	V	12	10	9	22	31
Whole province	H+V	1 221	1 072	589	2 293	2 882

The relative standard error of the total number of intersections then becomes

$$100 \cdot \frac{\sqrt{661}}{1\,221} = \frac{73.58}{\sqrt{1\,221}}\ \%$$

An estimate of the standard deviation per intersection amounting to 73.58 per cent is obtained.

The estimates of the standard deviation per intersection with the various types of roads and road nets are:

	σ_T^2	Number of intersections x	Standard deviation per intersection, per cent
Existing public roads	661.00	1 221	73.58
Existing private roads	593.00	1 072	74.38
Proposed roads	245.00	589	64.49
Present road net	1 369.00	2 293	77.27
Planned road net	2 438.00	2 882	91.97
Average			76.34

Exploiting the average standard deviation obtained per intersection, we get an approximate value for the relative standard error ε in an estimate based on x intersections (with one or several systems of lines independent of each other):

$$\varepsilon = \frac{76.34}{\sqrt{x}} \approx \frac{76}{\sqrt{x}} \tag{12}$$

On an average, the formula gives slightly too high standard errors in the open systems of roads and slightly too low values in the dense systems. For the two road systems that constitute the extremes in this respect, the following estimates are obtained directly on the basis of the survey result, as well as indirectly on the basis of the approximate formula:

	No. intersections x	Standard error of x in per cent	
		Acc. to survey	Acc. to formula (12)
Proposed roads	589	2.7	3.1
Planned road net	2 882	1.7	1.4

It is also possible to establish an approximate formula with improved adaptation by entering an expression of the density of the road system into the formula.

Naturally, the relative standard error of the number of intersections recorded between roads and a line system in a certain case also applies to the determination of the length of roads which is obtained as a product of the number of intersections and "the survey factor" (3.9270).

The approximate formula (12) is presented graphically in figure 22. By means of this presentation an approximate value for the precision of the road-length determination for instance for separate small areas, and for groups of such areas, can be obtained on the basis of the number of intersections shown in table 25.

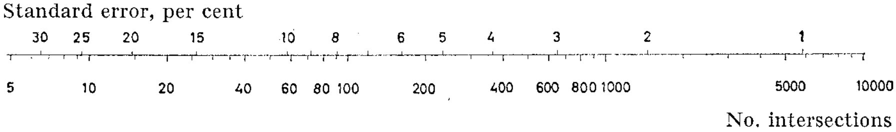


Fig. 22. Relative standard error of road length determination at various numbers of intersections with the line systems. Graphical presentation of the approximate formula (12).

Summary

The present report deals with some methods of calculating the relationship between the cross-country transport distance and the road net density, and procedures to estimate the length of a road net. The methods, which also apply to transport systems other than roads, are primarily intended for summary calculations pertaining to large areas such as management units or parts thereof, districts, catchment areas, and still larger areas. However, the methods are also partly applicable to small areas such as individual tracts of felling.

Calculations that require knowledge of the relationship between the mean cross-country transport distance and the road net density are usually based on a relationship which applies provided the roads are straight and parallel lines, the distances between the roads are equal, and the cross-country transport is straight-lined and perpendicular to the roads.

The first part of the report deals with a method of estimating empirically the adjustments that should be entered into the function for this "theoretical" relationship between the mean transport distance and the road net density in order to make the function applicable to the real conditions of road nets and transports. An adjustment, the road net adjustment, is then calculated for the relationship between the "theoretical" mean distance and the road net density of existing road nets. The distance adjustment for cross-country transport is calculated for the relationship between the distance of the "practical" cross-country course of the road and the "theoretical" one.

The "theoretical" cross-country distance to the road has been called the "geometric" cross-country transport distance, and it has been defined as the shortest straight-line distance from a given point to the nearest road.

The estimate of both "geometric" and "practical" mean distances has been made after systematic sampling by means of a lay-out of quadratic point patterns. The points in the patterns have been made points from which the distances to roads or other transport systems have been measured.

The road net adjustment has been studied on both models of road nets and maps of existing roads. It then appeared that the road net adjustment varies as the geometric design of the road net. Thus, it has been shown that the evenness in the distribution of the roads over the area is of essential importance for the magnitude of the road net adjustment. An increase in the road net

adjustment follows an increase in the unevenness of the distribution of the roads.

It has also been shown that the road net adjustment of a certain road net is usually changed very little even when the road net density has been raised considerably by road extension. However, when extension caused a marked change in the form of the road net, the road net adjustment has changed rather more clearly.

The investigations of the road net adjustment have also produced empirical values showing how the area within regions is distributed between various "geometric" cross-country transport distances in relation to the "geometric" mean distance.

Values of the cross-country transport adjustment have been computed empirically from maps and by combining field work and map measurements.

The adjustment has appeared caused by the following conditions:

— Cross-country transports from some parts of a transport area must often be carried out to some road other than that which is the closest one.

— The timber is usually hauled to special landings on the road side.

— The "practical" course of haulage deviates from the straight line between the point of loading and the landing on the road side.

An investigation carried out by means of a combination of measurements in the field and on maps produced a correlation to the regression of the cross-country transport adjustment on the geometric cross-country distance that was very weak. Under this assumption it should be possible to compute for an area the cross-country transport adjustment on the basis of relatively few determinations and to apply the arithmetic mean value to various distances. However, a clear trend has been obtained to the effect that a decline in value is associated with longer distances when the cross-country transport adjustment has been computed on the basis of maps only.

A separate section deals with descriptions of how the adjustments of road nets and of cross-country transports have been entered into the type of calculations of the road net density reported by *Sundberg*.

The second part of the report, which contains determinations of the length of roads, starts with statistics concerning the extent of the road net in Sweden at various times. It is then reported on a method of determining the length of roads by random sampling and collection of data. This method is based on an application of the "*Buffon* needle problem" described by *Matérn*.

The method has so far been most important when used at the road inventory annually carried out by the National Forest Survey from 1956 to 1963. A registration has then been made concerning the number of intersections between roads and the sides of so-called survey tracts. Classification has been applied to the various properties of the roads at the intersections.

The road inventory has been particularly aimed at a determination of the length of roads that affect the cross-country transport distance. Data on the total length of such roads have not previously been available.

The inventories of the years 1956—1960 report on data concerning the length of public and permanent roads of importance for the cross-country transports. Data have been presented according to their distribution between the regions I-V used by the National Forest Survey and according to their distribution between ownership categories within one northern and one southern part of Sweden. For the northern part has also been presented the length of seasonal roads travelled by tractor or lorry in winter. The road net density expressed in metres of road per hectare has also been calculated by means of areal data. The road net densities calculated in this way have been used together with assumed values of the adjustments for road nets and for cross-country transports in order to compute the mean cross-country transport distances within various ownership groups and areas of reporting.

In the last section of the work a report has been presented for the estimates of the length of roads within small areas in the province of Jämtland. The same procedure, a line survey on maps, has then been used as that used at the road inventory of the National Forest Survey. The report has been made rather comprehensive in order to make it useful as a guide for other estimates of a similar nature.

REFERENCES

(1) ANDERSSON, S, 1960. Några kalkylmodeller för bilbasvägar och permanenta bilvägar. (Some models of calculation concerning seasonal lorry roads and permanent roads.) — Statens skogsforskningsinstitut, avd. för arbetslära. Rapporter nr 11.

(2) ARVIDSON, B, 1955. Traktorvägen i småskogsbruket. (Tractor roads in woodland forestry.) — Skogsägaren 4:1955.

(3) —1959. Flygbilden vid kostnadsberäkning av skogsvägföretag. (Aerial photography used for cost control in forest road planning.) — Skogen 12: 1959.

(4) Det enskilda skogsbruket 1942—1950. (Private forestry in 1942—1950.) — Stockholm 1944—1952.

(5) Domänverket 1922—1960. (Summary: The Swedish Forest Service 1922—1960.) — Stockholm 1924—1961.

(6) FRÉDEN, E, 1957. Skogsbrukets biltransporter statistiskt kartlagda på grundval av Biltransportutredning 1955. Del I och II. (Lorry transports in forestry mapped statistically on the basis of the 1955 Lorry Transport Investigation, parts I and II.) — (Stencil.)

(7) GUSTAFSSON, B, 1963. Vägnätet den 1 januari 1963. (The Swedish road net of January 1st, 1963.) — Svenska Vägföreningens Tidskrift 7: 1963.

(8) HAGBERG, E, 1957. The New Swedish National Forest Survey. — Unasylva. Vol. 11. 1957: 1.

(9) HALL, B, 1959. Några synpunkter och exempel på olika sätt att uppskatta de transportvinster, som uppkomma vid byggande av skogsbilvägar. (Some views on and examples of various ways of estimating the transport gains obtained by the extension of forest roads.) — Medd. nr 3 från Inst. för fastighetsteknik, Kungl. Tekniska Högskolan, Stockholm.

(10) HJELMSTRÖM, L, 1961. Vägarna och skogstransporterna. Allmän översikt av problemställningarna. (Roads and transports in forestry. General summary of the problems.) — Sv. Skogsvårdsföreningens Tidskrift 1: 1962.

(11) Instruktion för arbetet å marken under riksskogstaxeringen år 1960. (Instruction to the field work at the National Forest Survey in 1960.) — Statens skogsforskningsinstitut, avd. för skogstaxering. (Stencil.)

(12) JANLÖV, C-E, 1954. Angående skogsbrukets transportnät. (Notes on the net of transport roads in forestry.) — Kungl. Skogshögskolans institutioner — skoglig arbetslära. Uppsatser nr 7.

(13) Kungl. Domänstyrelsens förvaltning åren 1911—1921. (Management of the National Board of Crown Forests and Lands in 1911—1921.) — Stockholm 1913—1923.

(14) Kungl. Maj:ts Befallningshafvandes femårsberättelser för åren 1856/60—1901/05. (Five-yearly reports of the county governors for the years 1856/60—1901/05.) — Stockholm 1863—1912.

(15) Kungl. Skogsstyrelsens underdåniga berättelser för åren 1870—1910 (fr. o. m. år 1883: Domänstyrelsens . . .) (Accounts of the National Board of Crown Forests and Lands for the years 1870—1910.) — Stockholm 1872—1912.

(16) Lag angående väghållningsbesvärets utgörande på landet. (Law concerning the execution of road maintenance in rural districts.) — Svensk författningssamling 1891: 68.

(17) LARSSON, G, 1959. Studies on Forest Road Planning. — Transactions of the Royal Institute of Technology, Stockholm. No. 147.

(18) LARSSON, G & RYDSTERN, O, 1961. Kalkylering av skogliga vägsystem. (Design of forest road systems.) — Medd. nr 6 från Inst. för fastighetsteknik, Kungl. Tekniska Högskolan, Stockholm.

(19) MATÉRN, B, 1960. Spatial variation. — Medd. från Statens skogsforskningsinstitut. Band 49: 5.

(20) MATTSSON MÅRN, L, 1950. Kalkylering av skogliga vägsystem. (Design of forest road systems.) — Kungl. Skogshögskolans institutioner — skoglig arbetslära. Uppsatser nr 1.
(21) NILSSON, N-E & VON SEGEBADEN, G, 1962. Undersökning rörande avsättningslägen för skog och skogsmark i Jämtlands län. (Summary: Investigation concerning the accessibility of forests and forest land in the province of Jämtland.) — Medd. från Statens skogsforskningsinstitut. Band 51: 4.
(22) Riksskogstaxeringen 1953—62; redovisningsområden, faktorer m. m. (The National Forest Survey 1953—1962; reported areas, survey factors etc.) — Statens skogsforskningsinstitut, avd. för skogstaxering, 1957. (Stencil.)
(23) VON SEGEBADEN, G & SUNDBERG, U, 1957. Några synpunkter på det praktiska utförandet av optimalkalkyler över vägnätets täthet. (Some views on the practical application of calculations of the optimum road net density.) — Cirkulärskrivelse U 17 från Skogsbrukets Transportutredning. (Stencil.)
(24) Skogsstatistisk årsbok 1951—1960. (Summary: Statistical Yearbook of Forestry 1951—1960.) — Stockholm 1953—1962.
(25) Skogstillgångarna i Jämtlands län. Förslag till bättre utnyttjande avg. av 1960 års skogskommitté för Jämtlands län. (Forest resources in the province of Jämtland. Proposals to improved utilization presented by the 1960 Forestry Committee for the province of Jämtland.) — SOU 1962: 1. — Stockholm 1962.
(26) SNORRE STURLASSON (1179—1241). Norges konungasagor. (Norwegian Tales of the Kings. First volume.) Translated by E Olsson. — Lund 1919.
(27) Statistisk årsbok för Sverige 1914—1961. (Summary: Statistical Abstract of Sweden 1914—1961.) — Stockholm 1914—1961.
(28) SUNDBERG, U, 1953. Studier i skogsbrukets transporter. (Summary: Studies of transportation in forestry.) — Forskningsstiftelsen SDA:s medd. nr 48.
(29) Utredning rörande skogsnäringens ekonomiska läge med förslag till åtgärder för höjande av näringens bärkraft. III. Skogsbrukets transportfrågor: vägar och järnvägar. (Investigation of the economic status of forestry and proposals to measures adapted to improve its profitability. III. Transport problems in forestry: roads and railways.) — 1936 års skogsutrednings betänkande nr 3. — SOU 1943: 4. — Stockholm 1943.

Sammanfattning

Studier över terrängtransportens längd och vägnätets utbyggnad

Föreliggande redogörelse återger i sammandrag innehållet i en licentiatavhandling i ämnet skoglig arbetslära, som något omarbetad publicerats i rapport nr 23 från institutionen för skogsteknik vid Skogshögskolan.

I avhandlingen redovisas vissa metoder för beräkningar rörande sambandet mellan tilltransportavstånd och vägtäthet samt för väglängdsbestämningar. Metoderna, som äger tillämpning även för andra typer av transportleder än vägar, är i första hand avsedda för överslagsberäkningar för större områden såsom revir eller delar därav, distrikt, ådalar och ännu större områden. De kan dock delvis tillämpas även för mindre områden såsom enskilda drivningstrakter.

Vid beräkningar, i vilka kännedom fordras om sambandet mellan medeltilltransportavstånd och vägtäthet, används i regel ett samband som gäller under förutsättning att vägarna är linjeräta och parallella, att det inbördes avståndet mellan dem är lika samt att transporten i terrängen (tilltransporten) är rak och sker vinkelrätt mot vägen.

Avhandlingens första del behandlar en metodik att empiriskt beräkna de korrektioner som bör införas i funktionen över detta »teoretiska» samband mellan medeltransportavstånd och vägtäthet för att denna skall kunna tillämpas även vid faktiska vägnäts- och transportförhållanden. En korrektion, vägnätskorrektionen, beräknas därvid för sambandet mellan det »teoretiska» medelavståndet och vägtätheten vid verkliga vägnät. För sambandet mellan längden av den »praktiska» körvägen i terrängen och den »teoretiska» beräknas terrängtransportkorrektionen.

Det »teoretiska» avståndet i terrängen till väg har i arbetet benämnts »geometriskt» tilltransportavstånd. Detta har definierats som det kortaste fågelvägsavståndet från en punkt till närmaste väg.

Uppskattningen av både geometriska och praktiska medelavstånd har skett i form av systematisk sampling genom utläggning av kvadratiska punktnät. De i näten ingående punkterna har utgjort mätpunkter för avstånden till väg eller annan transportled.

Vägnätskorrektionen har studerats både i modeller av vägnät och på kartor över verkliga vägnät. Därvid har det framkommit att den varierar med vägnätets geometriska utformning. Det har sålunda visats att jämnheten i vägarnas fördelning över arealen är av väsentlig betydelse för vägnätskorrektionens storlek. Med ökad ojämnhet i vägarnas fördelning följer en större vägnätskorrektion.

Det har även visats att vägnätskorrektionen för ett visst vägnät i regel ändrats endast obetydligt även när detta genom utbyggnad erhållit en betydligt ökad täthet. Då utbyggnaden medfört en markerad förändring av vägnätets form, har dock vägnätskorrektionen ändrats mera påtagligt.

Vid undersökningarna över vägnätskorrektionen har även erhållits empiriska värden på hur arealen inom områden fördelar sig på geometriska tilltransportavstånd i förhållande till det geometriska medelavståndet.

Värden på terrängtransportkorrektionen har beräknats empiriskt både på kartor och vid en kombination av fält- och kartarbete.

Korrektionen har befunnits vara orsakad av följande förhållanden:

— Från vissa delar av ett transportområde måste ofta terrängtransporten ske till annan väg än den närmast belägna.

— Virket transporteras i regel fram till särskilda lastplatser vid väg.

— Den »praktiska» körvägen avviker från fågelvägen mellan pålastningsplatsen i terrängen och lastplatsen vid väg.

Vid en undersökning, vilken utförts på kronoparken Kloten som en kombination av mätningar i fält och på kartor, har terrängtransportkorrektionens korrelation med det geometriska tilltransportavståndet varit mycket svag. Under denna förutsättning bör man för ett aktuellt område kunna beräkna terrängtransportkorrektionen med ledning av relativt få bestämningar och låta det därvid erhållna aritmetiska medelvärdet gälla för olika avstånd. — Då terrängtransportkorrektionen beräknats enbart på kartor har däremot en tydlig tendens till avtagande värde vid längre avstånd erhållits.

I ett särskilt avsnitt har beskrivits och exemplifierats hur vägnäts- och terrängtransportkorrektionerna införes i den typ av kalkyler rörande vägnätets täthet som redovisats av *Sundberg*.

Avhandlingens andra del, som behandlar väglängdsbestämningar, inleds med statistiska uppgifter rörande det svenska vägnätets omfattning vid olika tidpunkter. Därefter har redogjorts för en metodik att bestämma ett vägnäts längd genom stickprovsmässig insamling av data. Denna bygger på en av *Matérn* anvisad tillämpning av »*Buffons* nålproblem».

Metodiken har fått sin hittills viktigaste användning vid riksskogstaxeringens väginventering, som företagits åren 1956—1963. Därvid har en registrering skett av antalet skärningar mellan vägar och sidorna på de s. k. taxeringstrakterna. Med hänsyn till vägens beskaffenhet m. m. i anslutning till skärningspunkten med traktsidan har en klassificering utförts.

Väginventeringen har speciellt syftat till att bestämma längden av de vägar, som påverkar tilltransportavståndet. För den totala längden av dessa vägar har uppgifter ej funnits tillgängliga.

Från inventeringarna åren 1956—1960, kallad väginventeringen 1956—60, har bl. a. redovisats uppgifter om längden av allmänna och enskilda permanenta vägar av betydelse för tilltransporten. Uppgifterna har givits dels fördelade på riksskogstaxeringens regioner I—V och dels fördelade på ägarkategorier inom en nordlig och en sydlig regiongrupp. I den nordliga gruppen har dessutom angivits längden av vintertid med traktor eller lastbil farbara s. k. motorbasvägar. — Med hjälp av arealuppgifter har även vägtätheten i meter väg per hektar beräknats. De så beräknade vägtätheterna har tillsammans med antagna värden på vägnäts- och terrängtransportkorrektionerna legat till grund för en beräkning av medeltilltransportavstånd inom olika ägargrupper och redovisningsområden.

I arbetets sista avsnitt har redogjorts för uppskattningen av väglängden inom s. k. småområden i Jämtlands län. Därvid har samma metodik, i form av linjetaxering på kartor, tillämpats som vid riksskogstaxeringens väginventering. Redovisningen har gjorts relativt omfattande för att kunna tjäna till ledning vid andra uppskattningar av liknande karaktär.

A Method of Estimating the Total Length of Roads by Means of a Line Survey

by

BERTIL MATÉRN

The needle problem, the classical problem in geometric probabilities can be described as follows: A system of equidistant parallel lines is drawn on a sheet of paper. A needle of a length (a) less than the distance (b) between two neighbouring lines in the system is placed at random on the paper. What is the probability that the needle intersects the lines?

According to the French naturalist *Buffon*, the probability is

$$\frac{2}{\pi} \cdot \frac{a}{b}$$

Buffon published this solution in 1777 but stated the problem already 1733 (see *Castelnuovo*, 1925).

Consider a chain consisting of N links, each one with length a. The chain is placed on the paper so that the position of each individual link in relation to the line system is determined by a random procedure of the same nature as that governing the position of the needle in the *Buffon* problem. The expected number of intersections between the chain and the line system is then

$$\frac{2}{\pi} \cdot \frac{L}{b} \tag{1}$$

where $L = N \cdot a =$ the length of the chain. The same result is obtained if instead the chain is initially placed on the paper and the position of the line system then is determined by random translation and rotation in relation to fixed coordinate axes.

A simple argument (presented by *Barbier* in 1860) can explain why the factor $2/\pi$ occurs in formula (1). We consider the limiting case obtained when the links of the chain are made smaller and smaller in such a manner that the chain is finally transformed to a circle with a diameter equal to the distance between the lines (b). The circumference of the circle is $L = \pi \cdot b$, and the circle cuts the lines in two points, irrespective of position. Formula (1) also gives the value 2:

$$\frac{2}{\pi} \cdot \frac{L}{b} = \frac{2}{\pi} \cdot \frac{\pi \cdot b}{b} = 2$$

Consider then an area with a *network of roads* with the total length L. If a system of parallel equidistant lines (at distance b apart) is placed according to a randomly chosen direction, the expected number of intersections with the road net is $2L/\pi \cdot b$ as in formula (1). Conversely, let n be the number of intersections between the line system and the network of roads, and form the expression

$$Y = \frac{\pi}{2} \cdot b \cdot n \tag{2}$$

The expected value of this expression then equals L, or using E for mathematical expectation:

$$E\,(Y) = L$$

If the total length of roads is unknown, we thus have a possibility to estimate it by computing Y.

We can also write:

$$Y = \frac{\pi}{2} \cdot \frac{n}{t} \tag{3}$$

where t is the total length of the line system per unit area. (In the case now discussed $t = 1/b$). The formula $E(Y) = L$ also applies in the case of two line systems at right angles. It is also applicable to the tract sides of the third National Forest Survey which form a system of fragments from two line systems at right angles. In these cases we only have to substitute for t in formula (3) the value which corresponds to the form of survey concerned.

The formula $E(Y) = L$ applies if *the direction of the line system is determined at random.* In other words, the average value of Y for a very great number of systems with different directions equals L. For an individual system, however, Y may considerably deviate from L even if the lines of the system are spaced very closely. This situation may occur if the road net is located in such a way that most of the roads run in the same direction. However, the bias is considerably reduced if we have two systems at right angles, or fragments of two systems of this kind. This statement can be proved by formulas but it is perhaps better to illustrate it by some examples. The examples have been obtained by placing line systems on maps and by counting the number of intersections with some net of railways or roads.

Example	Length according to curve-meter km	Estimated length in kilometres		
		Formula (2) and line system in		Both systems
		W-E	N-S	
Province of Jämtland				
Railways	732	738	746	741
Main roads	1 835	1 806	1 885	1 846
Province of Göteborg and Bohus				
Railways	290	330	259	295
Isle of Öland				
Railways	146	192	90	140
Province of Malmöhus				
Railways	868	793	911	852
Total	3 871	3 859	3 891	3 874

As shown by the examples from the province of Göteborg and Bohus, and the isle of Öland, an one-sided orientation of the roads may lead to rather biased estimates if only one line system is used. No essential bias in estimating the road length has been encountered with two line systems at right angles, neither in the examples shown above, nor in other investigations not reported here.

If the surveyed line length in an area is small, large *sampling errors* may occur in the estimate of the total length of roads. This statement can be supported by an example. On a map of Östergötland, 15 different systems of survey tracts were laid out, each system having an extent equal to that of the annual set of survey tracts in the third National Forest Survey. The number of intersections between the different systems and the public roads shown on the map amounted to (the values arranged according to magnitude): 40, 42, 42, 43, 46, 47, 50, 50, 57, 57, 58, 58, 60, 62, 69. Using these observations, we then find for the relative standard error of the total length of the roads the estimate 17 per cent. This standard error applies to the estimate of road-length based on one year's tracts.

Postscript 1964. The investigations reported above were carried out in 1956. The author has briefly commented on the problem in a paper published in 1959. As mentioned in that paper, the theory of geometric probabilities has been applied to the estimation of the average fiber length (*Backman* 1934, *Kilpper* 1949, *Kane* 1956, *Kallmes & Corte* 1960) in a way analogous to the procedure described above.

For a more general discussion the reader is referred to an interesting paper by *Steinhaus* (1954) and the recent monograph *Kendall & Moran* (1963).

REFERENCES

BACKMAN, A., 1934. Bestämning av cellulosahalten i papper genom mätning av fibertätheten. (Determination of the cellulose content of paper by measuring the fiber density.) — In Swedish — Pappers- och Trävarutidskrift för Finland, 16: 302—308.

BARBIER, E., 1860. Note sur le problème de l'aiguille et le jeu du joint couvert. — J. Math. pures et appl. 5: 273—286.

BUFFON, G., 1777. Essai d'arithmétique morale. — Supplément à l'Histoire Naturelle, Vol. 4.

CASTELNUOVO, G., 1925. Calcolo delle Probabilità. I. — Bologna.

KALLMES, O. & CORTE, H., 1960. The structure of paper. I. The statistical geometry of an ideal two dimensional fiber network. — TAPPI 43: 737—752. (Errata: Volume 44: 448.)

KANE, M. W., 1956. The determination of average fiber length. — TAPPI 39: 478—480.

KENDALL, M. G. & MORAN, P. A. P., 1963. Geometrical probability. (Griffin's statistical monographs & courses, no. 10.) — London.

KILPPER, W., 1949. Entwicklung einer Schnellmethode zur Bestimmung der Faserlänge von Fasergemischen für betriebliche Zwecke. — Wochenblatt für Papierfabrikation, 75: 160—164.

MATÉRN, B., 1959. Några tillämpningar av teorin för geometriska sannolikheter. (Some applications of the theory of geometric probabilities.) — In Swedish — Svenska Skogsvårdsföreningens Tidskrift 57: 453—458.

STEINHAUS, H., 1954. Length, shape and area. — Colloquium Mathematicum 3: 1—13.