Preference Cloud Theory: Modelling Imprecise Preferences and a New Theory for Decision under Risk

Oben K. Bayrak
Centre for Environmental and Resource Economics (CERE)
Faculty of Forest Sciences
Department of Forest Economics
Umeå

Doctoral Thesis
Swedish University of Agricultural Sciences
Umeå 2016

Acta Universitatis agriculturae Sueciae

2016:44

ISSN 1652-6880
ISBN (print version) 978-91-576-8590-2
ISBN (electronic version) 978-91-576-8591-9

Print: SLU Service/Repro, Uppsala 2016

Preference Cloud Theory: Modelling Imprecise Preferences and a New Theory for Decision under Risk

Abstract

This study presents Preference Cloud Theory, a two-step model of decision making under risk. It also includes an experimental study on valuation gap which provides supporting results for the new theory. The new theory provides an explanation for empirically observed anomalies of Expected Utility Theory such as the Allais Paradox, valuation gap, and preference reversals. Central to the theory is the incorporation of preference imprecision, which has support in emerging literature, and challenges to the alternative models for Expected Utility Theory. Preference Cloud Theory assumes that preference imprecision arises because of individuals' vague understanding of numerical probabilities. The theory combines this concept with the use of the Alpha Model (which builds on Hurwicz's criterion) and constructs a simple model, helping us to understand various anomalies discovered in the experimental economics literature that standard models could not explain.

Keywords: decision under risk, preference reversals, valuation gap, Allais Paradox, willingness to pay and accept disparity, experimental economics, behavioural economics

Oben K. Bayrak, SLU, Department of Forest Economics,
P.O. Box 901 83 Umeå, Sweden
oben.Bayrak@ slu.se

Dedication

To my parents; Sevim and Şahin Bayrak for investing their whole life to me, to my sister Özge Bayrak for being my best friend, and to my wife, my significant other Elif Süslü Bayrak who suffered with me through this journey and never left my side, finally to my hairs that I lost; of course, no thanks to the hurdles (things and people) I faced and met, without them this book will be prepared earlier with less stress…

We're all mad here.

Lewis Carrol, Alice in Wonderland

The reasonable man adapts himself to the world; the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.

George Bernard Shaw

Imagination is a quality given to man to compensate him for what he is not and a sense of humor was provided to console him for what he is.

Oscar Wilde

Contents

List of Publications

I Oben K. Bayrak and Bengt Kriström (2016). Is There a Valuation Gap? The Case of Interval Valuations. *Economics Bulletin* 36(1), 218–236.

II Oben K. Bayrak and John D. Hey (2015). Preference Cloud Theory: Imprecise Preferences and Preference Reversals. *SSRN Scholarly Paper* No. ID 2633392. Social Science Research Network, Rochester, NY.

III Oben K. Bayrak (2015). Note: Imprecise Preferences, Preference Cloud Theory, Multiple Selves and Intrapersonal Planner Approach. *SSRN Scholarly Paper* No. ID 2669986. Social Science Research Network, Rochester, NY.

IV Oben K. Bayrak. Modelling imprecise preferences and a new theory for decision under risk: Preference Cloud Theory.

Papers I-III are reproduced with the permission of the publishers.

The contribution of Oben K. Bayrak to the papers included in this thesis was as follows:

I Bengt Kriström suggested using self-selected intervals in valuation surveys, I proposed the idea of using self-selected intervals in valuation gap experiment. I distilled the hypothesis to be tested, designed and conducted the experiment and do the statistical analysis, and finally wrote the paper. Bengt Kriström contributed in writing as well.

II I proposed the idea; throughout the discussions with John Hey, the idea is developed. I wrote the paper and John Hey contributed in writing as well.

Abbreviations

PCT: Preference Cloud Theory
EUT: Expected Utility Theory
BDM: Becker-DeGroot-Marschak mechanism
RDUT: Rank Dependent Utility Theory
WTP: Willingness to Pay
WTA: Willingness to Accept

Introduction

Decision theories are the building blocks of economic theory, used in modelling behaviour in various sub-fields and issues such as finance, management, insurance, health economics, game theory, welfare economics, life cycle income and consumption, and tax policy. Economists use decision models to explain observed behaviour, evaluate policy or market design schemes, and provide predictions, as do decision analysts providing consultancy services to the individuals and firms who want to make better and more coherent decisions.

This study focuses on decision making under risk, which can be described by simple lotteries with monetary outcomes and associated probabilities. In general, decision theories can be broadly classified as prescriptive (normative) and descriptive theories. Prescriptive theories suggest how a rational agent should act in a given circumstance, whereas descriptive ones provide insights about how actual individuals make decisions. When we talk about prescriptiveness, rational decision theory is synonymous with Expected Utility Theory, a standard theory in economics. Until the 1970s, it was also regarded as a descriptive theory. Although it had appealing mathematical properties and established axioms, new literature emerged as behavioural and experimental economics raised doubts about its descriptive validity and predictive power. Experimental studies documented various systematic and robust deviations, conventionally known as anomalies, from the behaviour predicted by Expected Utility Theory, such as the Allais Paradox, valuation gap, and preference reversals. In response to those anomalies, several alternative models have been proposed, such as Prospect Theory, Rank-Dependent Utility Theory, Regret Theory, and Cumulative Prospect Theory.

However, there is an emerging literature on preference imprecision which challenges the validity of these alternative theories. Experimental studies in this new strand of literature suggest that even intelligent and numerate individuals find it hard to know their own preferences precisely and are not able to state their choices and subjective valuations for goods and risky prospects with perfect confidence. Although alternative theories model individual behaviour in a non-standard way to explain these observed anomalies, they share a common implicit assumption with Expected Utility Theory that individuals can articulate their subjective valuations for goods and make choices in a precise manner. Therefore, the issues raised by this recently emerging literature are not covered by the existing models in the literature including both the Expected Utility Theory and its alternatives. These recent findings have critical implications as well: if, for example, consumers' preferences are imprecise and prone to being manipulated, this may be used against consumers' own best interests. Moreover, if the inherent characteristics of economic preferences are imprecise, the validity of the studies that evaluate and analyse the policies and/or market schemes based on the existing models of precise preferences should also be reconsidered. In order to reach solid conclusions about all of these issues, it is vital to have a better understanding and a better model of the imprecise preferences.

This study is organised as follows: Chapter 1 reviews the early attempts of modelling decision under risk, such as Pascal and Fermat's expected value concept and famous Expected Utility Theory, along with a review of the anomalies documented by experimental studies that raise doubts about the descriptive and predictive power of Expected Utility Theory. The chapter also includes the alternative models to Expected Utility Theory, which normalise these detected anomalies. Chapter 2 presents a critical review of the emerging hypothesis known as preference imprecision hypothesis: it includes a discussion of existing modelling approaches for imprecise preferences, i.e., stochastic preferences (Section 2.2), and the experimental studies designed to elicit imprecision ranges, particularly in valuation tasks (Section 2.3 and 2.4). Chapter 3 presents an extended version of Bayrak and Kriström (2016), an exploration and re-examination study on valuation gap from the imprecision perspective. Our study investigates the existence of the valuation gap when we allow subjects to state their subjective valuations as intervals. It extends the literature on the willingness-to-pay/willingness-to-accept (WTP/WTA) disparity by testing two hypotheses distilled from the literature. It also introduces an incentive compatible mechanism for eliciting the imprecision range in valuation tasks. Its incentive compatibility is an important contribution to the literature because the existing mechanisms in the imprecision literature rely on subjects' self-reporting in eliciting the imprecision ranges, which is not conventional to experimental economics. Finally, Chapter 4 presents a new decision theory for risk, Preference Cloud Theory, which incorporates preference imprecision and explains the anomalies of Expected Utility Theory. The new theory can be seen as an extension of Expected Utility Theory as it models behaviour over final wealth levels and can explain the observed anomalies without incorporating reference dependency and loss aversion.

I would like to thank the participants of the conferences and departmental (economics) seminars where this work is presented: ESA (European Science Association) European Meeting (2015), Public Economic Theory Association 16th Annual Conference (2015), Spanish Association of Law and Economics, Annual Conference (2015), The 6th Ulvön Conference on Environmental Economics (2014), University of Gothenburg (2014), University of Aachen (2014), University of Amsterdam (2014), Erasmus University Rotterdam (2014). I also thank Kim Kaivanto, Mehmet Bac, Prasenjit Banerjee, Fredrik Carlsson, Jack Knetsch, Mike McKee, Zahra Murad, Charles Plott, Bo Ranneby, Jason F. Shogren, Joep Sonnemans, Chris Starmer, Peter Wakker and Kathryn Zeiler for helpful discussions and comments and to my colleagues Dr. Kelly de Bruin, Brian Danley and Jinggang Guo for their help in conducting the experiments. Finally, I thank to my mentors Prof. Bengt Kriström and Prof. John D. Hey for their feedback and useful suggestions. All errors are mine. I thank Rachel Siegel (Cambridge Editors) for doing proofreading and polishing the language of this piece. Finally, financial support of Tore Browaldh Foundation for my PhD studies is also acknowledged.

1 Background: Expected Utility Theory, Anomalies, and Alternatives

This chapter presents the historical background of modelling decision making under risk, starting with Pascal and Fermat's expected value concept that soon led the development of Expected Utility Theory (Section 1.1). Section 1.2 introduces the anomalies observed in the literature that raised doubts about the descriptive and predictive validity of the Expected Utility Theory, providing a brief introduction to anomalies such as the Allais Paradox, preference reversals, and valuation gap. The detailed reviews for these anomalies are presented in Chapter 4. Section 1.3 presents the existing theories developed to incorporate and explain these reported anomalies in the literature. Finally, Section 1.4 paints the overall picture of the literature presented.

1.1 Early Developments and Expected Utility Theory

The origin of modelling decisions under risk can be traced back to the collaboration of Pascal and Fermat in 1694 on solving what is known as the problem of points; the solution led to the development of the mathematical foundation of probability concept. The problem of points is based on the problem of how to divide up the stakes of an unfinished game between two players who have equal chances of winning in each round. The rule is that the two players contribute equally to a prize pot and the first player to win a certain number of rounds collects the prize. However, unpredicted external circumstances interrupt the game before either of the two players wins the certain number of rounds. How then to divide the pot fairly?

The norm that they proposed is the expected value, which is the weighted sum of the monetary outcomes where the weights are the corresponding probabilities of each outcome. Formally, let $L = \{x_i, p_i\}$ represent a prospect which specifies the monetary outcomes for each state $i = \{1, \ldots n\}$ and the corresponding probabilities satisfying $p_i \geq 0$ and $\sum_1^n p_i = 1$. Then the expected value (EV) of L is calculated as:

$$EV(L) = \sum_1^n p_i \cdot x_i \tag{1}$$

Their underlying assumption is that the attractiveness of a gamble is linearly proportional to the outcomes and the corresponding probabilities.

The problem with this view was raised by Daniel Bernoulli, Swiss mathematician, in his paper *Specimen theoriae novae de mensura sortis* (1738) or 'Exposition of a new theory on the measurement of risk'. The main purpose of the paper is to show that different people may value the same lottery differently depending on their different risk attitudes. This view was a major breakthrough in the theoretical understanding of decision under risk because it accounts for the heterogeneity of individual preferences and personalities. He demonstrated his ideas by his famous St. Petersburg paradox, a gamble in which a fair coin is flipped repeatedly until it comes tails. If it comes up heads in the first toss, it pays \$1, then \$2 if it comes up heads in the second toss, \$4 in the third toss, etc. The prize is doubled with each toss until the first tails comes. The problem is to determine the willingness to pay for such a gamble. The expected value of this gamble sums to infinity, which is unreasonable as few individuals would forgo more than a moderate amount for a one-shot play:

$$EV = 1/2 \cdot 2 + 1/4 \cdot 4 + 1/8 \cdot 8 + ... = 1 + 1 + 1 + ... = \infty \tag{2}$$

Bernoulli proposed that individuals do not evaluate prospects by their expected value but rather by their expected utility, a subjective value. He explains the utility concept in his famous work as following:

> Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill-advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats.

The crucial point about utility is its concavity, which implies that \$200 does not necessarily mean that it worth double what \$100 is worth. Again Bernoulli explains this as:

> . . . the determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount.

This property is known as diminishing marginal utility, explained using a log function. Bernoulli then suggests a new method for calculating the value of a gamble:

> If the utility of each possible profit expectation is multiplied by the number of ways in which it can occur, and we then divide the sum of these products by the total number of possible cases, a mean utility (moral expectation) will be obtained, and the profit which corresponds to this utility will equal the value of the risk in question.

Unlike Pascal and Fermat's linear formulation, Bernoulli suggests that there is a nonlinear relationship between the value of a gamble and the payoffs in each state, but that relationship is still assumed to be linear in corresponding probabilities:

$$EU(L) = \sum_{1}^{n} p_i \cdot u(x_i) \tag{3}$$

This formulation offers a solution for the St. Petersburg paradox by assuming a concave utility function or risk aversion since the sum does not lead to infinity:

$$EU = \sum_{i=1}^{\infty} (1/2)^i \cdot \ln(2^i) = \sum_{i=1}^{\infty} i \cdot (1/2)^i \cdot \ln(2) = \sum_{i=1}^{\infty} (i/2^i) \cdot \ln(2) = 2 \cdot \ln(2) \tag{4}$$

The concavity of the utility function guarantees that, while the expected value of the gamble is infinite, its expected utility is finite. Bernoulli's ideas influenced Von Neumann and Morgenstern's foundational book *Theory of Games and Economic Behavior* (Morgenstern, 1976; Von Neumann and Morgenstern, 1944). They provided the necessary and sufficient conditions under which the Expected Utility Theory (EUT) holds, and this becomes the standard theory of decision under uncertainty and the core of game theory, used in a vast range of economic theoretical models. However, empirical evidence accumulated for more than four decades has revealed a variety of patterns in choice behaviour that appear inconsistent with EUT. I will follow the convention in the literature and refer to them as anomalies, such as the Allais Paradox, preference reversals, valuation gap, and Rabin's paradox. These anomalies raised concerns about the descriptive validity of the Expected Utility Theory and led researchers to develop alternative models which normalise these anomalies.

1.2 Anomalies

This section is a brief introduction to the most prominent detected anomalies in the literature. A more detailed review is presented in Chapter 4 while presenting Preference Cloud Theory (PCT).

1.2.1 Allais Paradox

The violation of the independence axiom was discovered by Maurice Allais (1953), and is now known as the Allais Paradox. His seminal work includes two hypothetical choice problems which are depicted in Table 1. In the first one, known as the common consequence effect, the task is formulated as a choice between two pairs of lotteries. The first pair includes choosing one of the two prospects: s_1 = (\$1M, 1) or r_1 = (\$5M, 0.1; \$1M, 0.89; 0, 0.01). The second pair includes the two prospects: s_2 = (\$1M, 0.11; 0, 0.89) or r_2 = (\$5M, 0.1; 0, 0.9). An individual whose preferences are compatible with EUT would either choose 's' or 'r' type of prospects in both choice problems, because according to EUT, common consequences added or subtracted to the two prospects should have no effect on the desirability of one prospect over the other because common consequences cancel out. A closer look would reveal that s_1 and r_1 includes a common consequence of \$1M with probability of 0.89, and that s_2 and r_2 are derived by subtracting this common consequence from s_1 and r_1, respectively. Therefore an individual who chooses r_1 (s_1) in the first problem should choose r_2 (s_2) in the second problem. However, Allais argued that most people might opt for s_1 in the first problem lured by the certainty of winning \$1M, and r_2 in the second problem since the odds of winning are very similar but the winning prizes are very different; \$1M and \$5M. Evidence that emerged after his study also supports his predictions; this anomaly is called as 'common consequence effect'.

Table 1. *Common Consequence Effect*

Experiment 1				Experiment 2			
s_1		r_1		s_2		r_2	
Outcome	Odds	Outcome	Odds	Outcome	Odds	Outcome	Odds
\$1 M	100%	\$1 M	89%	Nothing	89%	Nothing	90%
		Nothing	1%	\$1 M	11%		
		\$5 M	10%			\$5 M	10%

The second phenomenon discovered by Allais is the 'common ratio effect'. This problem has the same structure, there are two choice tasks and each task includes a pair of lotteries. The first pair includes: m_1 = (\$3000, 1) or n_1 = (\$4000, 0.8; \$0, 0.2) whereas the second pair includes m_2 = (\$3000, 0.25; \$0, 0.75) or n_2 = (\$4000, 0.2; \$0, 0.8). A fair amount of evidence and Allais's prediction suggests that many people would choose m_1 in the first task and n_2 in the second task, a pattern of choice inconsistent with EUT because the second pair is formed by multiplying the probabilities of the first pair's winning prizes by a common ratio of 0.25. In order to see it more formally, individuals who choose m_1 over n_1 imply that:

$$m_1 \succ n_1 \iff u(\$3000) > 0.8 \cdot u(\$4000) \tag{5}$$

$$1.25 > u(\$4000)/u(\$3000) \tag{6}$$

whereas in the second problem, if n_2 is chosen over m_2:

$$n_2 \succ m_2 \iff 0.2 \cdot u(\$4000) > 0.25 \cdot u(\$3000) \tag{7}$$

$$u(\$4000)/u(\$3000) > 1.25 \tag{8}$$

As it can be seen, (6) and (8) show a contradictory result. Allais's two famous examples challenged the independence axiom of EUT, the idea of expected utility being linear in probabilities, and finally contributed to the development of alternative models.

1.2.2 Preference Reversals

Preference reversal is another observed anomaly of the standard economic theory; it can be summarised as the dependence of the preference ordering on the method of elicitation such as choice and valuation. Conventional preference theory predicts that preferences should be independent of the method of eliciting them, thus the preferred lottery in the choice task should also be valued more than the other one. The phenomenon was first observed by psychologists (Lichtenstein and Slovic, 1971; Lindman, 1971), but it was later introduced to economics literature by Grether and Plott (1979a) who confirmed the existence of the phenomenon under well-designed, incentive-compatible experimental settings and defined it as a threat to the fundamental optimisation principles of economics.

In a typical setting, subjects are asked to make a choice between two lotteries and in another task they are asked to state their selling prices. The two binary outcome gambles in the preference reversals experiments have distinct features: one of them typically called the ‘P-bet’ offers a relatively better chance of winning a modest prize, whereas the other bet, the ‘$-bet’, offers a relatively small chance of winning a larger prize. Moreover, those two bets are constructed such that their expected values are the same or insignificantly different. The results show that a significant proportion of subjects choose the P-bet in the choice task but value the $-bet more. Moreover, the opposite inconsistency, in which the $-bet is chosen but the P-bet is valued more, is much less frequently observed. Further studies show that the phenomenon is not a special case for gambles but also observed for hiring practices and the provision of public versus private goods (Hsee, 1998, 1996).

The phenomenon is extended to marketing literature as well as typically defined as the inconsistency between competitive (joint evaluation of options) and monadic (separate judgement of options) elicitation designs. This is a major threat for pricing research, because if a subject states a higher value for Option A than for Option B in a monadic setting, prefers one option to another, it is assumed that the subject would choose Option A over Option B in a competitive setting. The experimental research shows that preference reversals also exist for consumer durable goods such as televisions, microwave ovens, toasters, and cordless phones (Nowlis and Simonson, 1997), thus confirming the concerns of the marketing literature as well.

For policy related issues such as for environmental goods, the phenomenon raises doubts about the reliability of the preferences elicited by choice and valuation-based surveys: if such a phenomenon exists, valuations elicited in contingent valuation surveys might favour the project that would not be chosen if the participants were asked to choose between the projects. Therefore it is vital to understand the exact nature of this phenomenon.

1.2.3 Valuation Gap

The disparity between Willingness-to-Accept (WTA) and Willingness-to-Pay (WTP) is one of the most prominent anomalies in standard economic theory. WTA and WTP should be similar if the goods in question have close substitutes and if the income effects are small (Hanemann, 1991) The gap between WTA and WTP was first documented by mathematical psychologists Coombs et al. (1967) and by Hammack and Brown (1974) in an early contingent valuation study. Knetsch and Sinden (1984) brought the issue into the laboratory using real monetary incentives and found a significant difference between WTA and WTP. Knetsch and Sinden (1984) demonstrated the disparity in an exchange experiment where the participants were endowed with either a lottery ticket or with $2.00. Then, each subject was offered an opportunity to trade the lottery ticket for the money, or vice versa. Results show that very few subjects chose to switch. Those who were given lottery tickets seemed to like them better than those who were given money. Since then, the disparity has been documented in an array of studies, contingent valuation surveys, and field and laboratory experiments for a wide range of goods: mugs, pens, movie tickets, hunting permits, nuclear waste repositories, foul-tasting liquids, and pathogen-contaminated sandwiches (Horowitz and McConnell, 2002). In a typical experimental setting, subjects are divided randomly into two groups as buyers and seller; where sellers are endowed with the good but the buyers not. Then the subjective valuations of the buyers and sellers are elicited under an incentive compatible mechanism such as a Becker-DeGroot-Marchak mechanism and a Second Price Auction. Under the typical incentive compatible setting, it is optimal to state the maximum buying price (WTP) for buyers and minimum selling price (WTA) for sellers.

The disparity between WTA and WTP has implications for the Coase Theorem and EUT. Most policies produce both winners and losers; thus, studies that assess policies by assuming reference independence are on shaky ground because the presence of a WTA-WTP disparity indicates that the assumption is false. The disparity also raises fundamental questions about, e.g., the stated preference methods that are used in environmental policy analysis, such as contingent valuation and cost-benefit analysis, because the latter requires welfare measurement (and thus information about WTA and/or WTP). Together with other anomalies (e.g., Preference Reversals and the Allais Paradox), the disparity raises further questions with regard to the power of standard preference models to describe the economic behaviour of ordinary people (Braga and Starmer, 2005). The dominant explanation in the literature seems to be the endowment effect coined by Thaler (1980), i.e., goods that one owns are valued more highly than identical goods not held in the endowment. Thus the lower WTP values are interpreted as the buyers' potential gain from acquisition, and are apparently smaller than the sellers' or owners' potential loss from sale. The endowment effect is commonly interpreted as the result of the 'loss aversion' notion of prospect theory (Kahneman and Tversky, 1979), which states that losses are weighted substantially more than gains at outcomes above the reference point.

1.3 Alternative Theories

This section presents the major alternative theories for EUT. The motivation to develop such alternative models is to explain the anomalies reviewed in the previous section.

1.3.1 Prospect Theory

In EUT, the value of the prospect equals the weighted sum of the utilities of the outcomes by associated probabilities in a linear manner. However, in the Allais Paradox setting this principle is commonly violated: individuals overweight the outcomes that are considered certain relative to the merely probable outcomes. In their seminal paper, Kahneman and Tversky (1979) present a new theory for decision under risk that provides explanations for the reported anomalies in the literature. They demonstrate the inconsistent behavioural patterns with EUT in a series of experiments, especially by focussing on the independence axiom, and introduce a new theory, which incorporates nonlinear probability weighting and asymmetric treatment of gains and losses.

Prospect Theory models decision making under risk as a two-step process where the initial phase includes the editing and the last step is the evaluation of the gamble. Editing includes the reformulation of the prospects by employing heuristics to simplify the decision problem. Therefore, this phase describes the underlying process of the individuals' perception, the way that individuals filter and reform the given information. One of the major operations of the editing phase is 'coding', which consists of the perception of the outcomes as gains and losses, rather than as final states of wealth, a major departure from the EUT. Gains and losses are defined relative to some reference point that usually corresponds to the current wealth level. However, Kahneman and Tversky also do not rule out the possibility that the perception of the reference point might depend on the presentation of the prospects and the expectation of the individual. The second operation of the editing phase is the 'combination', which includes the simplification of the prospects by combining the probabilities of the identical outcomes. As an example, consider the prospect X, which consists of two equal outcomes, each with a probability of 0.25. The combined version would be the prospect which gives X a probability of 0.5. The third operation is the 'segregation', which includes the perception of the riskless outcome separately than the risky outcome. For example, the prospect (200, 0.8; 100, 0.2) can be perceived as a sure gain of 100 and a risky gain of

100 with a probability of 0.8. The final operation, ‘cancellation’, can be explained by using a game which has two stages, in the first stage there is a probability of 0.75 to end the game without winning anything, and a probability of 0.25 to move into the second stage (Problem 10 in their study). If the game reaches the second stage, a decision maker has to choose one of the following prospects: (400, 0.80) or (3000). However, the choice has to be made before the game starts. If we combine the first and the second stage by multiplying the probability of continuation in the first stage and the probability of winning in the second stage, the prospects can be represented as: (0.20, 4000) and (0.25, 3000), respectively. In this binary choice problem, 78% of the 141 subjects chose the second option. However, when asked to choose between (0.20, 4000) and (0.25, 3000) when the problem is formulated as a single stage (in Problem 4), 65% of the subjects opt for the first prospect. Clearly the two problems include identical prospects, but differ in their presentation. In Problem 4, subjects are presented the combined prospects without complicating the problem by separating it into two stages. However while answering Problem 10, respondents ignored or cancelled out the first stage, which is common to both prospects, and evaluated the prospects by merely looking at the second stage. They also suggested additional operations for the editing phase such as the ‘simplification’ and the ‘detection of dominance’. The first refers to the rounding of the probabilities and outcomes whereas the second looks for dominance between prospects and, when detected, decides without making further evaluation. Kahneman and Tversky also suggest without making further assertions that the reason behind some of the observed anomalies and intransitivities can be the combined application of these operations in the editing phase.

After the editing phase, the individual subjectively evaluates the simplified decision problem in the evaluation phase, as all decision-making-under-risk problems have two elements: probability and outcome. However, Prospect Theory undertakes the judgement of these two elements differently than does EUT: objective probabilities are not taken into the calculation linearly but transformed by the individual in a nonlinear way. This is done by the weighting function $w(p)$. The weighting function $w(p)$ associates each objective probability p , with a decision weight $w(p)$ that reflects the subjective evaluation, the individual’s perception, of the objective probabilities. Decision weights do not obey the probability axioms and they should not be interpreted as measures of beliefs. Therefore, they are not the likelihood of the events, but instead imply the effect of the probabilities on the desirability of the prospect. They are depicted as a function of objective probabilities because in the simplest form, prospects are defined by their outcomes and their associated probabilities. Therefore, they assume that individuals consider the probabilities to be relevant information regarding the attractiveness of the prospect. However, in other contexts decision weights could be influenced by different factors such as ambiguity.

In Figure 1, the bold curve depicts Kahneman and Tversky's (1979) typical probability weighting function. According to Prospect Theory, low probabilities are overweighted whereas high probabilities are underweighted.

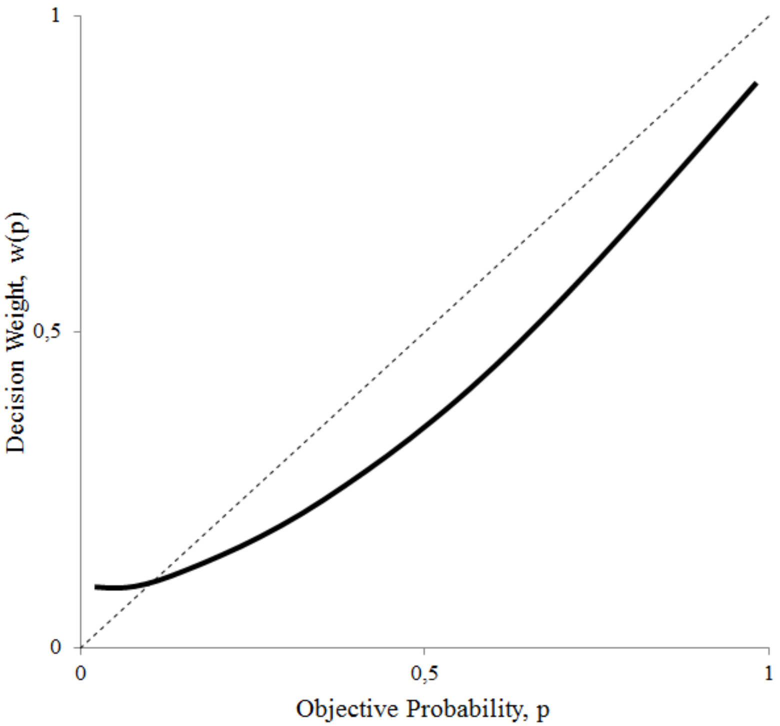


Figure 1. Nonlinear probability weighting of Prospect Theory

The outcomes are incorporated in the value function $v(x)$ that replaces the von Neuman-Morgenstern utility function that measures the deviations from the reference points. Therefore $v(x)$ measures the deviations from the reference point. The rationale for this departure is explained with an example from Adaptation-Level Theory (Helson, 1964): When individuals assess the attributes of a particular object such as temperature, brightness, or loudness, they perceive them as dependent on a reference point. For example, the temperature of an object might be judged as hot or cold depending on the temperature the individual is exposed to prior to the assessment. Kahneman and Tversky assert that the same propensity is valid for attributes such as wealth, prestige, and health. The same outcome, for example, might mean being worse off for some individuals and better off for others. They suggest that the value function under prospect theory is concave for gains and convex for losses, which is a major departure from the standard view (Figure 2).

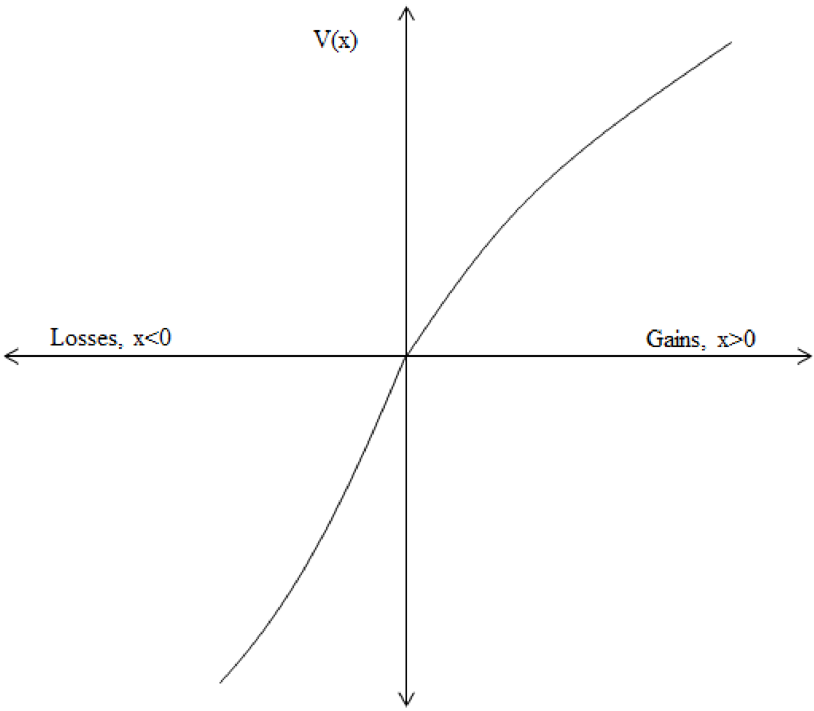


Figure 2. Hypothetical value function of Prospect Theory

The asymmetric shape of the value function is a result of loss aversion, which implies that losses loom larger than gains: most people would find unattractive a prospect that offers a gain and loss of an equal amount with equal probabilities. This can be interpreted as the value function being steeper for losses.

In Prospect Theory, the overall value of the prospect, denoted by V, is calculated by multiplying these two measures. However the model can only be applied to simple prospects, denoted by $(x, p; y, q)$, where x and y are the outcomes and p and q are the associated probabilities. Simple prospects are defined as the prospects with at most two non-zero outcomes. Therefore there can be an additional outcome that gives zero with a probability of *1-p-q*. Moreover, a prospect is strictly positive if both x and y are positive and probabilities p and q sum to one, whereas a prospect is strictly negative when both outcomes are negative and probabilities p and q sum to one. A regular prospect cannot be classified as strictly negative or positive. Formally, a regular gamble's value $V(x,p;y,q)$ is calculated as:

$$V(x,p;y,q)=w(p)\cdot v(x)+w(q)\cdot v(y) \tag{9}$$

Further, assume that $v(0)=0$, $w(0)=0$, and $w(1)=1$.

The model works with a slight modification for the strictly positive or strictly negative prospects if the segregation operation is applied in the editing phase. Remember that individuals segregate the riskless loss or gain from the risky component with this specific operation. For example, $V(400,0.25;100,0.75)$ can be transformed into a sure gain of 100 and a risky gain of 300 with a probability of 0.75. The formula for these kinds of prospects would be:

$$V(400,0.25;100,0.75)=v(100)+w(0.25)\cdot[v(400)-v(100)] \tag{10}$$

Notice that the value of the risky component is calculated as the difference between the subjective values of the outcomes, and not the subjective value of the difference in outcomes, 300. Also the decision weight is applied to the difference in outcomes for the risky component; it reduces to the equation (9) if and only if $w(p)+w(1-p)=1$, however this is not usually satisfied due to the structure of the decision weight component of the model. The issue is solved by Cumulative Prospect Theory.

Another crucial departure from the EUT is the treatment of risk attitudes. Notice that if the probabilities enter the expected utility calculation linearly then the risk attitude is solely determined by the utility/value function (in general the function that undertakes the evaluation phase in the decision problem) as in the EUT. Thus the curvature of the utility function that implies the diminishing marginal utility of money determines the risk attitude of the individual. On the other hand, Prospect Theory treats the probability component of the decision problem in a nonlinear way as well, therefore under Prospect Theory these two functions jointly determine the individual's risk attitude.

Beside the significant advances and departures from the standard theory, their original model has some limitations: i) it can only be applied to prospects with at most two non-zero outcomes; ii) it predicts that people will opt for the stochastically dominated gambles in some circumstances. Those two limitations have been tackled in the latter version of the theory, Cumulative Prospect Theory (Tversky and Kahneman, 1992), discussed in Section 1.3.3.

1.3.2 Rank-Dependent Expected Utility Theory

As mentioned in the preceding section, the problem with Prospect Theory was that, since the decision weights do not obey the probability axioms, i.e., they do not sum to one, the theory allows for the violation of first order stochastic dominance.

The solution for this problem is offered by Anticipated Utility Theory (Quiggin, 1982), which soon became known as the Rank-Dependent Utility Theory. This theory makes the decision weights dependent on the rank of the outcomes and calculates them using the cumulative distribution, instead of individual probabilities. This property ensures that the decision weights sum to one, therefore solving the problems of violation of dominance.

Before proceeding with the general formula of the Rank-Dependent Utility Theory, it is useful to explain the probability transformation technique. According to the theory, individuals rank the outcomes of the prospect from worst to best $(x_1 \leq ... \leq x_n)$, and the corresponding probabilities are $(p_1,..., p_n)$. As in Prospect Theory, it is assumed that there exists a probability weighting function, $w(p)$, a strictly increasing mapping from the interval [0,1] onto itself. The endpoint probabilities, 0 and 1, are transformed as they are, therefore no distortion occurs for these two special objective probability measures. The decision weights of the probabilities associated with the outcomes of a prospect are calculated in a cumulative way but the probability of the best outcome is transformed directly. For the rest, the decision weight of a particular outcome is calculated as the difference between the cumulative transformation of the probabilities associated with getting equal or better outcomes and the transformation of the probabilities associated with strictly better outcomes. At this point it is necessary to introduce new notation: as in the previous sections small letter $w(p)$ implies the individual objective probability transformation, cumulative transformation is depicted as the capital $W(p)$, and cumulative transformations are the ones used in calculating the subjective value of the prospects:

$$V\left(x_1, p_1;...;x_n, p_n\right) = \sum\nolimits_{i=n}^{n} W\left(p_i\right) \cdot u\left(x_i\right) \tag{11}$$

Where the cumulative transformation, $W(p)$ is calculated as:

$$W(p_i) = w(p_n), \text{ if } i = n \tag{12}$$

$$W(p_i) = w\left(\sum_{k=i}^{n} p_k\right) - w\left(\sum_{k=i+1}^{n} p_k\right), \text{ if } 1 \le i < n \tag{13}$$

Subscript n corresponds to the best outcome, thus (12) indicates that the transformation occurs directly by inputting the probability of the best outcome into the weighting function, whereas (13) shows the transformation method for the other outcomes. As an example, consider the second best outcome, denoted with subscript $n-1$; the cumulative decision weight associated with the second best outcome $W(p_{n-1})$ then will be:

$$w\left(\sum_{k=n-1}^{n} p_k\right) - w(p_n) \tag{14}$$

The first element of (14) corresponds to the cumulative probability transformation of the outcomes weakly better than the second best outcome, therefore we sum the probabilities of the outcomes from the second best outcome to the best outcome, and then transform using the weighting function, $w(p_n + p_{n-1})$. Since there is only one better outcome than the second best outcome, we subtract $w(p_n)$ to find the cumulative probability weighting of the second best outcome, $W(p_{n-1})$.

This process of transformation avoids the valuation of monotonicity and also adds a less appealing new feature to the model: the subjective weight attached to the probability of a particular outcome depends on the ranking of the outcome within a prospect, therefore it depends on how good or how bad the outcome is within a prospect. It might be problematic because a slight change in the magnitude of an outcome can change the Rank-Dependent Utility of a particular prospect significantly if it changes the ranking of the outcome. Moreover, a significant change in the magnitude of an outcome will not change the value of the prospect, if the rank of that outcome remains unchanged.

Rank-Dependent Utility Theory can be seen as the generalised version of the classical EUT, because it does not include any nonconventional notions such as reference dependency and loss aversion but it does incorporate the distortion of the objective probabilities. Since the utility part of the expectation formula is identical to the EUT, the crucial element of Rank-Dependent Utility Theory is probability transformation, i.e., that the predictions of the model depend highly on the shape of the probability weighting function. A concave weighting function will result in overweighting the probabilities of the high ranked outcomes (good outcomes), whereas a convex weighting function leads to underweighting of those outcomes. This curvature is intuitively explained by the pessimism and optimism levels of the individuals (Quiggin, 1982; Yaari, 1987; Diecidue and Wakker, 2001).

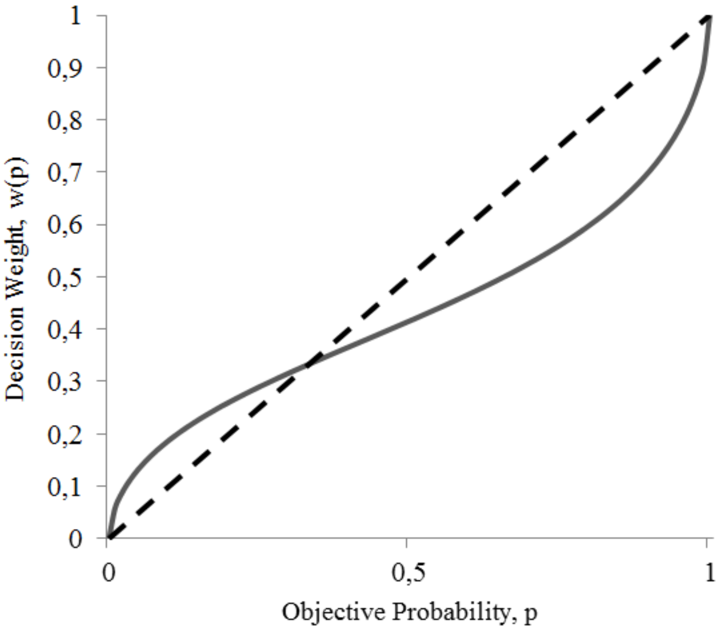


Figure 3. Inverse S-shaped probability weighting function

Figure 3 shows one of the suggested forms for the weighting function, which is inversely S-shaped and has a switching point at p*. The function is concave below the switching point and convex above it (Prelec, 1998; Preston and Baratta, 1948). Therefore probabilities below p* are overweighted and above p* are underweighted. Quiggin (1982) proposed 0.5 to be the switching point, in order to explain the anomalies of EUT such as the Common Consequence and Common Ratio Effect. However, empirical studies consistently suggest the switching point to be around 0.4 (Wu and Gonzalez, 1996).

Since the weighting function is assumed to be nonlinear, it determines the risk attitude of the individual, together with the utility function. Therefore, unlike in EUT, we cannot make direct inferences about the risk attitude of the individual by looking solely at the curvature of the utility function. For example, a pessimistic individual with a concave utility function will exhibit a universally risk-averse attitude. However, an individual who has convex utility function can be risk averse as well, if he or she is sufficiently pessimistic (Chew, Karni, and Safra, 1987; Chateauneuf and Cohen, 1994).

Although the incorporation of probability weighting enables Rank-Dependent Utility Theory to explain Allais anomalies, the theory fails to explain other anomalies such as valuation gap and preference reversal. The next section focuses on Cumulative Prospect Theory that is the synthesis of Original Prospect Theory and the Rank-Dependent Utility Theory.

1.3.3 Cumulative Prospect Theory

Kahneman and Tversky (1992) presents the cumulative version of Prospect Theory; in the new version they solved the problems with monotonicity by featuring the cumulative transformation technique of Quiggin's Rank-Dependent Utility Theory separately for gains and losses. With the help of this advance, the Original Prospect Theory can be applied to prospects with any number of outcomes. Moreover they discarded the editing phase in the new version of the theory, providing a mathematically tractable model, as it is very difficult to determine which operations are employed by individuals in the editing phase. Cumulative Prospect Theory retains the important notions of Original Prospect Theory such as reference dependency and loss aversion. The general form of the Cumulative Prospect Theory is as follows:

$$V\left(x_1, p_1;...;x_m, p_m;...;x_n, p_n\right) = \sum_{i=1}^{m} W^-\left(p_i\right)\cdot v\left(x_i\right) + \sum_{i=m+1}^{n} W^+\left(p_i\right)\cdot v\left(x_i\right) \qquad (15)$$

where the losses are indexed from 1 to m thus the gains are from $m+1$ to n and, as previously noted, we assume $(x_1 \leq ... \leq x_n)$. $W^-\left(p_i\right)$ and $W^+\left(p_i\right)$ are the cumulative decision weights for losses and gains, respectively, and are defined by:

$$W^-\left(p_i\right) = w^-\left(p_1\right), \text{if } i = 1 \qquad (16)$$

$$W^-\left(p_i\right) = w^-\left(\sum_{k=1}^{n} p_k\right) - w^-\left(\sum_{k=i+1}^{n} p_k\right), \text{ if } 1 \leq i < m \qquad (17)$$

$$W^+\left(p_i\right) = w^+\left(p_n\right), \text{ if } i = n \qquad (18)$$

$$W^+\left(p_i\right) = w^+\left(\sum_{k=i}^{n} p_k\right) - w^+\left(\sum_{k=i+1}^{n} p_k\right), \text{ if m+1} \leq i < n \qquad (19)$$

Tversky and Kahneman (1992) suggested the inverse S-shaped probability weighting function that implies that individuals exhibit diminishing sensitivity for probability changes near 0.5 and they are relatively more sensitive to the changes near the endpoints, 0 and 1. Value function, $v\left(x_i\right)$, has the same properties as in the original version of the theory, convex for the losses and concave for the gains. Loss aversion is maintained by the following property:

$v'(x) < v'(-x)$ where $x \geq 0$. This feature of the theory is also known as the 'diminishing sensitivity' because it implies that, while comparing a 10 gain (or loss) with a 20 gain (or loss) has a significant utility impact, comparing a 100 gain (or loss) with a 110 gain (or loss) has a smaller impact. The concavity over gains captures the finding that people tend to be risk averse over moderate probability gains and risk seeking for losses.

Although loss aversion has a central role in explaining anomalies such as preference reversals and valuation gap, it is a seriously limiting property, because it limits the number of functional forms that can be used under Cumulative Prospect Theory. For instance, many of the functional forms used in expected utility such as negative exponential do not meet these requirements.

Cumulative Prospect Theory departs from Rank-Dependent Utility Theory by the incorporation of loss aversion and reference dependency and also by the specification of different probability weighting functions for gains and losses. In that sense Rank-Dependent Utility Theory is a more flexible model in terms of the required functional forms, since it does not require diminishing sensitivity or loss aversion. However, the trade-off is that Rank-Dependent Utility Theory cannot explain procedural anomalies such as preference reversals and valuation gap.

There are also variants of Cumulative Prospect Theory developed recently (Baucells and Heukamp, 2006; Davies et al., 2004; Schmidt and Zank, 2008; Trepel et al., 2005; Wu et al., 2005), but the notable one is the Third Generation Prospect Theory (Schmidt et al., 2008), which allows reference points to be uncertain while decision weights are specified in a rank-dependent way. In the Original and Cumulative Prospect theories, the reference points are assumed to be certainties. The criticism raised by Schmidt et al. is that if reference points are restricted to certainties then these theories cannot be applied to problems in which a decision maker is endowed with a lottery and has the opportunity to sell or exchange it. They accomplish that by defining the preferences over acts following a Savage (1972) style framework and borrowing the state-contingent, reference-dependence concept of Reference-Dependent Subjective Expected Utility Theory (Sugden, 2003): consider an individual endowed with a reference lottery and asked to evaluate another lottery. The gains and losses are calculated as the difference between the outcomes of the evaluated lottery and the reference lottery for each state of the world. Therefore the reference for each state is the outcome of the reference lottery in that state. This implies that gains or losses are defined as the relative state-wise attractiveness of the lottery that is being evaluated. The remaining operations such as rank-dependent nonlinear probability weighting associated with gains and losses are identical with Cumulative Prospect Theory.

1.3.4 Regret Theory

Regret Theory (Bell, 1982; Loomes and Sugden, 1982) provides explanations for anomalies by accommodating neither reference-dependence nor nonlinear probability weighting, which makes it the most distinct compared to those theories explained above. Central to the theory is evaluating a prospect by comparing its outcomes with the outcomes of an alternative prospect in a state-wise manner. Individuals would feel regret for the states in which the outcome of the alternative prospect is higher, whereas the individual would feel joy for the states in which the alternative gives lower payoff. This intuition later contributed the idea for Gul's Disappointment Theory (Gul, 1991).

To see how the theory models the decision under risk, consider two prospects: $X\left(x_1, p_1;...;x_n, p_n\right)$ and $Y\left(y_1, q_1;...;y_n, q_n\right)$, where x_i and y_i denote the outcomes of state i that is one of the possible states of the world S ; and p_i and q_i are the corresponding probabilities. Then, an individual chooses X instead of Y and i^{th} state of the world occurs. Thus, the realised consequence is x_i, instead of y_i, had he chosen differently. In Regret Theory the satisfaction for this choice is denoted as $M(x_i, y_i)$ which is an increasing function of x_i, and decreasing function of y_i. Loomes and Sugden (1982) suggest the following form for the modified utility function: $M(x_i, y_i) = c^{x,i} + R(c^{x,i} - c^{y,i})$ where $c^{x,i}$ is analogous to the standard Bernoulli conception of utility function, i.e., the psychological experience of pleasure related to the prospect X if the ith state of the world occurs. $R(.)$ is called the 'regret-rejoice function', $R(0) = 0$ and non-decreasing. This function exhibits disutility of regret if $x_i < y_i$ or a positive utility of rejoice if $x_i > y_i$.

Notice that in the theories discussed so far, the nature of the available options does not affect the level of satisfaction attained from the choice being made. If the alternative option has a higher payoff for the realised state, regret decreases the utility or the psychological experience of pleasure related to that particular choice. The opposite of regret is 'rejoice' in the theory's terminology. Therefore the psychological experience of pleasure related to a particular prospect incorporates not only 'what it is' but also 'what might have been'. They also assume that if $x_i = y_i$, then the individual feels neither regret nor rejoice for choosing x_i if state i occurs. Therefore, $M(x_i, y_i)$ equals only the utility of getting x_i, $c^{x,i}$. Individuals maximise Expected Modified Utility denoted as $E_X^Y = \sum_{i=1}^{n} p_i \cdot M(x_i, y_i)$, which is the evaluation of prospect X when the alternative option is Y. As in EUT, the Expected Modified Utility is the weighted sum of the modified utilities, $M(x_i, y_i)$, where the weights are the objective probabilities, p_i , in a linear manner. Thus, Regret Theory accommodates EUT as a special case for situations in which the individual does not feel any regret or rejoice. We can now write the preference relation between X and Y, for example, if the individual weakly prefers X over Y, $X \succsim Y$, if and only if $E_X^Y \geq E_Y^X$:

$$\sum_{i=1}^{n} p_i \cdot [c^{i,X} - c^{i,Y} + R(c^{i,X} - c^{i,Y}) - R(c^{i,Y} - c^{i,X})] \geq 0 \tag{20}$$

The most apparent limitation of the theory is that unlike the theories mentioned before, it cannot be a conventional theory that assigns values independently to individual prospects because it has to allow comparisons between available choice options (Starmer, 2000). Another limitation is that the original form can only be applied if there are only two prospects, however Sugden (1993) and Quiggin (1994) suggest ways of generalising it to multiple choice problems. Loomes and Sugden (1982) suggest reducing the alternative options into one single option by calculating the weighted average of all the remaining options, and the weights are action weights according to their appellation. Therefore if there are more than two options available, an individual calculates the Expected Modified Utility of Option X with respect to Y; in this case Y is the weighted average of the alternative options. However, their approach requires developing a sound and solid theory for the action weights as well.

Less attention is given to the question of stochastic dominance. Loomes and Sugden (1982) note that regret-theoretic preferences do not preserve first-order stochastic dominance in the sense of Hadar and Russell (1969), but that state-wise stochastic dominance is preserved. Quiggin (1994) shows that violations of stochastic dominance are pervasive in regret theory, in the sense that for any prospect with more than two distinct outcomes, there exists a preferred prospect which is first-order stochastically dominated by the initial one.

1.4 Conclusion

Problems of decision under risk are simply represented as lotteries with two elements: outcomes and their associated probabilities. As mentioned in Section 1.1, the story of modelling the issue starts with Pascal and Fermat's notion of expected value, which is basically the weighted sum of the outcomes of a lottery with the weights being the probabilities associated with each outcome. This notion is challenged by Bernoulli's St. Petersburg paradox; the solution is offered by the first crucial departure, that is, EUT treats the outcome in a non-decreasing but concave function called the utility function. The important property is the nonlinear and concave nature of this function, known as the diminishing marginal utility. As outlined in Section 1.3, a series of anomalies are reported in the literature that challenged EUT. Those anomalies led researchers to develop alternative models. The alternative theories are different in terms of treating the probabilities or outcomes or both. For example, Prospect Theory differs both in the probability and outcome parts of the decision problem. For the outcome, it introduces the concept of loss aversion, which asserts that losses loom larger than gains. Moreover, unlike EUT, Prospect Theory assumes that individuals do not use objective numerical probabilities directly, but instead use the transformed versions of them. Another example is Rank-Dependent Utility Theory, which is an extension of EUT because the only difference is the nonlinear cumulative probability transformation notion. The monetary outcomes are treated in the same manner as EUT.

Overall, the progress in the literature seems to have been led by the anomalies and challenges to whatever the existing theory is. The story of modelling the decision under risk started with the linear incorporation of the elements into calculations, i.e., the expected value concept, but this concept is challenged by St. Petersburg paradox. Next, EUT is offered as a solution to the paradox, which treats the outcome element of the decision problem nonlinearly, but is soon challenged by anomalies such as the valuation gap, Allais Paradox, and preference reversal. These anomalies have led to new theories that generally assume that individuals do not perceive the objective numerical probabilities linearly, but instead perceive them in a nonlinear manner, such as underweighting the high probabilities and overweighting the low probabilities. However, those alternative theories are challenged by the recently emerging evidence of preference imprecision.

2 Alternative to the alternatives: Preference Imprecision

This chapter reviews the emerging hypothesis of preference imprecision, which challenges both the EUT and the alternative theories. Although the alternative theories reviewed in Chapter 1 can explain the anomalies of EUT, they ignore preference imprecision, implicitly assuming that individuals can articulate their preferences precisely. Section 2.1 presents an introduction for the new hypothesis by explaining how it contradicts the standard notions of economic preferences. Section 2.2 presents the attempts in the literature to model the imprecision as a stochastic component, added to the existing theories. Section 2.3 and 2.4 reviews the experimental studies in the literature, which elicit the imprecision intervals in valuations tasks. These experimental studies investigate preference imprecision as an alternative explanation for observed anomalies such as preference reversals, valuation gap, and the Allais Paradox.

2.1 Introduction

The central idea of decision theories is to use the attributes of risky prospects, such as outcomes and their associated probabilities, and calculate a single number that can reflect the subjective attractiveness of these prospects. A natural way to think about the theories of decision under risk is that these attributes are the inputs and the summary statistics—such as expected utility and expected value—produced by these theories are the outputs. Alternative theories such Prospect Theory, Rank-Dependent Utility Theory, and Cumulative Prospect Theory reviewed in Section 1.3 have also their own summary statistics, assumed to be the criteria that individuals take into account while making decisions. Those statistics represent a measure of expected satisfaction or pleasure associated with the risky prospect, and individuals are assumed to prefer more to less, such that they make decisions to maximise their total satisfaction.

Since Pascal and Fermat, researchers have been suggesting different ways of incorporating the inputs and calculating the summary statistic that is assumed to be the criterion for decision making under risk. In general, based on the theories reviewed in Section 1.3, the conventional approach can be summarised as individuals (i) take the elements of the prospects such as outcomes and probabilities as inputs; (ii) calculate a summary statistic of the prospect; and (iii) use this summary statistic to choose among options and/or assign monetary valuations to them. Although these theories provide different ways to use the two inputs and calculate a summary statistic, varying in their treatments of the two inputs, they share one important and implicit assumption that individuals can form their subjective valuations precisely.

Considering the limitations of human perception and cognitive abilities, one might see this as a strong assumption. However, when modelling human decision making, economists have conventionally assumed that individuals have well-behaved preferences that do not allow for preference imprecision. For example, Savage (1972) assumes that for any two acts, f and g, either $f \precsim g$, or $f \succsim g$, or $f \precsim g$ and $f \succsim g$, which implies $f \sim g$. This assumption states that the individual either prefers f to g or g to f or is indifferent between them, ruling out the possibility that the individual prefers f to g and g to f simultaneously. It also ignores the possibility of observing neither $f \precsim g$, nor $f \succsim g$; the individual is assumed to be have defined preferences over all sets of options and is not allowed to provide inconsistent rankings. This assumption ensures that there is no situation where an individual feels indecisive and vacillates; therefore it does not allow the incommensurability of the options. In reality, individuals might end up in a situation where they cannot determine their preferences confidently.

The standard approach also assumes that every risky prospect has a certainty equivalent, a precise amount of money that is equally desirable. This might be true for an individual who has sufficient familiarity and expertise in risky situations, but ordinarily it is more likely that the certainty equivalent would be a range of rounded numbers rather than a precise estimation.

The related notion of the conventional approach with this precision is 'betweenness': consider the three acts f, g, h; g is ranked as between f and h, i.e. either $f \succsim g \succsim h$, or $f \precsim g \precsim h$. Savage (1972) Theorem 4 states that there exists only one $\alpha \in (0,1)$ such that $\alpha f + (1-\alpha)h \sim g$. To see how this axiom is connected to the concept of precision of the certainty equivalent, consider a risky prospect $X : (f, \alpha; h, 1-\alpha)$, which has two possible outcomes f and h and associated probabilities α and $1-\alpha$, respectively. To avoid confusion, note that we can see f and h as two degenerate lotteries which give the amounts, f and h, with certainty. Therefore prospect $X : (f, \alpha; h, 1-\alpha)$, is a compound lottery that is a mixture of the two degenerate lotteries. Finally we can view g analogous to the certainty equivalent of lottery X, since g is the degenerate lottery which promises sure amount of money that is equally desirable as X. As a natural conclusion, there exists another degenerate lottery g', which gives a more certain amount of money than g such that $g' > g$, should be strictly preferred to $X:(f,\alpha;h,1-\alpha)$. This way of thinking leads to two important conjectures about the certainty equivalent of a simple binary lottery: (i) The certainty equivalent of a binary outcome lottery should lie between the two outcomes, and (ii) two different amounts cannot be the certainty equivalent, simultaneously. The second point is important for the concept of preference imprecision, because if an individual has imprecise preferences, i.e., cannot articulate the subjective evaluations as single amounts, the individual will end up with a range of certainty equivalents and be unable to state a precise estimate confidently. As the conventional assumptions and axioms seem to be problematic for preference imprecision, I offer a new understanding for the imprecision concept in Section 4.3.3.

2.2 Modelling Imprecision as Stochastic Preferences

Two prominent findings of experimental literature lead economists to focus on stochastic preferences. The first is that when subjects face the same pairwise choice more than once, a considerable portion of the subjects seem to be behaving inconsistently on different occasions in a given experiment (Ballinger and Wilcox, 1997; Camerer, 1989; Hey and Orme, 1994; Starmer and Sugden, 1989). Second, the existing theories of decision under risk seem to be only partially successful in explaining the behaviour observed in experiments (Loomes and Sugden, 1998).

The idea of imprecision dates back to 19th century, investigated in the works of Fechner and Weber who are considered to be the founders of the psychophysics and experimental psychology (Gescheider, 2013). They investigate the relation between stimulus and sensation, particularly focusing on judgments about stimuli such as light, sound, weight, and distance. Those early works suggest that human judgement of stimuli is subject to errors, therefore expecting a perfect evaluation from individuals is not realistic (Fechner, 1966). Moreover, upon comparing, e.g., the weight of two objects, the probability of making a mistake is higher when the weights are very close, such as 1 kg and 1.05 kg.

Psychophysics studies focus on the physical stimuli, however in the realm of economics, individuals deal with evaluations of risky prospects, which are the main focus of this study. Therefore, risky prospects or lottery tickets in economics are the counterparts of the physical stimuli concept in psychophysics. Finally, the ‘preference imprecision hypothesis’ is the idea which claims that as individuals’ judgements about objects are subject to mistakes, the choices among options and valuations of the goods are also liable to imprecision and noise.

The history of imprecision in economics dates back to 1950s in the form of probabilistic choice and random preferences models (Becker et al., 1963; Georgescu-Roegen, 1958; Luce, 1959; Luce and Suppes, 1965; Mosteller and Nogee, 2006). As reviewed in Chapter 1, researchers tried to explain the observed anomalies by developing alternative models, however they do not consider the noise and imprecision accounting for these anomalies. Beginning in 1990s, the idea of imprecision began to receive attention by researchers in the form of modelling it as the stochastic component of a deterministic theory such as EUT and/or alternative theories (Harless and Camerer, 1994; Hey and Orme, 1994; Loomes and Sugden, 1998, 1995; Sopher and Gigliotti, 1993). The common approach employed by these studies is to incorporate the imprecision as the stochastic component—the random and/or error part—of a core deterministic theory, but these studies do differ in the interpretation of the source of randomness. The logic that is employed is to reject a theory if the observed behaviour systematically departs from the core theory, if the anomalies cannot be explained by random errors or deviations from the core theory. However, it seems that no single combination of deterministic core and stochastic specification can explain the significant portion of the anomalies (Loomes, 2005). There are three major approaches so far in the literature for modelling the imprecision as stochastic preferences: the random error approach, the trembling hand approach, and the random preference approach.

2.2.1 Random Error Approach

At this point it is useful to review the prominent approaches to modelling the imprecision and/or noise in the literature; I start with Hey and Orme (1994), inspired by Fechner’s (1966) ideas of individuals’ imprecise judgements of the stimuli modelled as white noise, normally distributed with a mean of zero. The reason for such an error might be the subjects’ misunderstanding the nature of the experiment or operational mistakes during the experiment, e.g., pressing the wrong key by accident. Moreover, subjects’ inattentiveness, such as being in a hurry to complete the experiment and/or having another motivation rather than maximising their welfare from participating in the experiment, might be the reasons behind those errors.

Hey and Orme’s (1994) idea is that the preferences can be represented by a core theory plus a random error term:

$$V(f) - V(g) + \varepsilon > 0 \Leftrightarrow f \succ g \qquad (21)$$

where $V(.)$ is the preference functional of a deterministic core theory, f and g are the two options and ε is the stochastic component with a constant variance and mean of zero. An individual prefers f over g if the difference between the utility of the two options plus some random error is positive. When $\varepsilon = 0$, the choice solely depends on the core deterministic theory part of the model. Notice that if ε is sufficiently high in the opposite direction of the deterministic part, although $V(f) - V(g) > 0$, the model predicts that the individual prefers g over f. Moreover, the greater the difference in the deterministic part, the less likely it is that the preferences predicted by the core theory will be reversed by the error term.

Hey and Orme's (1994) data was composed of 100 pairwise choice questions answered by 80 subjects; details about the lottery pairs are listed in Table 2.

They estimated eleven different preference functionals including: risk neutrality (expected value), Subjective Expected Utility Theory, Disappointment Aversion Theory, Prospective Reference Theory, Quadratic Utility Theory, Regret Theory with dependence and independence, rank dependence with the power weighting function and 'Quiggin' weighting function, and Yaari's Dual Model which is a special case of the Rank Dependent Utility Theory with the probability function left as general and the utility function assumed to be linear.

Their results provide insights about the winner and loser theories. For example risk neutrality is rejected in favour of EUT; on the other hand, at the 1% level, EUT is rejected in favour of the remaining nine preference functionals. Overall, for approximately 39% of the subjects, EUT does not perform worse than any of the alternative models. For the remaining portion of the subjects, Rank Dependent Utility Theory functionals and Quadratic Utility Theory seem to be the strongest models. Next, they find that Regret Theory with independence performs better than the one with dependence, which suggests that the subjects perceived the two lotteries as being statistically independent. Among the remaining nine models Yaari's Dual Model and Disappointment Aversion Theory are the poorest. However, they emphasise that the sample consisting of the responses of 80 subjects should not be taken as representative. On the other hand, their analysis strongly supports the importance of the errors and suggests that deterministic core models do not describe the significant portion of the observed behaviour.

Table 2. *Pairwise choice questions*

p_1	p_2	p_3	q_1	q_2	q_3
0.6250	0.0000	0.3750	0.3750	0.6250	0.0000
0.3750	0.6250	0.0000	0.5000	0.2500	0.2500
0.0000	1.0000	0.0000	0.1250	0.5000	0.3750
0.1250	0.7500	0.1250	0.2500	0.5000	0.2500
0.5000	0.3750	0.1250	0.6250	0.1250	0.2500
0.2500	0.7500	0.0000	0.3750	0.0000	0.6250
0.2500	0.6250	0.1250	0.3750	0.2500	0.3750
0.2500	0.2500	0.5000	0.1250	0.6250	0.2500
0.1250	0.3750	0.5000	0.0000	1.0000	0.0000
0.1250	0.2500	0.6250	0.0000	0.5000	0.5000
0.1250	0.8750	0.0000	0.2500	0.6250	0.1250
0.2500	0.7500	0.0000	0.5000	0.0000	0.5000
0.6250	0.3750	0.0000	0.7500	0.1250	0.1250
0.1250	0.5000	0.3750	0.2500	0.0000	0.7500
0.1250	0.7500	0.1250	0.3750	0.1250	0.5000
0.3750	0.3750	0.2500	0.5000	0.1250	0.3750
0.0000	0.7500	0.2500	0.1250	0.3750	0.5000
0.5000	0.1250	0.3750	0.3750	0.5000	0.1250
0.7500	0.0000	0.2500	0.6250	0.3750	0.0000
0.2500	0.3750	0.3750	0.3750	0.0000	0.6250
0.0000	0.8750	0.1250	0.1250	0.6250	0.2500
0.0000	0.6250	0.3750	0.1250	0.2500	0.6250
0.2500	0.5000	0.2500	0.1250	0.8750	0.0000
0.5000	0.5000	0.0000	0.6250	0.1250	0.2500
0.2500	0.5000	0.2500	0.3750	0.2500	0.3750

Notes: The 100 questions were composed of 4 sets of (the same) 25 questions, each set applied to 3 of the 4 amounts £0, £10, £20, and £30.

2.2.2 Trembling Hand Approach

Harless and Camerer (1994) suggest a simpler error-generating mechanism analogous to the game theoretic idea of the 'trembling hand'. Their approach assumes that individuals have true underlying preferences characterised by a core deterministic theory, but they make the wrong choice with a fixed probability of w. Although this assumption offers a simpler way, intuitively it seems a quite insufficient way to incorporate the stochastic nature of human behaviour, since the likelihood of making an error is expected to increase when the difference in satisfaction between the options decreases (see Loomes et al. (2002) for further discussion). Notice that in their approach, the probability of making an erroneous decision is independent of the features of the options, in other words, they assume that individuals choose the less preferred option with a probability of *w*, no matter how much the difference between the utility of the options is according to a core deterministic theory.

They conducted their analysis on 23 data sets consisting of approximately 8,000 choices that subjects made in Allais type of problems. Overall, they found that none of the existing theories perform significantly better than others: all theories are rejected by a chi-square test. This implies that the variation that is not predicted by the existing core theories can be explained by another theory as yet undeveloped, because for every theory the 'trembling' part is systematic variation rather than being an error. However, they can identify some dominated and dominant patterns: the dominated theories are generally the ones which assume betweenness rather than independence and assume fanning in Machina triangle, whereas the dominants are mixed fanning, Prospect Theory, EUT, and expected value. Interestingly, EUT is never dominated, but it is never selected as the best model according to the several selection criteria such as the Schwarz criterion.

Another important observation is that the theories like EUT and Weighted EUT can be improved by further generalisations to incorporate commonly observed patterns in the literature. Moreover, the alternative models such as Rank-Dependent Utility Theory seem to allow patterns that are rarely observed. Thus, the results suggest not abandoning EUT but extending it.

2.2.3 Random Preference Approach

The Random-Error and Trembling-Hand approaches model the imprecision as deviations from the true preferences due to the errors that people make in their calculations and judgements, and thus can be categorised as Fechner type of models. The final approach that I will discuss is known as the 'random preference model', first discussed by Becker et al. (1963), then generalised by Loomes and Sugden (1995). This approach assumes that individuals decide according to a core theory, but the parameters of the theory are determined randomly for each action. For example, if the core theory is EUT, then the risk aversion parameter will be randomly drawn with replacement for each task, thus it might not be the same for another action. In other words, this modelling approach sees preference imprecision as a set of preference functions that are consistent with a theory, rather than as some white noise added to a core theory. Intuitively, it views the individual as a collection of multiple selves, where the self that is deciding for each task is randomly chosen. Notice that in this approach imprecision is not viewed as an error added to a core theory; this is a major departure from the standard notion of economic preferences, because under the standard view the individual is assumed to have stable preferences, i.e., to exhibit the same parameter values for each action. As an example, consider the EUT with a simple power utility function, $u(x) = x^a$, where the parameter a determines the curvature of the utility function, i.e., the risk attitude of the individual. Now suppose a equals 0.8, implying that the individual exhibits risk aversion. The standard view of preferences sees this parameter value as an inherent characteristic of the individual and assumes that independent of the task type such as buying, selling, or choice, and the available options, the individual employs the same value for a upon making decisions. Whereas in the 'random preference model', the value of a is allowed to change, therefore the personality of the individual is not assumed to be stable.

Compared to the Random Preference Model, one obvious limitation of the Fechner type of models is the violation of dominance. We know from the previous experimental literature that individuals seldom violate dominance at least when it is transparent, i.e., they frequently choose the stochastically dominant option (Loomes, 2005). For example Loomes et al. (2002) analyses the data presented by Loomes and Sugden (1998), in which the binary choices of 92 subjects for 45 lottery pairs are collected. The distinct feature of the data is that each pair is presented twice in different orders. Among the 45 different lottery ticket pairs, in 5 of them one option stochastically dominates the other such as offering a slightly higher chance of winning the same amount or lower chance of losing the same amount. What they find is that, although the Fechner-type error models predict 10-15% of subjects will violate dominance, the ratio was less than 1.5%. Therefore, when we include the dominance cases, Fechner models perform poorly, however one can interpret this result as individuals not behaving according to these models when the dominance is transparent. In other words, in those 'easy' decision problems, individuals behave according to the predictions of EUT, but when there is not dominance between pairs, the stochastic nature of the preferences is more applicable to describe the behaviour.

It is easy to see that the Random Preference Model does not incorporate any violations of dominance, because the stochastic component is inherent in the individuals' preferences (Loomes and Sugden, 1995). This means that the Random Preference Model underpredicts the observed rate of the violation looking at the experimental evidence provided by Loomes et al. (2002), as discussed before. Remember that according to the model, for each choice task the individual draws the parameters of the core theory randomly, and if the core theory predicts behaviour consistent with the dominance notion, then the Random Preference Model does too. Consider two binary outcome prospects f and g, which gives the same amount X and zero, but the first one has slightly higher probability of winning X, such as 0.25 and 0.20, respectively. The expected utility of the prospects are:

$$EU(f) = 0.25 \cdot u(X) + 0.75 \cdot u(0) \tag{22}$$

$$EU(g) = 0.20 \cdot u(X) + 0.80 \cdot u(0) \tag{23}$$

Clearly, expected utility of f is higher than g, since the first one stochastically dominates the other. The choice problem can be represented as:

$$0.05 \cdot [u(20) - u(0)] > 0 \tag{24}$$

Thus, regardless of which parameters are drawn randomly for $u(.)$, if the core theory behaves according to the dominance notion, so does the Random Preference Model. On the other hand, Fechner-type of models incorporate the error term separately:

$$0.05 \cdot [u(20) - u(0)] + \varepsilon > 0 \tag{25}$$

If the error term is negative and sufficiently high, the inequality will be reversed and model predicts the dominated option g will be preferred over f .

Loomes and Sugden (1998) find that the trembling hand approach of Harless and Camerer (1994) performs poorly. Moreover, the frequency of cases that exhibit violation of dominance is overpredicted by the random error approach of Hey and Orme (1994), where random preference fails to predict any violation of dominance. They suggest a future direction can be to incorporate a type of trembling notion into a random preference model. Subsequently, Loomes et al. (2002) implemented this trembling modification to a random error model and a random preference model. They compared EUT and Rank-Dependent Utility Theory using different stochastic specifications such as a random preference model with trembles, and a random error model with and without trembles. Results show that the trembling modification significantly increases the explanatory power of the two stochastic specifications. The best fitting menu seems to be the Rank-Dependent Utility Theory together with a random preference model with trembles.

Moreover, they find that the trembles disappear as subjects gain experience towards the completion of 90 choice questions. This implies the interesting conclusion that the tremble can be seen as a type of error due to the calculation or misunderstandings, but the variation incorporated by the random preference model is stable and does not decay. They further speculate that this part might be attributed to the preference imprecision and might be inherent. Notice that this is the only study in the literature so far where there is a clear distinction being made between imprecision and errors: Loomes et al. (2002) view the decaying part of the variation as a result of errors but the stable part of the variation as imprecision. The latter is the issue that this study develops around and presents a new theory for.

2.3 Experiments in Direct Elicitation of Imprecision Intervals

Previous sections review the empirical works in which the parameters of the various existing models are estimated together with a stochastic specification using the choice data. In this section, I review the experimental studies that use direct elicitation methods of imprecision intervals mainly relying on the subjects' self-reporting. Self-reported data is often used in social and behavioural sciences and in environmental valuation or happiness studies; however, it is unconventional in experimental economics to rely on unincentivised methods. As a principle, unlike in psychology, intrinsic motivation is not seen as sufficient for subjects to reveal their true preferences, as it is not a costly action for subjects to lie about their offers (see Camerer and Hogarth (1999) for a detailed discussion). In Chapter 3, I introduce a new mechanism that is incentive compatible under the given assumptions to overcome these problems. Generally, relying of self-reporting is not a desired method for experimental economics, but so for it seems difficult to develop a better way to elicit imprecision intervals.

There are two methods used in the literature: the Response Table (Cohen et al., 1987; Cubitt et al., 2015), and the Iteration Procedure (Butler and Loomes, 2007, 2011; Dubourg et al., 1997, 1994). In the first method subjects are asked to respond to a series of binary choice questions between a risky prospect and a sure amount of money by filling a response table similar to the Table 3:

Table 3 *Example response table*

Certain Amounts	I definitely prefer the good	Not sure	I definitely prefer the certain amount
0	✓		
1	✓		
2	✓		
3		✓	
4		✓	
5			✓
6			✓

For the example depicted in Table 3, a subject prefers the risky prospect for the certain amounts up to and including 2, whereas the risky prospect is preferred for the amounts 5 and 6. The imprecision interval corresponds to the values 3 and 4, for which a subject cannot confidently state a preference between the risky prospect and these amounts. Cohen et al. (1987) included a fourth column that states equivalence (indifference) between the two options. However, due to participants' misunderstandings, they combined the imprecision and equivalence column in their data analysis. More recently, Cubitt et al. (2015) also use the reduced version of the response table discarding the equivalence statement. Another difference between the two studies is related to payoff determination: in Cohen et al. (1987), if a subject stated an imprecision interval, the experimenter randomly determines which option is picked, whereas Cubitt et al. (2015) leave the choice to the subjects by asking them to determine a switching point inside the imprecision interval.

The second method relies on an iterative process. For example Dubourg et al. (1994) used a numbered disk, which has a small window showing only single value at a time. For each value, subjects state their preference by choosing one of the three phrases: definitely willing to pay, definitely not willing to pay, or not sure. If the response was 'willing', the interviewer rotates the disk to reveal a higher value through the window, whereas if the answer is 'not willing', the interviewer reveals a lower amount. The experiment continues until there is a maximum amount that subjects are definitely willing to pay and not willing to pay. If the two amounts are different, then the interviewer asks for a 'best estimate' of the subject for determining the 'switching point' in Cubitt et al. (2015).

Butler and Loomes (2007) elicited the valuations for risky prospects using a similar method which they call the 'incremental choice method'. They focused on preference reversal phenomenon, so they elicited value and probability equivalents for a series of P-bets and $-bets. The procedure is very similar to the method described before but this time they included four categories instead of three to describe the subjects' confidence in their choice: definitely preferring A, probably preferring A, probably preferring B, and definitely preferring B.

Overall, the existing methods in the literature are the slightly modified versions of the ones mentioned above; all rely on subjects' self-reporting.

2.4 Patterns Found in Experiments

Cohen et al. (1987), one of the early studies that used response table method, observed that 10% of the subjects exhibiting imprecision, the lowest ratio observed in the literature. Butler and Loomes (1988) focus on decision difficulty by using the experimental data first described in an earlier paper Loomes (1988) that compares behaviour under three kinds of elicitation procedures for certainty equivalents. The major finding is that although subjects are allowed to state their valuations in increments of one penny, a majority of them preferred to round their valuations to the nearest 50 pence. This can be interpreted as a support for the argument that individuals do not have cognitively costless access to their precise preferences. Butler and Loomes (1988) focus on the iterative elicitation procedure by which they elicited the certainty equivalents of the following four binary outcome lotteries in Table 4:

Table 4. *Lotteries used by Butler and Loomes (1988)*

Lottery	p_1	x_1	p_2	x_2
A1	0.2	£30	0.8	£0
A2	0.4	£15	0.6	£0
A3	0.6	£10	0.4	£0
A4	0.8	£7.5	0.2	£0

For each lottery in Table 4, subjects answered a series of binary choice questions where the second option was a sure amount of money. If a subject chooses the risky option, the sure amount is increased in the next question; on the other hand, if a subject chooses the sure amount, the value is updated downwards in the next question. Additionally, subjects are asked to use a cursor to state their confidence about their decision. The cursor can be moved to 51 different positions, corresponding to the feeling of confidence between 'very confident' to 'very unsure'. Despite the limitation of their data and experimental design, they conclude that as we move from A4 to A1, i.e., as the winning amount increases without changing the other outcome, the size of the imprecision range increases. However, the balance between the probabilities of the two outcomes also affects the size of the imprecision range. Second, their analysis provides some support for the hypothesis that as the variance gets higher the imprecision range also widens.

Morrison (1998) tested the prominent explanations of valuation gap such as the endowment effect, substitutability, or imprecise preferences using the experimental data presented first in Morrison (1997). In this study, three responses for each WTA or WTP question were elicited: a lower-bound, an upper-bound, and subjects' 'best estimate'. They tested the imprecision hypothesis by looking at whether the ranges for WTA and WTP intersect significantly or not. Results reject the imprecision hypothesis as an explanation for the valuation gap, because the lower bound of WTA is significantly higher than the upper bound of WTP; in other words, the ranges of the two measures do not overlap. Similarly, Dubourg et al. (1994) elicited WTP and WTA values for changes in the risk of nonfatal road injuries using an iterative procedure. They found that individuals exhibit a significant amount of imprecision, however this imprecision alone is not insufficient to explain the observed disparity between WTA and WTP.

Another important study is Butler and Loomes (2007), which focuses on investigating to what extent preference imprecision can explain preference reversals, using a similar iterative mechanism as Butler and Loomes (1988). In the 2007 study, rather than a 51-point scale of confidence, they use a refined version that only consists of four categories or phrases: ‘definitely choose A’, ‘probably choose A’, ‘probably choose B’, and ‘definitely choose B’. They ground the theoretical side of their study on an unpublished but influential paper by MacCrimmon and Smith (1986) that conjectures that individuals might have interval values rather than precise amounts for the risky prospects and claims that preference reversal phenomenon can be explained by $-bets having a wider interval than P-bets. They refrain from suggesting a formal structure about how individuals form these interval valuations, but they suggest that as the risky prospect become more dissimilar to the certainty or degenerate lottery the interval widens. Butler and Loomes (2007) found that the imprecision argument can be seen as one of the explanations of the preference reversals phenomenon, since the intervals found for the $-bet is significantly higher than the P-bet, and that more importantly they overlap, which is in line with the conjectures of MacCrimmon and Smith (1986).

Another important study conducted by Butler and Loomes (2011) proposed and tested preference imprecision as an explanation for the observed violations of independence and betweenness axioms. They use MacCrimmon and Smith’s (1986) model of imprecise preferences and demonstrate it on the Marchak-Machina triangle. Their results confirm the fanning out hypothesis and favour preference imprecision as an explanation of the violations of EUT.

The most recent study is Cubitt et al. (2015), which elicited the imprecision intervals by using the response table method; they also asked subjects to state their best estimate from the interval. They found that the best estimates of the subjects move coherently with the attractiveness of the lotteries, in other words, as the attractiveness of the risky prospect increases, subjects’ best estimates also increase. Overall, they found that 87% of the subjects exhibit imprecision in their preferences. The size of the interval does not seem to be dependent on the outcomes of the lotteries. Furthermore, the size of the stated intervals is found to be the constant proportion of the distance between the best and the worst outcome of the lotteries, which is in line with Butler and Loomes (2011). Their design also enables tests of stability, i.e. whether the size of the intervals changes with repetition or not. It is important, because if imprecision is merely a result of errors or unfamiliarity with the experimental mechanisms, it should disappear with repetition and experience. However, they found no evidence for imprecision declining with experience. Their analysis supports that imprecision is stable and not temporary; it seems to be the inherent characteristic of individuals' preferences. Although they could not find any evidence that imprecision accounts for the violation of betweenness and independence, it seems to be an important phenomenon of individual behaviour, and the nature of it should be understood further.

2.5 Conclusion

Imprecision in economics literature seems to be seen as the errors that individuals make in their calculations. This idea is influenced by early works in psychophysics literature. I reviewed three major approaches that emerged in the economics literature. These approaches model imprecision as a stochastic element added to a core deterministic theory. However, as the results indicate, even with these specifications, none of the deterministic models seems to explain a significant portion of the observed behaviour. The stochastic component seems to be systematic, suggesting that new theories should be developed to explain this residual. Another important result presented by Loomes and Sugden (1998) found that the imprecision should not merely be interpreted as the errors that subjects make; instead a portion of it seems to be an inherent part of the preferences that does not diminish as subjects gain experience. In Chapter 4, I present a new deterministic core theory that focuses on this inherent part.

3 A Re-examination: Valuation Gap and Interval Valuations

This chapter includes an extended version of the published paper Bayrak and Kriström (2016) which is an experimental study on valuation gap. It can be seen as an exploration study, which provides insights about the interval valuations specifically focusing on the valuation gap. Interval valuation concept is directly related to preference imprecision. As mentioned in Chapter 2, individuals with imprecise preferences cannot state a precise amount confidently as their subjective valuation of a good, therefore will end up having ranges.

We extend the literature on the willingness-to-pay/willingness-to-accept (WTP/WTA) disparity by testing two hypotheses, distilled from the literature. We also introduce a modified mechanism for eliciting the subjective valuation range if the individual cannot articulate the subjective value as a precise amount confidently. We elicited valuations for four goods: three ordinary market goods and a lottery ticket. Under the conventional setting in which subjects are asked to state a single precise amount, we observed a significant disparity for the lottery ticket. On the other hand, our key finding is that the disparity disappears under the intervals treatment, suggesting that response format is important, given that earlier experimental studies invariably uses point values (i.e. open ended questions about WTP/WTA). Moreover, for the risky prospect we observe that from their admissible range the buyers state the lower bound as their WTP whereas sellers state the upper bound as their WTA. We conclude that this type of behavior can to some extent explain the observed disparity at least for the risky prospects. The results lead to the development of PCT, which is a new decision theory for under risk and is presented in Chapter 4. Section 3.1 provides and introduction and explains the motivation of investigating the issue with interval valuations. Section 3.2 reviews the literature on valuation gap, evaluates the results of three meta-analysis of the issue and presents the three different explanations of the issue existing in the literature. Section 3.3 introduces our working hypothesis. Section 3.4 provides the details of our experimental design. As mentioned before, we also introduce a new experimental mechanism; a modified version of Becker-DeGroot-Marschak mechanism for eliciting imprecise preferences, Section 3.4 presents the incentive compatibility analysis of our new mechanism. Finally, Section 3.5 presents our results and Section 3.6 concludes.

3.1 Introduction

The "valuation gap" refers to the empirically found disparity between WTP and WTA. It remains one of the most prominent anomalies in standard economic theory, because we expect that WTP and WTA should be similar if the goods in question have close substitutes and if the income effects are small (Hanemann, 1991). The gap was first documented by mathematical psychologists Coombs et al. (1967) and by Hammack and Brown (1974) in an early contingent valuation study. Knetsch and Sinden (1984) brought the issue into the laboratory using real monetary incentives and found a significant difference between WTP and WTA. Since then, the disparity has been found in an array of studies, including contingent valuation surveys and in field and laboratory experiments for a wide range of goods: e.g. mugs, pens, movie tickets, hunting permits, nuclear waste repositories, foul-tasting liquids, and pathogen-contaminated sandwiches (Horowitz and McConnell, 2002).

The gap has many implications for the application of economic theory, but also for theory proper. For example, if a cost-benefit analysis is conducted on a proposed policy that generates both winners and losers, estimated net benefits will then depend on whether WTA or WTP was used in the assessment. At a more fundamental level, the gap raises questions about the power of standard preference models to explain economic behavior (Braga and Starmer, 2005).

Explanations of what may drive the disparity include the endowment effect which suggests that preferences are reference dependent and losses loom larger than gains. Thus sellers perceive giving away the good as a loss and ask for more as a compensation for their loss (Thaler, 1980). Theorists have also developed alternative models of economic behavior that address the disparity and several other anomalies[1].

[1] Briefly, Prospect Theory (Kahneman and Tversky, 1979), Cumulative Prospect Theory (Tversky and Kahneman, 1992), Third Generation Prospect Theory (Schmidt et al., 2008), Rank Dependent Utility Theory (Quiggin, 1982), and Regret Theory (Bell, 1982; Loomes and Sugden, 1982).

Yet, emerging evidence suggests that, under certain types of procedures and settings, the WTP-WTA disparity is smaller than previously observed. Shogren et al. (1994) find that the size of the gap depends on the type of good that is used in the experiments (e.g., mugs, candies, lottery tickets, and tokens). Other researchers find that the disparity declines with trading experience (List, 2004a, 2003; Loomes et al., 2003; Shogren et al., 2001). Sayman and Onculer (2005) conducted a meta-analysis of 39 studies and found that studies that employ iterative bidding exhibit smaller disparities. These findings suggest that experimental design features are critically important. Indeed, in the most recent meta-study, Tuncel and Hammit (2014) find that studies that were published after the first meta-study (Horowitz and McConnell, 2002), exhibit lower WTP-WTA ratios and interpret this as the improvements in study design characteristics. This begs the question of what an “improvement” entails. We suggest two criteria that can be used to assess an experiment:

i. The experimental instructions and procedures should be clear to the subject.
ii. The response format should be close to the “natural way” that people think about their valuations.

The first item has been covered by Plott and Zeiler (2005), who conducted experiments to control for subject misconceptions about the experimental mechanisms, such as the Becker–DeGroot–Marschak mechanism (BDM). Their design employs numerical examples, paid and unpaid training rounds, anonymity of the subjects’ identities, and verbal explanations of how to obtain the optimal response. The disparity is not observed for ordinary market goods when procedures to eliminate subjects’ misunderstandings about the experimental mechanism are employed: Their result weakened the loss aversion explanation of the disparity. However, Isoni et al. (2011) pointed out that the disparity persists when using lottery tickets, so the issue extends beyond subject misconceptions.

Our second criterion has not yet been sufficiently explored in valuation gap studies in an experimental setting. In the contingent valuation literature, a substantial number of papers have been published on the subject of elicitation mechanisms. One strand of this literature compares open-ended and dichotomous choice formats(Loomis et al., 1997; Reaves et al., 1999). In the open-ended format, subjects are simply asked how much they are willing to pay, whereas in the latter, subjects are asked to accept or reject a series of pre-selected prices. More recent elicitation mechanism allows for respondent uncertainty in various ways; see Mahieu et al. (2014) for a recent survey. In short, experimental studies that find a disparity, invariably uses an open-ended valuation question. This format is not currently much used in contingent valuation, the most important reason being that the response rates are typically low.

The contingent valuation literature rather converged on finding a response format that is allegedly closer to the way that individuals think about their valuations (Brown et al., 1996). For most individuals, valuation (of the maximum/minimum buying/selling price) is not a routine task. Therefore, asking individuals for precise estimates of their subjective valuations can be cognitively challenging (Mitchell and Carson, 1989), especially for complex and unfamiliar goods (Gregory et al., 1995; Ready et al., 1995). We also know from the psychology literature that when individuals are faced with difficult tasks, they have a tendency to employ heuristics to facilitate them (Shah and Oppenheimer, 2008). For example, McCollum and Miller (1994) found that 44% of the respondents reported $0 due to their inability to provide a precise WTP even when they indicated a positive attitude towards the good.

If the same behavioral pattern is also present in experiments on disparity, then it might, for example, cause buyers to understate their subjective valuations and cause the observed disparity. A particularly useful alternative mechanism caters for imprecision, without compromising the possibility to state a precise amount. In this variation, individuals are asked to state interval valuations, in case they are unable to come up with a point[2].

In a related literature which focuses on imprecise preferences, subjects are assumed to have an admissible range of subjective valuations from which they cannot state a precise amount confidently (See Cohen et al. 1987; Butler and Loomes 1988, 2007, 2011; and Morrison (1998)). Butler and Loomes (2011) claims that preference imprecision could explain anomalies within EUT. For example, Butler and Loomes (2007)[3] explore imprecision as a way to understand preference reversals. They argue that many individuals' choices and valuations involve a degree of uncertainty or imprecision, and their findings suggest that imprecision explains a significant portion of the preference reversal phenomenon[4].

---

[2]Some researchers in the contingent valuation related literature have suggested the use of self-selected intervals in surveys. The basic idea behind self-selected intervals dates to at least Morgan and Small (1992), who suggested them as a way of overcoming "overconfidence" in surveys and to address the anchoring problem. There is also a connection to symbolic data analysis (Billard and Diday, 2007), in which intervals play an important role. Detailed statistical theory for handling this unusual kind of interval censoring has been developed by Belyaev & Kriström (2015).

[3]See Gal (2006) and Neilson (2008) for a theoretical approach to imprecision and empirical studies that can be classified similarly but that used non-incentivized elicitation methods for strength of preference (Dubourg et al., 1994; Loomes and others, 1997).

[4]They asked the subjects to state their preferences in a series of binary choices in which one option (A) was held constant and the other (B) was adjusted upwards or downwards by $1,

Interval valuation as a response format is yet to be tested thoroughly in an experimental setting. Banerjee & Shogren (2014) explore the bidding behavior in second price auctions using an induced value experiment in which subjects are given point or interval values and are asked to state point or interval bids. In their *point value/interval bid* treatment, they find that even though the value of the object is given exogenously as points, most of the subjects tend to state their values in terms of intervals. It appears that subjects form these intervals in a way that the expected value of the interval equals the point value. This observation is important because it is natural to expect imprecision to be case for uncertain and/or unfamiliar goods; yet their subjects prefer to state their bids in intervals although the goods has an exogenously given point value.

As a simple remedy for the problem observed by McCollum and Miller (1994), we frame the response format as intervals of which the bounds are determined by subjects: if they cannot provide a precise estimate, they are allowed to state an interval for their WTP and WTA, and we test whether the disparity survives under this framing. If individuals are stating some amount lower than they would pay for the good merely because they cannot provide a precise amount, but the experimental design asks them to do so, framing the response format as intervals can decrease the cognitive burden and make subjects think more carefully about their valuations (*Response Format Framing Hypothesis;* RFFH). This is called *framing* here, because only the buyer's upper bound and the seller's lower bound are incentivized; the trade is determined by comparing only the incentivized bound with the randomly selected market price. Consider a buyer who states a range: the subject buys the good, if the market price is within or below the stated range. For the seller role, trade occurs if the market price is within or above the stated range. We do not observe a disparity when we use interval framing, whereas we observe a significant disparity for the lottery ticket, when we asked subjects to state single points (See Section 3 for details).

Gregory et al. (1995) found that individuals display a surprisingly large WTP range, and when they are asked to state a single amount, they are likely to state an amount closer to the middle of their range. As sellers, subjects tend state a point close to the upper bound of their admissible range. This behavioral pattern might produce the observed disparity and gives rise to a hypothesis we

---

depending on the starting point. (In one treatment, they started from $1 and increased, whereas in another treatment they started from a positive payoff of the first lottery and gradually decreased.) In each binary choice problem, the subjects stated which option they chose and selected one of the following phrases that reflected the strength of their decision: definitely prefer A, prefer A but not sure, prefer B but not sure, and definitely prefer B. However, "preference strength elicitation" is not incentivized under their design.

call the *Preference Cloud Hypothesis* (PCH). The Preference Cloud Hypothesis posits that individuals cannot intrinsically determine precise single points, but able to identify a range of values for their personal valuation of the good. If the experiment forces them to state a point, they employ a heuristic: buyers state the lower bound while sellers state the upper bound in their admissible range.

To test this hypothesis we first have to focus on the good which we observe a significant disparity under the conventional setting as our baseline, in our case it was only for the lottery ticket that we observed a gap. For comparison we use the bounds of the intervals elicited in another treatment (*Buyer-Seller Uncertainty*, BS_{int} hereafter) in which subjects are allowed to state intervals, not knowing in advance whether they are buyers or sellers (the role is determined randomly after they stated their offers). Statistical tests confirm our hypothesis that WTP in baseline treatment and lower bound of the offers in the intervals treatment comes from identical distributions, whereas WTA in the baseline treatment and upper bound of the interval treatment comes from the identical distributions (See Section 3 for details).We now turn to empirical analysis and begin by explaining the experimental design.

3.2 Literature Review

There is no consensus on the size and the existence of the disparity in the literature. In order to show this, we begin by presenting findings of three meta-analyses. This will paint a useful overall picture of factors behind the existence (and size) of the disparity. We then outline three major strands of the literature, in order to focus sharply on the various disagreements.

3.2.1 Three Meta-Analysis Studies

There are three notable meta-analysis studies about the disparity and their findings are reported in Table 5. The earliest is Horowitz and McConnell (2002) (HM, hereafter) who analyzed a set of 45 studies and found that experiments with real incentives do not give significantly different results compared to hypothetical experiments. Interestingly, experiments that use incentive compatible designs find higher WTA\WTP ratios. Another important finding is that the experiments with public and non-market goods have higher ratios than ordinary market goods. HM report no systematic difference between the studies using student or non-student subjects.

Table 5. *Results of three meta-analysis*

Study Design Characteristics	HM (2002)	SO (2005)	HT (2013)
Public vs. Ordinary private goods	Larger for public	Larger for public [a]	Larger for public
Incentive compatible designs	Larger [b]	Smaller	Smaller
Student vs. Non-student subjects	No difference	NA	No difference
Within subjects design	NA	Smaller in Within	No difference
Out of pocket payments	NA	Larger	NA
Iterative bidding	NA	Smaller	No difference
Experience	NA	Smaller	Smaller

Notes: HM: Horowitz and McConnel, SO: Sayman and Onculer, HT: Hammit and Tuncel. NA implies that corresponding study does not include that specific factor as an explanatory variable so result is "not available".
[a] The disparity is larger also for the health-related goods.
[b] They find that experiments includes real payoffs do not result in significantly different disparity compared to the hypothetical payoff experiments.

Sayman and Onculer (2005) analyzed data from 39 studies, focusing on the effects of design and method on the size of the disparity. They found that iterative bidding and within-subjects designs decrease the disparity and, on the other hand, out of pocket payments increase it. In the case of iterative bidding, subjects are asked whether or not they would pay a given amount (starting point or bid) for the good described. If the participant is willing to pay something, the interviewer revises the bid upwards until a maximum willingness to pay (or downwards until a minimum willingness to accept compensation) is reached. This method seems to contribute the subjects' learning their personal values because they are asked to state "yes" or "no" answers for several amounts during the interview. Therefore they are lead to think about them more carefully.

The most recent meta-study is Hammit and Tuncel (2014) which can be seen as an updated version of HM, since they include more recent studies and some methodological refinements such as using the natural logarithm of the WTA-WTP ratio. Their findings are in line with HM about the type of good (public and ordinary private goods) and the subject profile (student and non-student). By contrast, they find that incentive compatible mechanisms result in a lower disparity. Most importantly, they find that the studies published after HM, exhibit lower WTA-WTP ratios. Their interpretation of this result is that this reflects an improvement in study design characteristics.

To summarize, the meta-studies suggests that the disparity is lower for ordinary market goods and with iterative bidding. The size and the existence of the disparity are strongly dependent on the exchange mechanisms, the experimental procedures and subject experience.

3.2.2 Three Strands of the Previous Literature

We continue the literature review by defining -in the broadest manner- three main strands in the literature on explaining the disparity: (i) the psychology based approach, (ii) standard economic theory based approach, and (iii) experimental design effects and learning based studies.

The Psychology Based Approach

These studies seek explanations from a psychological perspective and explain the observed gap using the loss aversion concept of Prospect Theory. Most studies accept the existence of the gap, concluding that it is in the nature of individual preferences. Thaler's (1980) "endowment effect" suggests that individuals value goods more when they own it; this is directly related to loss aversion notion of Prospect Theory, which posits that losses loom larger than gains (Kahneman et al., 1990). In other words, sellers' perceive transferring the ownership of the good as a loss and this causes them to ask for more compensation.

There is another psychological explanation, which is related to moral attitudes and ethical concerns, but this type of explanation is not applicable to inexpensive ordinary market goods such as mugs and candies; it is more applicable to environmental goods, such as preserving species or environmental amenities. This is because emotions and responsibility concerns are more relevant to these issues (Boyce et al., 1992; Irwin, 1994; Peters et al., 2003). For example, Boyce et al. (1992), using pine trees as the good for their experiment, told the subjects that the trees would be killed if they do not purchase the tree (or if they sell the tree to the experimenter). Their results show that both WTP and WTA and the ratio between them are higher than in the control group (no kill treatment). They explain the higher WTA value in the "kill treatment" with the owners' feeling of responsibility towards preserving the pine trees.

Standard Economic Theory Based Approach

Another group of studies seeks explanations within the standard economic theory. Hanemann (1991) suggested that the gap should be smaller or disappear if the good has a close substitute. Therefore the gap should be higher for goods such as a national park or a market commodity which has no close substitute. Adamowicz et al. (1993) tested Hanemann's proposition using a closed-ended CV format to value the tickets for a particular movie. In one group a close substitute was available, but not in the second group. Although having a substitute decreases the gap by 30%, they observed a significant gap.

Experimental Design Effects and Learning

The third stream of the existing literature is the newest and focuses on experimental design and procedures such as repeated setting, different exchange mechanisms, institutional and value learning. The underlying question is whether the disparity is robust against the mentioned features of experimental design or not. A maintained hypothesis is stable and well defined preferences; the observed anomalies thus stem from subject errors due to design features such as the lack of opportunity to learn the mechanism (institutional learning) and value learning (Braga and Starmer, 2005).

One of the first studies that investigated the disparity in a repeated setting was Coursey et al. (1987). They used a variant of the Vickrey auction and asked subjects about their WTP and WTA to avoid tasting an unpleasant substance. Results indicate that averages of WTA and WTP converged with repetition. However, Kahneman et al. (1990) report a significant and persistent gap even in the repeated setting using the BDM mechanism[5]. Shogren et al. (1994) conducted a repeated setting experiment with the Vickrey second-price sealed-bid auction. Their result suggests that the WTA and WTP values do converge by repetition in the market goods case (mugs and candies). However, for a nonmarket good with no close substitutes such as “reduced health risk”, persistent difference is observed even with the repeated market participation. In a related paper, Shogren et al. (2001) compares different auction mechanisms effect on the disparity. Results suggest that except for the BDM mechanism, the values seem to converge with repetition.

Plott and Zeiler (2005) focus on subject misconceptions and experimental procedures. They replicated the gap with the procedures of Kahneman et al. (1990) and conducted additional experiments which control for several misconceptions that might rise from specific design features. They implemented modifications such as paid, unpaid training rounds and subject anonymity. Under these modifications they observed no gap between WTA and WTP. However, Isoni et al. (2011) claim that the lottery valuation data which is not published in Plott and Zeiler (2005), shows a persistent and significant disparity. Isoni et al. (2011) replicated Plott and Zeiler (2005) procedures that control for subject misconceptions. Again, there were no gaps in the case of mugs, but a significant gap for the lottery valuation task. Why do the control procedures for misconceptions eliminate the gap in the mugs round but not in lottery ticket tasks? Subject misconceptions apparently play a very

[5] See Knetsch et al. (2001), Brookshire and Coursey (1987) and Price and Sheremeta (2011) for further discussion about the effect of repeated setting on endowment effect.

important role; the results of Plott and Zeiler (2005) and Isoni et al. (2011) still leaves some unturned stones when trying to come to grips with the disparity.

Summary Of The Three Strands

The psychology based and standard economic theory explanations of the disparity are, quite naturally, disparate. Studies in the third strand of the literature generally observe converging WTA and WTP values in repeated lab-settings or find no evidence for the existence of the disparity. Although these findings seem to give support to standard economic theory, there is also another way to read the evidence: preference uncertainty or imprecise preferences (See Chapter 2). This could be the reason why results are dependent on certain experimental settings and procedures.

3.3 Hypotheses

Before presenting our hypotheses, we summarize the findings reported in the literature. This summary is designed to help the reader understand the support for our hypotheses:

- The disparity can be eliminated or minimized under certain types of procedures that enable subjects to understand both the experimental mechanism and how to find their optimal response (Plott and Zeiler, 2005).
- Individuals exhibit a significant amount of imprecision, especially in their valuations of goods (Cohen et al. 1987; Butler and Loomes 1988, 2007, 2011; Dubourg et al. 1994, 1997; and Morrison 1998).
- Response format affects valuation behavior: the “correct” format should be close to the subjects’ natural mode of thinking. Otherwise, valuation task will be cognitively challenging for the subjects (Brown et al., 1996; Mitchell and Carson, 1989; Gregory et al., 1995; Ready et al., 1994). When individuals face difficult problems, they have a tendency to employ heuristics to facilitate the decision-making process (Shah and Oppenheimer, 2008). McCollum and Miller (1994) found that a significant portion of the $0 responses were due to the individuals’ inability to arrive at a precise amount.
- Gregory et al. (1995), found that for WTP, subjects are more likely to state an amount that is close to the lower bound of their range.

From the findings in the literature mentioned above, we distill and test two hypotheses: i. *Response Format Framing Hypothesis* (hereafter, RFFH) and ii. *Preference Cloud Hypothesis* (hereafter, PCH).

3.3.1 Response Format Framing Hypothesis (RFFH)

The RFFH states that the WTP-WTA disparity is an artifact of the response format. All studies on the disparity have employed an open-ended format, which is simply to ask for precise WTP and WTA amounts. Based on the findings highlighted especially in the second and third bullet points, we argue that allowing subjects to state intervals is closest to the way that people naturally think about their subjective valuations. Therefore, using intervals as the response format can be the next extension of the experimental study design characteristics that fulfill criterion ii in Section 3.1. This design approach will ease the decision-making process and thus eliminate the effect that results from the burden of having to determine a precise estimate of WTP and WTA.

3.3.2 Preference Cloud Hypothesis (PCH):

Drawing from especially the third and fourth bullet point, we propose the PCH: individuals cannot intrinsically determine precise single points, but identify a range of values for their personal valuation of the good. If the experiment forces them to state a point, they employ a heuristic: buyers state the lower bound while sellers state the upper bound in their admissible range. Thus, if subjects are asked to state a precise amount from their range, they begin to play a “guess your true personal value” game. Note that we refer “game” as a metaphor to explain the intuition of PCT. Depending on the role, individuals draw different values from the range: being in the buyer role causes individuals to employ a distribution that is skewed to the right, while those in the seller role tend to employ a distribution that is skewed to the left. They project these distributions onto their admissible range of their subjective valuations and calculate the mean accordingly. The main motivation for this type of behavior can be seen as caution. If this hypothesis is confirmed, it will open up a new way of modeling the decision-making process. This new theory should answer two questions: i. How do individuals form these intervals? ii. From this interval, how do they decide on a single value as their WTP or WTA if they are asked to state a precise amount?

To sum up the two hypothesis: RFFH states we will not observe disparity if we give subjects the freedom and flexibility to state in terms of intervals or points, we will not observe disparity, because most individuals can come up with a range of values not single precise amounts as their subjective value for a good. Therefore, thinking and response format compatibility is important. PCH states that when we ask them to state single points depending on their role, they state different bounds of their true personal value (if they are buyers they state the lower bound, if they are seller they state the upper bound of the true range).

In addition to this, we expect this hypothesis to be more applicable to the good, which include uncertainty such as lottery tickets.

Next section details our experimental design and explains how we test these two hypotheses.

3.4 Experimental Design

We conducted a between-subjects experiment with two treatments: *Points* and *Intervals* (See Table 6 for an outline of the experimental design). The only difference between the two treatments is that in the *Intervals* treatment subjects were allowed to state their valuations in terms of ranges. Subjects are allowed to state single amounts if they prefer. In *Points* only single amounts are allowed, there were the usual two groups: Buyers and Sellers denoted B_p and S_p, respectively. In *Intervals*, we use three groups: Buyers, Sellers, and *Buyer-Seller Uncertainty* (B_{int}, S_{int}, and BS_{int}, respectively).

In the *Points* treatment: subjects state their offers, and then a market price is determined randomly. If the market price equals or is below the stated offer, a buyer pays the market price and buys the good. For sellers, if the market price equals or is above the stated offer, the seller gets the amount of money equal to the market price and gives away the good. As noted, B_{int} and S_{int} groups of *Intervals* treatment is a new type of framing, because only the buyer's upper bound and the seller's lower bound are incentivized; the trade is determined by comparing the incentivized bounds with the randomly selected market price. For the buyer only the upper bound of the stated range is binding. For the seller, trade occurs if the market price is within or above the stated range. Therefore only the lower bound of the range is binding. The only difference between *Points* and *Intervals* is the response format; thus, any difference in the results is due to this feature. We compare the values elicited by B_p, S_p, B_{int}, S_{int} to test the RFFH, (WTP_p, WTA_p, WTP_{int}, WTA_{int}; respectively). If we observe a statistically significant difference between WTP_p and WTA_p but not between WTP_{int} and WTA_{int}, RFFH is supported.

Testing PCH is not straightforward; we need to compare the point offers with ranges that are elicited in an incentive compatible way. Remember that PCH claims that buyers state the lower bound, whereas sellers state the upper bound of their admissible range (this is the underlying reason for observing the disparity). We elicit the usual point offers in B_p and S_p however we cannot use the ranges elicited in B_{int} and S_{int} because only one bound of those ranges are incentivized. They are only appropriate to test RFFH which is a hypothesis focusing on the framing of the response format.

To accomplish this we developed BS_{int} which is a modified version of BDM in which both the lower and upper bounds were incentivized (See the appendix for details): At the end of the experiment, roles were determined randomly; the probability of being a buyer is ½ (likewise, the probability of being a seller is ½). If subjects overstate their valuations, there is a 50% chance of being a buyer and a risk of paying an undesirably high amount. If they understate their values, they might be a seller and would have to sell the good for an undesirably low amount.

Table 6. *Summary of the Experimental Design*

Anonymity	Assigning subjects an ID number randomly
Instructions	Also read aloud
	Numerical examples to explain optimal response
	Hypothetical Training Round
Goods	Four goods with real incentives
Good 1	Premium bitter chocolate
Good 2	Created their own package of three cans, from five different flavours of a beverage brand
Good 3	Select one of the ten different flavours of a chocolate brand
Good 4	Lottery ticket: winning 30 SEK with a probability of 0.5, zero otherwise
Incentives	Show-up fee of 100 SEK≈$12
	One of the four goods and a market price selected randomly
	Only in BS_{int} group, subject role (buyer, seller) is also selected randomly after value elicitation

The roles were determined after the four tasks were completed using the following procedure: The experimenter wrote "*buyer*" and "*seller*" on two separate pieces of paper, placed them in two separate envelopes, one of them is picked from an opaque bag. In addition, the procedure was explained to the subjects in detail when the instructions were provided. To see the incentives under this mechanism see Section 3.5.

We recruited the subjects by announcement (flyers and posters) from Umeå University and the Swedish University of Agricultural Sciences (SLU)[6]. In total, 38 subjects participated in points, and 54 subjects participated in intervals, most of whom were master's degree students from various fields of study. The sessions lasted approximately 40 minutes, and the average earnings were 108 SEK[7] (including a 100 SEK show-up fee). Each subject chose an

[6]These two universities are very close to each other and can considered the same campus area. Umeå University has over 20,000 students, whereas SLU is a much smaller university.

[7]1 SEK is approximately 0.15 US Dollars.

envelope marked with an ID number upon entering the room. We told the subjects to keep these ID cards and to use them to retrieve their earnings after the experiment. The instructions were read aloud, and the participants were instructed not to communicate with each other or react verbally to any events that occurred during the experiment.

In both experiments, following Plott and Zeiler (2005), certain training procedures were employed to minimize or prevent subject misconceptions, i.e., anonymity was ensured and numerical examples were used to explain the mechanism[8] together with examples to show the subjects why stating their true value is the dominant strategy. In addition, the participants were provided with an unpaid training round in which the good was a candy. As indicated in Plott and Zeiler (2005), the provision of paid practice rounds is not an essential procedure: No disparity is however found between bids submitted in the paid and unpaid practice rounds.

After the training round, the subjects were encouraged to ask questions. They wrote their questions on pieces of paper and raised their hands; the experimenter silently read the questions and answered them by writing on the same piece of paper.

The practice round was followed by four tasks (goods), and the subjects were told that these four tasks had an equal chance of being selected and the payoffs will be determined according to the randomly selected task.

In task 1, the good was a premium bitter chocolate. In task 2, the subjects were given a list of five different flavors (regular, light, zero, vanilla, and cherry) of a nonalcoholic beverage brand. They were asked to create any package of three cans; thus, they were allowed to mix and match among the five types. Then, they stated offers for their created package. The good in task 3 was similar: In that case, 10 different flavors of the same brand of chocolate were provided, and we asked the subjects to select one of the flavors.

Goods 2 and 3 are homogenous for all subjects, since prices in local shop do not vary with the flavors and these two goods can be considered as vouchers providing the right to choose a favorite flavor. We included these to contribute the literature by re-examining the disparity with a new type of goods. Also, the endowment effect might be stronger for these goods since the subjects picked their favorite flavors; thus, they might have felt more attached to these goods.

[8] The numbers that are used in the examples are completely unrelated to the possible range of prices in the experiment to avoid any anchoring effects (e.g., 1000–1020 SEK, whereas the experiment market price can be between 1 and 30 SEK). The numerical examples were part of the written instructions provided, and they were explained on a board.

The participants were not provided with any information about market prices during the experiment. The prices of the goods in tasks 1, 2, and 3 were 19 SEK, 24 SEK, and 22 SEK, respectively, at a local shop.

Finally, the fourth good was a lottery ticket with the following prospects: winning 30 SEK with a probability of 0.5 and winning nothing with a probability of 0.5. The lottery outcome was determined by using one hundred ping-pong balls that were numbered from 1 to 100 and placed in an opaque bag. At the end of the experiment, a ball was selected from the bag. If the number on the ball was 50 or below, the lottery paid 30 SEK; otherwise zero.

After a task had been completed, the response sheets for that task were collected, and the next response sheet was handed out to prevent cheating. The subjects were given the goods and told to examine them before recording their offers. The sellers were told that they owned the good; the buyers were told that they could inspect the good but did not own it.

When all four tasks were completed, one task was chosen as "real," and the market price was drawn for that task. In all of the tasks, including the unpaid training round, the subjects were told that the market price would be randomly selected from a range of 1 to 30 SEK using the ping-pong balls. The market price was determined by picking one ball out of 30, each with a single price written on it. To avoid any bias that might result from the potential market price range, we used the 1–30 SEK range as a potential market price range for all of the tasks (see Bohm et al. (1997) for a comprehensive discussion of this issue).

At the end of the experiments, the subjects were given both a questionnaire requesting demographic information and test of their understanding of the instructions. Only the subjects who answered all quiz questions correctly were included in the analysis.

3.5 Incentives under Buyer-Seller Uncertainty Mechanism

Before presenting the results, it is useful to analyze the incentives under the new mechanism. In order to accomplish that we have to make three assumptions: the first assumption we make is that there are three possible cases or groups of people [A1]:

i. Individuals who have a precise estimate of their WTA and WTP and they exhibit no endowment effect therefore behave according to the Standard Economic Theory: The optimal response for them is obviously to state the precise estimate as a single point and they are allowed to do so in BS. Note that for this type WTP equals WTA.

ii. Individuals who have a precise estimate of WTA and WTP but exhibit loss aversion, therefore their WTA is higher than their WTP:

$$u\left(y_0 + WTA - \lambda X\right) - u\left(y_0\right) = 0 \tag{26}$$

$$u\left(y_0 - WTP + X\right) - u\left(y_0\right) = 0 \tag{27}$$

where y_0 is the wealth, X is the good in question and λ lambda is the loss aversion parameter. Obviously when $\lambda > 0$, we have WTA>WTP, individual exhibits a WTA WTP disparity. So far is standard in studies which explain endowment effect (WTA-WTP disparity) with loss aversion concept. However, under BS mechanism, individual does not know whether his or her role is buyer or seller in advance (both is equally likely, determined by a random mechanism). Optimal offer $u(offer^*)$ under this setting is given by:

$$u\left(offer^*\right) = \tfrac{1}{2}u\left(WTA\right) + \tfrac{1}{2}u\left(WTP\right) \tag{28}$$

Note that for type ii individuals, an optimal offer does not guarantee a positive payoff in all cases: Consider an individual who has a WTA of 10 and WTP of 5, thus states 7.5. Now suppose the randomly selected market price is 8 and the individual is designated as seller, randomly. Thus, trade occurs: the individual sells the good for 8 which is lower than 10 (WTA). However stating the mid-point is still optimal: optimal does not mean that the payoff will be positive in all states of the world; it means it is the best strategy among the possible ones. If the subject stated a bid equal to 10 which is the WTA, then there is a ½ probability that the subject would be a buyer: Subject would buy the good for 8, although WTP is 5, thus ending up having a loss of 3. Stating an interval is not optimal for this group, because in the buyer role the trade occurs if the market price is inside or below the stated range and in the seller role the trade occurs if the market price is inside or above the stated range. Obviously, the bounds which are valid for payoff determination is lower bound for selling and upper bound for buying. This rule ensures that the subject cannot state a selling price higher than the buying price, the best he or she can do is to minimize the expected loss and state the weighted average of his or her WTA and WTP, where the weights are probability of being a buyer and seller (in our case 1/2 for each).

iii. Now consider the case in which individual cannot come up with a precise estimate of his or her subjective value but a range from which cannot confidently state a single amount. For this case, we make another assumption

[A2] that individuals with imprecise preferences have "equivalence intervals" rather than having precise points of indifference between alternatives (MacCrimmon and Smith, 1986). This suggests that individuals assign interval values to the goods instead of precise points and individual is indifferent between the good and the values inside this range. For example consider an individual comes up with a range of values between 5 and 10 dollars but cannot state one of them confidently. This implies that individual is indifferent between \$5 and the good, \$6 and the good, and so on. For a theoretical discussion of the issue see Luce (1956) which discusses the notion of *just noticeable difference* and *semiordering*. To understand the intuition, suppose you are given several cups of tea which have different amount of sugar in it, and the difference between the cups are very small amounts such as 1 mg. You start tasting the cups of tea starting from the one which has the lowest amount of sugar to the highest one. You might not be able to distinguish the difference between them, therefore not be able to state your preference between cups, confidently. However, you would be able to state your preference between two cups confidently if the difference was large enough, which is called the *just noticeable difference*.

We develop a much simpler understanding of the imprecision range to demonstrate the incentives under *Buyer-Seller Uncertainty* mechanism. Denote the equivalence interval as $[L, H]$, where $L \leq H$ corresponds to the lower bound and upper bound of the range, respectively. For any good X, an individual stating $[L, H]$ as his or her subjective valuation for the good implies:

$$X \sim L \sim ... \sim H \tag{29}$$

Which means that, individual is indifferent between the good and the sure amount of monetary amounts between L and H.

Denote the surplus from the trade as $S(X, p) = u^{-1}(X) - p$ for buying task, and $S(X, p) = p - u^{-1}(X)$ for the selling task, where $p \in [a, c]$ is the randomly determined market price. For our experiment, a is 0 SEK and c is 30 SEK. Thus, the expected surpluses for buyer and seller role are[9]:

$$E[S(p, X)] = \int_0^b \left(u^{-1}\left(X\right) - p\right) f\left(p\right) \partial p \tag{30}$$

$$E[S(X, p)] = \int_0^b \left(p - u^{-1}\left(X\right)\right) f\left(p\right) \partial p \tag{31}$$

[9] The similar approach of analyzing the incentive compatibility of a mechanism can be found in Kaas and Ruprecht (2006). They analyzed BDM and Vickrey auction and we adapted their approach to *Buyer-Seller Uncertainty* mechanism.

where b is the stated bid of the individual and $f(p)$ denote density function of the market price. Note that in our experiment we employed uniform distribution, and market price can be any value from the range of [0 SEK, 30 SEK]. Figure A1 shows the surplus for each market price, separately in the case of buyer and seller roles. When the market price equals the values in the equivalence interval, surplus is 0, which follows from A2. To calculate the surplus outside the imprecision range, we need to make further assumption [A3]: we assume that individual takes the mean of the range as the benchmark for the good X, denoted by μ. This may seem problematic when we look at the issue from the standard view of "well behaved" preferences; however one should note that we are in the realm of preference imprecision which indeed implies that preferences are not "well-behaved" objects[10].

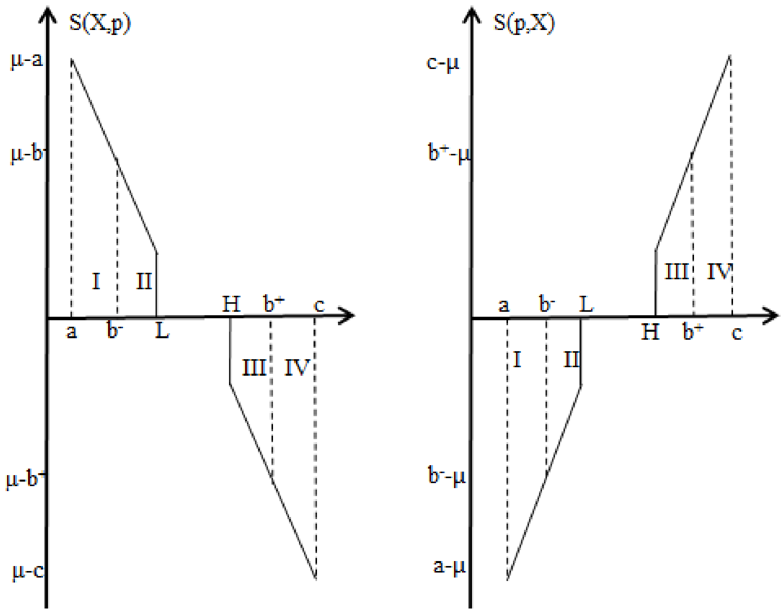


Figure 4. Trade surplus for individuals with imprecise preferences

[10] Note that the theoretical aspect of the issue is not central to our study, but one way to connect the nonstandard terminology with the imprecise preferences or equivalence intervals can be the following: Denote the *just noticeable difference* as $\eta = L - \mu = h - \mu$, therefore, if we see μ as the true subjective value, the equivalence interval is constructed around it by adding and subtracting η.

In Figure 4, left panel shows the surplus of buyer and the right panel shows the surplus of seller. Note that in *Buyer-Seller Uncertainty* mechanism, both roles have equal likelihood. After stating the bids, if the individual is assigned to the role of being buyer the payoffs are calculated according to left panel; whereas if the selected role is being seller the right panel is applicable. Remember that individual is allowed to state either as a point or interval, if individual stated a point and assigned to be a buyer; transaction takes place when the bid is higher than the randomly market price. This part is identical to the standard BDM mechanism. On the other hand if individual stated an interval, he or she buys the good when the randomly selected market price is inside or below the stated range. For the seller role, if individual states a point, selling transaction occurs when the market price is higher than the bid. Finally, if individual states a range, he or she sells the good when the market price is inside or above the stated range.

[L,H] is the privately known true interval, we see that it is weakly dominant strategy for individual to state either the true range, [L,H], or any single point from this interval. The expected surplus will be the areas I and II if the assigned role is being buyer, and III and IV if the role is being seller. Since we employed uniform distribution, we can suppress the probability part for simplicity.

Stating narrower intervals or points from the true range: However, if an individual with imprecise preferences, states a point from the true range or a narrower interval but keeping it inside the true range, still obtains I and II as surplus in the case of buying and III and IV in selling but he or she decreases the chances to buy or sell the good at a desirable price. Suppose the individual has a true range of 5 and 10 dollars but he or she overstates the lower bound such as a range between 7 and 10 dollars; if the individual is randomly assigned as being a seller at the end of the experiment, the individual loses the chance to sell the good for prices between 5 and 7 dollars. Now consider the case of understating the upper bound: Suppose the individual states a range between 5 and 8 dollars; if the individual is assigned as a buyer at the end of the experiment and the market price is between 8 and 10 dollars, individual misses the chance to buy the good for these prices which are inside the individual’s acceptable range.

Misstatement outside the true range: If individual understates the true value either as a range or point such as b^-, the surplus shrinks to area I for the buyer role and for the seller role it becomes III+IV-II which is lower than III+IV. If individual overstates such as b^+, the surplus from buying is I+II-III which is lower than I+II and the selling surplus shrinks to III from III+IV.

To summarize, individuals belong to either group i or ii, prefer to state precise points. Additionally some of the subjects who have imprecise preferences (group iii) might also prefer to state points. Because of this we conduct the test of PCH by eliminating the point responses too, but the results still confirm PCH. Looking at the analysis above, subjects who stated an interval definitely belongs to group iii, thus having imprecise preferences. Looking at table 2, the ratios in parenthesis in second column, we can say that more than half of the subjects have imprecise preferences except for good 3 for which the ratio is 46%. Considering the possibility of some subjects stating a point from their range although they have imprecise preferences, we can speculate that the ratio of subjects having preference imprecision can be even higher than the observed ratios.

As reviewed in Chapter 2, existing studies rely on the subject's self-reporting which is certainly not incentivized. Typically subjects are asked to fill a response table as in Table 7, where the first column includes sure amounts increasing incrementally. For each amount subject states his or her preference by choosing one of the three phrases which are in the remaining columns.

Table 7. *Example Response Table*

Certain Amounts	I definitely prefer the good	Not sure	I definitely prefer the certain amount
0	✓		
1	✓		
2	✓		
3		✓	
4		✓	
5			✓
6			✓

As an example, in Table 7, we see that for sure amounts equal and below 2, good is definitely preferred over the sure amounts for an imaginary individual. However, for the sure amounts, 3 and 4, the individual exhibits imprecision by stating "not sure" about his or her preferences. Finally, for the sure amounts 5 and 6, the individual definitely prefers the certain amount. Looking at this example response table, we say that the imprecision range corresponds to the value between 3 and 4. After the subject fills the response tables, usually a random mechanism draws a single amount from the imprecision range to determine the payoffs. Another procedure that can be used under this scheme is to let the subject determine the single amount chosen from the imprecision range. However, the disadvantage of this method is that subjects do not have a monetary incentive to reveal the true bounds of their imprecision range, if it exists. The reason is that the payoffs are determined by looking at a single amount either chosen randomly or by the subject from the imprecision range, and the information about the bounds of the imprecision range relies on the subjects' self-reporting.

We do not claim that *Buyer-Seller Uncertainty* mechanism is the perfect method for eliciting the imprecision range, but considering the hypothetical nature of the existing methods reviewed before, *Buyer-Seller Uncertainty* mechanism is superior in terms of incentive compatibility. We hope it finds the fruitful applications in the literature will be developed more in the future studies.

The following numerical examples are provided to help the reader to understand the incentives under this mechanism intuitively:

a. Overstating the lower bound: Suppose the individual states a range between 7 and 10 dollars; if the individual is randomly assigned as being a seller at the end of the experiment, the individual loses the chance to sell the good for prices between 5 and 7 dollars and remember that these are inside the true subjective valuation range (5-10 dollars).
b. Understating the lower bound: Suppose the individual states a range between 3 and 10 dollars. If the individual is randomly assigned as being a seller at the end of the experiment and the market price is randomly determined as some amount between 3 and 5 dollars, then the individual sells the good for an undesirably low price. Note that the true range is between 5 and 10 dollars.
c. Overstating the upper bound: Suppose the individual states a range between 5 and 12 dollars; if the individual is assigned as a buyer at the end of the experiment and the market price is between 10 and 12 dollars, individual has to buy the good for an undesirably high price.
d. Understating the upper bound: Suppose the individual states a range between 5 and 8 dollars; if the individual is assigned as a buyer at the end of the experiment and the market price is between 8 and 10 dollars, individual misses the chance to buy the good for these prices which are inside the individual's acceptable range.

3.6 Results

Summary statistics are reported in Table 8. The second column indicates the percentage of subjects that preferred to state intervals. Except for the BS_{int} group for good 3, a majority preferred intervals.

Table 8. *Summary Statistics*

	Treatment	Mean $_L$	Mean $_U$	Median $_L$	Median $_U$	σ_L	σ_U
Good 1							
(Premium bitter	B_{int} (71%)	17.3	**20.2**	18.0	**22.0**	6.6	5.4
chocolate)	S_{int} (83%)	**18.9**	21.8	**20.0**	22.0	4.5	4.7
	BS_{int} (62%)	**14.9**	**17.8**	**15.0**	**17.8**	3.2	3.9
	B_p	**14.3**		**15.0**		6.3	
	S_p	**13.5**		**14.0**		5.1	
Good 2							
(3 cans of Coke)	B_{int} (65%)	15.0	**18.4**	15.0	**18.0**	9.4	9.5
	S_{int} (56%)	**18.6**	20.5	**17.5**	20.0	7.2	7.9
	BS_{int} (62%)	**14.6**	**17.5**	**15.0**	**18.0**	6.7	6.6
	B_p	**13.9**		**15.0**		8.0	
	S_p	**19.1**		**18.0**		6.8	
Good 3							
(Chocolate)	B_{int} (75%)	13.3	**16.6**	13.5	**15.0**	5.6	4.5
	S_{int} (50%)	**19.0**	20.7	**17.0**	20.5	6.5	5.7
	BS_{int} (46%)	**14.3**	**16.2**	**15.0**	**16.0**	5.3	5.7
	B_p	**16.4**		**15.0**		6.4	
	S_p	**19.3**		**19.5**		6.8	
Good 4							
(Lottery ticket)	B_{int} (59%)	11.8	**14.5**	10.0	**15.0**	8.7	9.3
	S_{int} (61%)	**14.3**	17.2	**15.5**	18.0	6.9	7.4
	BS_{int} (54%)	**12.5**	**18.2**	**14.0**	**15.0**	6.2	6.7
	B_p	**12.5**		**11.0**		5.6	
	S_p	**20.2**		**20.0**		7.2	

Notes: The subscripts *L* and *U* denote the *lower* and *upper* bound, respectively. σ denotes the standard deviation. The values in bold are the incentivized ones. In the treatment column, the percentages in parentheses denote the portion of subjects stating a range of values for the specific task. Sample size for each treatment: B_{int}=17, S_{int}=18, BS_{int}=13, B_P=19, S_P=14.

Overall, statistical tests confirm both PCH and RFFH. To test the PCH, we should look at the first and second set of results in Table 9: For good 2, a ratio of 1.20 is significant with a p-value of 0.0449; the W statistic is 86.0 according to the Wilcoxon-Mann-Whitney rank sum test. For good 4, the ratio is 1.82, which is significant with a p-value of 0.0014 and a W statistic of 51.0.

Since the significance of the gap for Good 2 is on the edge (p-value=0.0449), we focus on good 4 (p-value=0.0014) and compare the point bids with the bounds that were elicited in the BS_{int}. The second set of results presents these comparisons, showing that the Wilcoxon-Mann-Whitney tests support our hypothesis: We cannot reject the hypothesis that the mean WTP in *points* and the mean lower bound of BS bids were drawn from the same distributions as the mean WTA in *points* and the upper bound of BS bids.

To examine the support for RFF, we look for the existence of the WTA-WTP disparity in *points* and its absence for *intervals*. For the *Points*, we observed a significant disparity for good 2 (p=0.0449) and good 4 (p=0.0014) (beverage and lottery ticket respectively). For the *intervals*, test results comparing the incentivized bounds (3.S_{int}^{L}/B_{int}^{U}) suggests that although the ratio of WTA to WTP is not exactly one, the difference in *Intervals* is not statistically significant.

Table 9.

		Wilcoxon-Mann-Whitney rank sum test (Null hypothesis: identical distributions)		
	Ratio[a]	W	*p*-value	Conclusion (α = .05)
1. S_p / B_p				
Good 1	0.93	141.5	0.6299	Cannot reject null
Good 2	1.20	86.0	0.0449	Reject null
Good 3	1.30	97.5	0.1005	Cannot reject null
Good 4	1.82	51.0	0.0014	Reject null
2. Good 4[b]				
B_p / BS_{int}^{L}	0,79	115.5	0.7699	Cannot reject null
B_p / BS_{int}^{U}	0,73	64.0	0.0211	Reject null
S_p / BS_{int}^{L}	1,43	145.0	0.0087	Reject null
S_p / BS_{int}^{U}	1,33	111.5	0.3233	Cannot reject null
3. $S_{int}^{L} / B_{int}^{U}$				
Good 1	0.91	176.5	0.7871	Cannot reject null
Good 2	0.97	168.0	0.6959	Cannot reject null
Good 3	1.13	122.5	0.1594	Cannot reject null
Good 4	1.03	148.0	0.4407	Cannot reject null

[a] Median ratios. [b] Two sided

In order to explore the power of our statistical tests we used the method of Plott and Zeiler (2005).We test the null hypothesis of WTA=2·WTP for the results obtained in the *intervals* treatment (See Table 10). The reason for multiplication by two is the same that Plott and Zeiler (2005) suggested. In the previous literature several authors claim that WTA is twice the WTP (e.g. Dubourg et al., 1994 and Knetsch et al., 2001). A t-test assuming unequal variances led to a rejection of the null in favor of the alternative, WTA < 2·WTP for all goods (See Table 4 first two columns). A two-sample Wilcoxon-Mann-Whitney rank-sum test gives the same result (See Table 4 last two columns).

Table 10. *Power of the tests*

Goods	T-test (Unequal Variances)		Wilcoxon-Mann-Whitney rank-sum test	
	T	p-value	z	p-value
1	-7.5642	0.0000	4.810	0.0000
2	-3.6934	0.0007	3.255	0.0011
3	-5.4225	0.0000	4.437	0.0000
4	-3.0386	0.0032	2.267	0.0234

Our design also enabled us to draw further conclusions which are useful for contingent valuation studies (See Table 11). We test whether subjects in buyer and seller role stated the true bounds for their valuations although only single bound is incentivized i.e. affect their final payoff. We compare these with the bounds elicited in *Buyer-Seller uncertainty* mechanism in which both bounds affect the subjects' final payoff from the experiment.

Additional Result 1: Binary comparison between BS group and buyers shows that, although the lower bound of buyers is not incentivized, subjects do not misrepresent (over or understating) their values both in terms of lower and upper bounds.

In order to see the support for this result, we should look at the first ("1. B_{int}^{L} / BS_{int}^{L}") and second ("2. B_{int}^{U} / BS_{int}^{U}") set of results. In the first set, the comparison is between lower bound of buyers (B_{int}^{L}) and that of the BS group (BS_{int}^{L}). In the second set of results, we consider upper bound of buyers (B_{int}^{U}) and the BS group (BS_{int}^{U}). Wilcoxon-Mann-Whitney test results support the hypothesis that the two samples come from identical populations.

Additional Result 2: Binary comparison between BS group and sellers shows that, although the upper bound of sellers is not incentivized, subjects apparently do not lie about their values (over or understating).

Consider the third ("3. S_{int}^{L}/BS_{int}^{L}") and fourth ("4. S_{int}^{U}/BS_{int}^{U}") set of results. In the third set, the comparison is between the lower bound of the sellers (S_{int}^{L}) and that of the BS group (BS_{int}^{L}); whereas, in the fourth set, it is between the upper bound of sellers (S_{int}^{U}) and that of the BS group (BS_{int}^{U}). Wilcoxon-Mann-Whitney test results again support the idea that the two samples come from the identical populations, except for good 1 (Wilcoxon-Mann-Whitney tests reject the null hypothesis with a p-value of 0.019).

Table 11. *Additional Results*

	Ratio[a]	Wilcoxon-Mann-Whitney rank sum test (Null Hypothesis: Identical Distributions) W	*p*-value	Conclusion (α = .05)
1. B_{int}^{L} / BS_{int}^{L}				
Good 1	1.20	146.5	0.1346	Can't reject null
Good 2	1.00	113.0	0.9325	Can't reject null
Good 3	0.90	89.0	0.5198	Can't reject null
Good 4	0.71	107.0	0.8994	Can't reject null
2. B_{int}^{U} / BS_{int}^{U}				
Good 1	1.24	145.0	0.1525	Can't reject null
Good 2	1.00	125.0	0.5568	Can't reject null
Good 3	0.94	103.5	1.0000	Can't reject null
Good 4	1.00	86.5	0.3197	Can't reject null
3. S_{int}^{L} / BS_{int}^{L}				
Good 1	1.33	171.5	0.0299	Reject null
Good 2	1.17	148.5	0.2126	Can't reject null
Good 3	1.13	158.5	0.0998	Can't reject null
Good 4	1.11	147.0	0.2328	Can't reject null
4. S_{int}^{U} / BS_{int}^{U}				
Good 1	1.24	175.5	0.0190	Reject null
Good 2	1.11	139.5	0.3739	Can't reject null
Good 3	1.28	161.5	0.0772	Can't reject null
Good 4	1.20	116.5	1.0000	Can't reject null

Notes: Rank sum tests are done by using R. The same analysis was also carried out with using different programs such as "Minitab", "Instat" and "Stata". Although p-values are slightly different across these programs, the outcomes about the hypothesis are same. All tests are two-sided. [a] Median ratios

3.7 Conclusion

Allowing subjects to state their sentiments using any interval on the line (of which a point is a special case) essentially has an effect on the observability of the disparity: When we use the conventional point response format, in line with Plott and Zeiler's (2005) findings, we observe disparity for the risky prospect, but not for the ordinary market goods[11]. Moreover, when we allow subjects to

[11] Except "three cans of coke" for which we observe a disparity significant on the edge (p=0.0449).

state intervals i.e. framing the response format as intervals, the gap disappears for all goods we used in our experiment.

As pointed out by Plott and Zeiler (2005), experimental procedures minimizing the subject misconceptions and misunderstandings are crucial. We have added the response format; taken together, this raises doubts about interpreting the disparity as an evidence for an endowment effect. In short, the endowment effect may not be the only explanation of the disparity, when we consider the total effect of selected experimental procedures.

In contrast to our results, Morrison (1998) observed a large gap between the two ranges; lower bound of WTA being more than one and a half times the upper bound of WTP. However, he did not use any procedures to minimize the subject misconceptions. In short, our results suggests that preference imprecision should not be discarded as a potential explanation of the observed anomalies. In the instructions, Plott and Zeiler (2005) included a guideline which explains subjects how to find their optimal offers: For example for buyers they suggest them to start thinking about a low amount such as 1 SEK, and ask themselves whether they are willing to pay 1 SEK for the good or not. If the answer is no, record 1 SEK as WTP. If the answer is “YES”, they are suggested to think about a higher amount such as 2 SEK, repeat the process until they reach an amount which they would not pay for the good and record that amount as WTP. This sequential process is similar to the “iterative bidding” scheme, but without an interviewer, in other words subjects interview with themselves silently. Similarly, Sayman and Onculer (2005) found that the disparity is lower in an iterative setting; the sequential process helps subjects to discover their optimal responses. Our results suggest that together with Plott and Zeiler procedures, allowing subjects to state intervals lead them think about each value more carefully like a sequential process, decrease tendency of biases and heuristics.

Consequently, many questions are left to be explored in more detail. For example, why do we observe a disparity for lottery tickets but not for ordinary market goods, when we ask for single amounts? How do individuals form admissible ranges? Why do buyers/sellers state different bounds? Thus, the area is fertile ground for development of new theory and additional testing. This could lead to an improved understanding of a long-standing controversy regarding the WTA-WTP disparity and potentially to the development of novel designs of survey instruments. Because the bulk of empirical research in e.g. social science is based on surveys, we do believe that there are good reasons to further explore the elicitation mechanisms studied in this paper.

Inspired by the results from this experimental study, I present a new decision theory for risk in Chapter 4, which incorporates the imprecision and is capable for explaining the anomalies detected in the literature.

4 Preference Cloud Theory

This chapter, an extended version of the working paper by Bayrak and Hey (2015), introduces a new theory for decision under risk that maintains the property of modelling the decision over final wealth levels, as in EUT. The theory is characterised as two stages: the first stage describes how individuals form the imprecision ranges, whereas the second stage is the selection of a single amount from that range taken as the criteria for the decision task.

Central to the first stage of the model is the incorporation of preference imprecision, which arises because individuals perceive each numerical probability only vaguely and therefore map them to a range of probabilities. The size of the range depends on how sophisticated the individual is in terms of understanding the probability concept, and thus depends on an inherent characteristic of the individual. For example, an individual with a great knowledge of probability and familiarity with the concept, exhibiting lower imprecision, will have a narrower range. Since each numerical probability is mapped to a range, this leads individuals to calculate a range of expected utilities for each risky prospect.

In the second stage, the problem can be seen as a form of decision making under ambiguity since the outcomes—the expected utility range from the first stage—are known, but the individual has no prior knowledge—probabilities are unknown—about which expected utility from the range is the true one, i.e., the individual cannot confidently determine a single expected utility from the range. Therefore, the individual forms beliefs and calculates the weighted average of the range according to those beliefs. Preference Cloud Theory (PCT) uses a simple formulation for the beliefs, which is similar to the Hurwicz's alpha criterion.

Section 4.1 sets the background for the new theory, and Section 4.2 presents the original version of it. We also considered alternative modelling schemes for PCT in Section 4.3. Section 4.4 shows how PCT can explain the anomalies;

and finally Section 4.5 presents a discussion about the new theory and compares it with the other theories.

4.1 Introduction

In order to understand the intuition of the conceptual framework, consider the following example: suppose you are asked to state your subjective value for a lottery ticket which gives $10 with a probability of 0.3 and zero otherwise. Can you pin your value down to a single precise number or do you end up with a range of values? Some, especially who are familiar with decision theory, might be relatively more likely to pin it down to a precise amount, but the ordinary person (the majority) is more likely to come up with a range. However, in real life we don't pay and get paid in terms of ranges, therefore while modelling the preference imprecision we also have to answer the following question: How do we decide whether to buy a good in the market which has a precise price tag on it when we have imprecise values for the good?

We assert that when individuals think the true subjective value of the gamble is somewhere within a range but cannot confidently state a single amount from this range, they form beliefs about the distribution of the true subjective value within this range. As they don't know the probability distribution of their 'true' subjective value inside this range, the situation they are in can be seen as decision under ambiguity: known outcomes but unknown probabilities for them. This scheme can explain the valuation gap too: suppose that you are endowed with this gamble and asked to state your WTA, what value are you more likely to state from this range? It is more likely that you will state a value close or equal to the upper bound of the range, conversely if you are assigned to be a buyer you are more likely to state a value from the lower bound of the range.

The name of the theory is inspired from the Electron Cloud Model, a product of quantum mechanics wherein electrons are no longer depicted as particles moving around the nucleus in a well-defined orbit. Instead, their probable location around the nucleus is described as a cloud that represents most probable regions with fuzzy boundaries. On the other hand, its predecessor, Newtonian mechanics, claims to predict both the location and the momentum of a particle with certainty. Quantum mechanics is devoid of that luxury. In Figure 5[12] the left panel depicts the classical model of the atom where the electron is a precise particle and travels along the well-defined orbit around the nucleus. On the other hand, next to it the Electron Cloud Model of an atom shows the electron as a fuzzy region instead of a particle.

[12] http://faculty.wcas.northwestern.edu/~infocom/The%20Website/plates/Plate%201.html

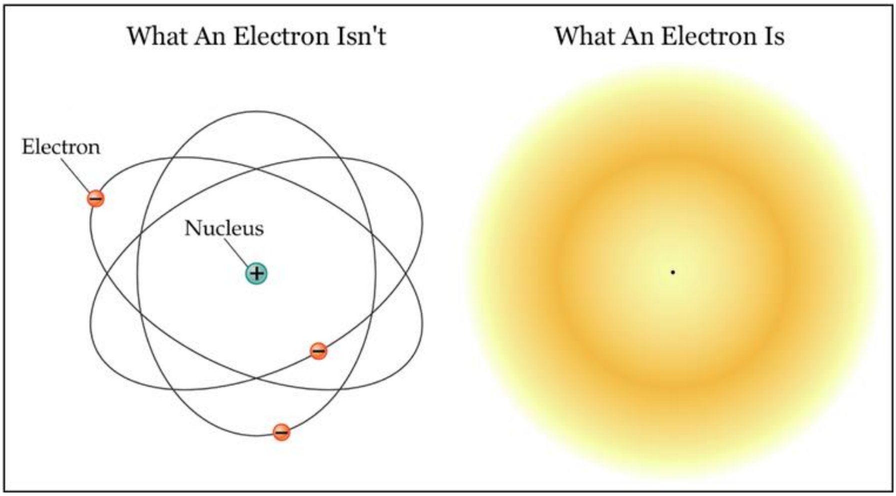


Figure 5. Physics vs. economics

Readers should note that we provide this example in order to give the intuition of our theory, and to help the reader with a visual. There is no resemblance or link between PCT and Quantum mechanics in terms of their mathematical and technical aspects. Standard economic theory and alternative theories such as Prospect Theory and its variants claim precision in individual preferences, i.e., they assert that individuals can confidently pin down their subjective valuations of a good to a single precise amount; likewise Newtonian Mechanics models the behaviour of electrons and claims to predict the location and the momentum of the particle with certainty.

Analogous to quantum mechanics, there is another line of literature in economics that asserts that people might have imprecise preferences and cannot articulate their subjective valuations of the goods precisely (see Chapter 2 for a detailed discussion and review). The idea of imprecision goes back to the 1950s and can be found in the stochastic choice models.[13] However this view sees imprecision as noisy preferences.[14] As psychological mechanisms related to two concepts, noisiness and imprecision can be seen as connected; noisiness relates to the errors that subjects make (Harless and Camerer, 1994; Hey and Orme, 1994), while imprecision can be viewed as 'incommensurability' (Cubitt et al., 2015) or indecisiveness. Moreover, for the latter there is an accumulating literature where the main finding is that

[13] The older literature includes prominent papers by, for example, Quandt (1956), Luce (1959, 1958), Block and Marschak (1960) and Becker et al. (1963).

[14] Harless and Camerer (1994), Hey and Orme (1994), Loomes and Sugden (1998, 1995) and includes Loomes et al. (2002), Hey (2005), Loomes (2005), Blavatskyy (2007) and Wilcox (2008)

individuals often exhibit imprecision (Cohen et al., 1987; Butler and Loomes, 1988, 2007, 2011; Dubourg et al., 1994, 1997; and Morrison, 1998). For example, Butler and Loomes (2007) elicited preference strength in a pairwise choice experiment, for each task individuals stated their preferred gamble and revealed their preference strength by simply choosing one of the four phrases: ‘I definitely prefer A’, ‘I think I prefer A but I’m not sure’, ‘I think I prefer B but I’m not sure’, or ‘I definitely prefer B’. The results favour imprecision as an account for preference reversals. Butler and Loomes (2011) take this account further by asserting that imprecision might explain other observed anomalies of EUT. This assertion has crucial importance because imprecision might have been the explanation of the anomalies *ab initio*, especially after the literature’s focus on explaining the anomalies with precise but non-standard preferences for at least the last four decades (e.g., Original, Cumulative, and Third-Generation Prospect Theory etc.).

However, the current status of the preference imprecision literature is about allowing subjects to state how sure they are about their stated values or choices. The related literature does not provide a preference functional or a formal theory that incorporates preference imprecision; we provide formalisation of the theories floating around in the soft form and a mathematically tractable model. Instead the literature focuses on modelling the imprecision as a stochastic component of a deterministic theory, as reviewed in Chapter 2.

Throughout this section, the focus is on decision under risk, which has two elements: probability and outcome. Therefore imprecision can arise at the perception process of one or both of these elements. Perception in psychology is defined as the ‘conscious recognition and interpretation of sensory stimuli that serve as a basis for understanding, learning, and knowing or for motivating a particular action or reaction.’[15] One reasonable claim is that it is more likely that the imprecision comes from the probability element rather than the outcome element. Suppose an individual is asked to value a gamble that gives \$10 with a probability of 0.40 and zero otherwise. Individuals can interpret the monetary outcome easily as, e.g., the cost of a lunch. In other words, \$10 is \$10. Yet, the perception of probabilistic information is convoluted for the ordinary individual.

4.2 Original Version of Preference Cloud Theory (βα model)

In the original version of the PCT (βα model), we assumed that imprecision arises due to the decision maker’s vague understanding of the numerical

[15] Perception (psychology). (n.d.) Mosby's Medical Dictionary, 8th edition. (2009).

objective probabilities involved. The empirical support for this assertion comes from the psychophysics literature; see Budescu et al. (1988). In their experiment, subjects were asked to state bids for lotteries where probabilities were represented numerically, graphically, or verbally. The results suggest that bids and attractiveness ratings are almost identical for the different representations of the same lotteries (see Budescu and Wallsten (1990) and Bisantz et al. (2005) for further evidence). Wallsten and Budescu (1995) explain that the similarity of behaviour under different representation modes is due to similarities in the vague understanding of numerical and verbal representation of probabilities. We therefore argue that a numerical, objective probability is perceived as corresponding to a range of probabilities and that individuals use this range in their calculations[16].

Zimmer (1984) introduced a useful insight from an evolutionary perspective: he noted that the numerical probability is a relatively new concept, appearing as recently as the 17th century. However, people were communicating uncertainty via verbal expressions long before probability was codified in mathematical terms. Zimmer further suggested that people process uncertainty verbally and make their decisions based on this processed information, not on the numerical information. We therefore assume that decision makers perceive each numerical and objective probability in a vague manner and the perceived versions can be modelled as if they map any given objective probability into an interval. This implies that individuals end up with a range of expected utilities (EUs) and they do not have prior knowledge about their 'true' EU from this range. For the second question, pinning down this range to a single value can be modelled as a decision problem under ambiguity. We use the Alpha Model (embodying Hurwicz's criterion) to provide a valuation of the prospect, given as the weighted average of the worst and the best possible EU.

Consider the following bet, which gives \$100 with a probability of 0.3 and zero otherwise $K:\left(x_1=\$0,\pi_1=0.7;x_2=\$100,\pi_2=0.3\right)$. As mentioned in the introduction, individuals perceive the objective numerical probabilities in a vague way, therefore they map each probability to a range: $[\pi-\beta,\pi+\beta]$. Imprecision level $\beta\left(\pi,\phi\right)$ is a function of objective probability $\left(\pi\right)$ and the individual specific sophistication parameter $\left(\phi\right)$. Figure 6, presents an illustrative example of different imprecision levels derived from different sophistication levels (depicted with different curves) and for different probabilities. A relatively unsophisticated individual would display a relatively high β, resulting in more imprecise preferences. For example, stock brokers

[16] Verbal expressions include 'rarely', 'very likely' etc. Each expression can be interpreted as a range of probabilities that may vary from individual to individual.

and gamblers who are expected to be more familiar with the concept of probability exhibit lower imprecision than the ordinary individual.

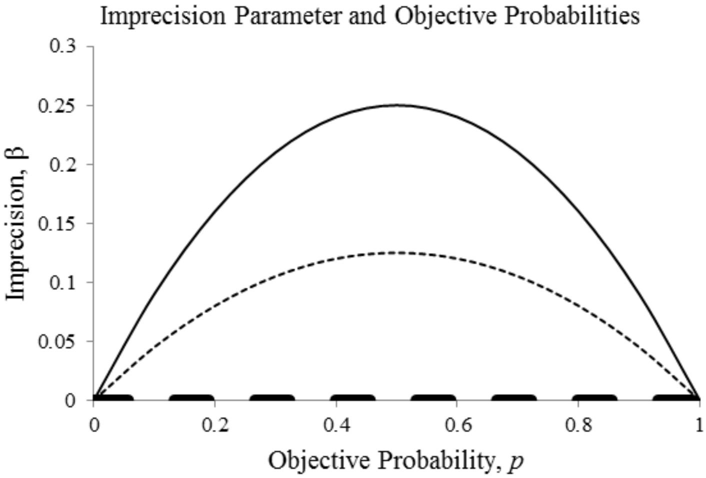


Figure 6. Imprecision parameter and different sophistication levels

PCT assumes there is not imprecision if the probability is 0 or 1 since the events occurring with these probabilities are not probabilistic events in daily language, that is, the event either never happens or always happens. Therefore the perception of these probabilities is a relatively easy cognitive task compared to the perception of 0.5, because it implies the event is neither likely nor unlikely, and this 'incommensurability' makes it difficult to derive a meaning from this probability. Therefore PCT assumes imprecision reaches its maximum if the probability is 0.5. Finally, for simplicity $\beta(\pi,\phi)$ is assumed to be symmetric around $\pi = 0.5$.

For our simple lottery example, consider that an individual with a zero initial wealth has an imprecision level of 0.1 for the winning probability of 0.3 in the previous example. This is interpreted as '0.3 as perceived by this individual' by mapping 0.3 to the range: $[0.2, 0.4]$. Next the individual calculates the lower bound of the risky prospect's expected utility by allocating $0.2\left(\pi_2 - \beta = 0.3 - 0.1\right)$ to the winning state and the remaining probability 0.8 to losing state $\left(1 - \pi_2 + \beta = 1 - 0.3 + 0.1\right)$. Similarly, the upper bound is when 0.4 is allocated to the winning state and 0.6 to the losing state (without loss of generality, normalise: u(100)=1 and u(0)=0)). Thus the vague perception causes the individual to end up having a range of expected utilities with the following lower $\left(EU_L\right)$ and upper $\left(EU_H\right)$ bounds:

$$EU_L(K) = u(100)0.2 + u(0)0.8 = 0.2 \tag{32}$$

$$EU_H(K) = u(100)0.4 + u(0)0.6 = 0.4 \tag{33}$$

The second step of PCT includes the selection of a single expected utility from this range as the criterion of decision making: the individual's problem is that the 'true' EU lies somewhere in this range, but the individual does not have information about the distribution of it, thus this step can be seen as decision under ambiguity. PCT models this process of selecting one expected utility from the range by a criterion similar to Hurwicz's α -criterion. α is the weight attached to the worst case and can be seen as pessimism parameter. For simplicity, we assume it is universal and individual specific. In other words, an individual exhibits the same α for all decision problems, and that can vary from individual to individual.

Therefore the expected utility that the individual considers for this lottery under PCT, $\alpha EU(K)$ is calculated as follows:

$$\alpha EU(K) = \alpha EU_{worst}(K) + (1-\alpha) EU_{best}(K) \tag{34}$$

Another way of interpreting this step is that the individual is playing an ambiguous binary lottery where expected utility from K is either 0.2 or 0.4, and the probability of each outcome is unknown to the individual. At this stage of the decision problem there are two states of the world: 'High Utility' and 'Low Utility'. In the first state, K provides a utility of 0.4 whereas in the second state it provides 0.2. The crucial point about the theory is that unlike EUT or theories that assume procedure invariance, PCT allows for different expected utilities to be withdrawn from the admissible range for different type of tasks such as choice, buying, and selling.

When individuals are presented with a gamble, they are most likely to end up having a range of subjective values. Withdrawing a single amount from this range to be the criterion of the decision making for the individual depends on the task presented: if it is a buying task the individual would select a value closer to the lower bound whereas the opposite is true for the selling task. Thus an individual sees the worst case for a buying or choice task as being the lower bound of this range as the upper bound for a selling task. Note that we employ pessimism/optimism concepts to formulate the individual's belief about which is the 'true' expected utility of the good in the imprecision range. However, in order to understand how PCT predicts individual's withdrawal of different values from the imprecision range depending on the task, we need to first understand the pessimism/optimism concept.

These concepts are not defined over the risky prospects' bounds of imprecision range directly; instead they are defined as the weights attached to the worst and best cases, the final wealth levels that the individual is likely to reach. The nuance is that the lower bound of the imprecision range is not always considered as the worst case. Being worst and best cases depend on the task type, whether it is a buying or selling task. Therefore $EU_L(.)$ and $EU_H(.)$ is determined at the first period depending on the good and the imprecision parameter. At the second stage of the PCT, the individual decides which bound is the best case and worst case depending on the final wealth levels to be reached, which is determined according to the task type. For a buyer, the worst thing is that the good has a utility equal to $EU_L(.)$, whereas for a seller the worst thing is that the good has a utility of $EU_H(.)$. Therefore the ranking of the bounds is different under PCT for different tasks. It also is possible to articulate the intuition in different way: for a buyer, buying a good which has a high quality is better than buying the one which has a lower quality. For a seller, giving away a higher-quality good is worse than giving away a lower-quality good.

Table 12 shows how individuals assign best and worst cases depending on the task type by focusing on the example of lottery K.

Table 12. *Worst and best cases under Preference Cloud Theory*

Task Type	Initial wealth	Worst possible final wealth	Best possible final wealth	Desirability ranking of the bounds
Buying	M	$M+u^{-1}(0.2)$-WTP	$M+ u^{-1}(0.4)$-WTP	$EU_H(K) \succ EU_L(K)$
Choice	M	$M+ u^{-1}(0.2)$	$M+ u^{-1}(0.4)$	$EU_H(K) \succ EU_L(K)$
Selling	M	M+WTA- $u^{-1}(0.4)$	M+WTA- $u^{-1}(0.2)$	$EU_L(K) \succ EU_H(K)$

Task types such as choice, buying, and selling are shown in the first column whereas M denotes the initial wealth shown in the second column. The third and fourth columns are the worst and the best possible final wealth levels, respectively. In the buying task, for an individual with an initial wealth level, M, the worst case is to pay WTP, and the true expected utility of the good is 0.2 which is the lower bound of the imprecision range formed in the first stage of PCT. On the other hand, for the selling task in which the individual is endowed with the lottery ticket M, the worst case is different: the individual receives the WTA and gives away the good which has an expected utility equal to 0.4, which is the upper bound of the imprecision range. For the choice task, the worst case is similar to the worst case from the buying task: an individual has an initial wealth of M and the good has an expected utility equal to the lower bound of the imprecision range, 0.2. The last column shows the desirability of the imprecision bounds formed in Stage 1 of PCT according to the final wealth levels by simply comparing the third and the fourth columns. For the buying task, the upper bound of the imprecision range is more desirable since $M+u^{-1}(0.2)-WTP<M+u^{-1}(0.4)-WTP$ for all $WTP\geq 0$; similarly for the choice task $M+u^{-1}(0.2)<M+u^{-1}(0.4)$. On the other hand, for the selling task where the good is to be given away, the lower bound is more desirable since $M+WTA-u^{-1}(0.4)<M+WTA-u^{-1}(0.2)$, for all $WTA\geq 0$. Therefore, the desirability of the imprecision bounds are not done according to the bounds, but instead according to the resulting final wealth levels. As a result of this, individuals view buying and selling differently under PCT.

4.3 Alternative Frameworks

In this section we present alternative modelling frameworks for both stages of PCT. These extensions provide different ways of understanding preference imprecision and its behavioural foundations under PCT. A possible future study is to test the relative performance of these schemes empirically with a similar approach used in the stochastic preferences literature (reviewed in Section 2.2). Another possible research question can be centred on explaining the anomalies and testing and comparing the existing theories together with the models of PCT in terms of predicting the anomalies. In order to do that, a binary choice experiment must be used to estimate the parameters of the models, and then these parameters are used to predict the behaviour of the individuals in settings such as preference reversals, valuation gap, and the Allais Paradox. Finally, comparison between the predicted and actual behaviour observed in the anomalies' setting might be compared to make the necessary assessments.

4.3.1 Fixed Bucketing

In the previous sections we modelled the first stage of PCT by assuming that individuals perceive numerical objective probabilities in a vague way: each numerical objective probability is mapped to a range of probabilities modelled by β (imprecision parameter). In the fixed bucketing scheme, individuals use the verbal correspondences of the numerical objective probabilities and calculate the EU of the goods according to the verbal correspondences.

In daily language most people use the phrases such as 'most likely', 'less likely', or 'you never know' to express the probability or randomness of an event. It is very rare to see people communicating the probability of an event as ‘with a probability of 0.4’; while most people prefer to receive information about the probabilities of chance events quantitatively, they prefer to express such information qualitatively. One explanation for this is that individuals’ cognitive capacity is more suitable for the qualitative correspondence of probabilities, not the numerical ones, because the former is more natural and familiar (see Zimmer, 1983). Quantitative probability concepts date back only to 17th century, whereas human beings have been communicating and dealing with uncertainty for thousands of years. It is natural to assume that individuals are more familiar with the verbal expressions of probability rather than the numerical ones. For ordinary individuals, it is rather difficult to perceive what a probability of 0.4 means, however it is relatively easy to understand the verbal representations such as ‘less likely’.

The standard economic view assumes people can understand probabilistic information, so they know what 0.36 or 0.70 means and they can distinguish the difference between 0.30 and 0.36. This is a strong assumption because it expects the ordinary person to understand the frequentist approach, e.g., that a probability of 0.4 should be understood as the event occurs 4 out of 10 times. An individual who is more sophisticated and familiar with probability concepts such as gamblers or stockbrokers can understand the mathematical expression, as mentioned before, however for the ordinary person, the majority of people, it is difficult. Without an understanding of the numerical information, how can a person use it in expected utility calculations? This problem is not specific to the EUT, indeed, all of the existing decision theories for risk incorporates the probability in a precise manner, even the alternative models reviewed in Chapter 1. These theories assume that individuals understand the numerical probability, but overweight or underweight it and then use the transformed version of the probabilistic information. But, the transformed version of the probabilistic information is also a precise number.

In the fixed bucketing scheme, we assume individuals interpret the probabilities according to their predefined buckets, which are verbal correspondences such as ‘less likely’, ‘likely’, ‘more likely’, etc. In addition, each correspondence is defined as a range of probabilities. Thus, the number of verbal correspondences that an individual is able to define spans the unity probability line, such that if we add the individually defined verbal correspondences they will cover all the probabilities from 0 to 1. Similar to the model suggested in Section 4.2, we assume that the number of verbal correspondences that an individual can define depends on how sophisticated the individual is about probability concepts and how familiar with the nature of uncertainty. For example, an individual who is not familiar with the nature of probability might be able only to define two buckets: for the events that occur with a probability less than 0.6 individuals sees them as *less likely* events and *highly likely* for the events which occur with a probability higher than 0.6. Thus, if a lottery pays out \$10 with a probability of 0.4 and pays 0 otherwise, an individual derives a meaning to be used for utility calculations by assigning it to the corresponding bucket. The probability of winning is perceived as less likely because it is between the corresponding boundaries for the bucket of 'less likely' (0,0.6). On the other hand, an individual who is highly sophisticated in probability concepts might be able to define more buckets for the unity probability range, 0 to 1. As the number of buckets that an individual can define increases, the imprecision decreases.

One problem with this framework is that it is difficult to extend it to lotteries with more than three outcomes. Another problem is the violation of monotonicity: the individual who can only define two buckets will end up having the same expected utility range for the two lotteries: A gives \$10 with a probability of 0.4 and zero otherwise, and B gives \$10 with a probability of 0.5 and zero otherwise. We can overcome this problem by assuming an editing phase, similar to that of the Prospect Theory, in which individual eliminates the stochastically dominated options before calculating the Expected Utility range. Another possible solution can be assuming the pessimism parameter not only depends on individual characteristics but also on the winning probability of the lottery. Thus, the pessimism parameter employed to withdraw a single amount for B will be lower than for A since the winning probability of B, 0.5, is higher than A, 0.4.

4.3.2 $k\sigma$- Model for Imprecision Range Formation

In this section, we depart from the probability perception argument for forming the imprecision range; instead we assume that individuals take dispersion into account, which can be measured simply with standard deviation. The idea of dispersion affecting utility is not a new idea: Allais (1979) proposed a model in which the expected utility depends on the variance of the risky prospects. Moreover, Hagen (1979) incorporated the third moment of utility, i.e., the skewness. The experimental evidence provided by Butler and Loomes (1988) find that the higher the variance of a lottery, the broader the admissible range of valuations for a lottery (see Chapter 2 for details). Taking this experimental evidence into account we assume that the imprecision range is proportionate to dispersion. Thus, for any lottery *X*, the bounds in the first stage of the PCT are calculated as:

$$EU_L(X) = EU(X) - k \cdot u(\sigma) \tag{35}$$

$$EU_H(X) = EU(X) + k \cdot u(\sigma) \tag{36}$$

where $k \geq 0$, a measure of an individual's ability to be precise about preferences. Notice that the individual has precise preferences and behaves in the way that standard theory predicts when $k = 0$. As k increases, the imprecision range also increases. This parameter has a similar intuition regarding the sophistication level of the individual as in the original version of PCT, however in this scheme, imprecision is assumed to be not only caused by the probability, but also by the outcome. This scheme might seem counterintuitive at first, but if we consider the second stage of the PCT as well, the picture becomes clearer: a pessimistic individual will withdraw an amount close to the lower end of the imprecision range $EU(X) - k \cdot u(\sigma)$, whereas an optimistic individual will be closer to the upper bound $EU(X) + k \cdot u(\sigma)$. The optimists will attain extra utility of $k \cdot u(\sigma)$ from how much dispersion the prospect has, because they see the dispersion as the opportunity not to be missed: they see the glass half full. The pessimists want to avoid dispersion, because the dispersion would cause them a disutility of $k \cdot u(\sigma)$: they see the glass half empty.

The advantage with this scheme is that it is easy to extend the theory to the cases that include more than two outcomes, whereas to extend the original version of PCT, the rank-dependent cumulative probability transformation technique can be used. On the other hand, under the $k\sigma$ model the extension is easy and straightforward, since the bounds are formed around the standard expected utility of the lottery, $EU(.)$ by adding and subtracting $k \cdot u(\sigma)$.

4.3.3 Multiple-Selves and Intrapersonal-Planner Approach

In this section, our focus is providing an alternative framework for the second stage of PCT in place of the pessimism/optimism approach of the original version of PCT.

In order to achieve this, it is first useful to discuss how problematic is to represent the imprecision with the standard preference relations ($\sim, \succ, \succsim$)which we argue is not an adequate way to represent the preferences in the case of imprecision. It is not sufficient because it does not reflect what exactly is happening inside the imprecision range. The data collected in the experiments related to preference imprecision usually takes the following form: individuals make binary choices and state how sure they are about their choice (Butler and Loomes, 2007). In this kind of task one option is usually the risky prospect whereas the other is a sure amount of money. Alternatively, in valuation experiments, an interviewer asks subjects whether they are willing to pay the amount or not and also asks how sure they are (Dubourg et al., 1994). The process continues iteratively for a series of amounts. Data produced by this method includes a lower bound that the subject is definitely willing to pay and an upper bound that the subject is definitely not willing to pay. Thus the values in between constitute the imprecision range. Standard preference relations are incapable of representing the data elicited in the preference imprecision literature, as they do not include such information about how sure or how much an individual is willing to pay a particular amount stated in the imprecision range, e.g., 'I am 80% sure that my WTP is $20'.

For example, an individual thinks that the WTP for good X is a range between $5 and $10, but the individual cannot state a precise amount confidently. Therefore, in the standard way, one can interpret the admissible range of this individual's WTP for good X as:

$$u(X) \sim u(5GBP),...,u(X) \sim u(10GBP) \tag{37}$$

This is also problematic because it leads us to the following conclusion:

$$u(5GBP) \sim ... \sim u(10GBP) \tag{38}$$

This suggests that for the individual there is no difference between any amount of money between $5 and $10, which is not plausible at least from the monotonicity assumption, i.e., individuals should not be indifferent between different amounts of money, they should always prefer more to less.

To overcome the problem with the standard way of representation and incorporate the type of data that is collected in preference imprecision experiments (see Section 2.3 and 2.4), we suggest a different scheme that captures the difference for the each value within the imprecision range by incorporating the level of willingness, denoted as *w*.

To accomplish that, we need to define a few more concepts related to our approach that is, seeing the imprecision range as the collection of the subjective valuations by multiple selves within a self. For example, suppose a decision maker ends up having a range of expected utilities for good X equal to the utility of the range between $5 and $10. For simplicity, assume that the smallest monetary increment is $1, therefore the range implies that there are six selves within a self (decision maker), which compete with each other in terms of the true subjective value of the good. For example, the most generous self thinks that the good is worth $10, whereas the most parsimonious self thinks the good is only worth $5. Under this scheme, the decision maker acts as an intrapersonal planner, which is analogous to the social planner of welfare economics. For simplicity, assume the individual weights each multiple self uniformly. Therefore, each self has equal importance for the decision maker, but this assumption can be relaxed. Next, define the level of willingness as the ratio of the multiple selves who agree with the decision maker's ultimate decision to the total number of multiple selves. Consider the previous example, where the individual have a range of subjective valuations between $5 and 10. If the market price is $7, it means multiple selves who value the good at less than $7 are not agreed on the price, but the ones who value the good at equal to and above $7 are convinced to buy the good, thus the level of willingness will be 4/6 for $7 and can be denoted as $u(X) \sim_{4/6} u(7GBP)$ where the subscript of the preference relationship is the level of willingness $(\sim_w)$. The preference relation can be represented in the following way:

$$u(X) \sim_{1/6} u(5GBP), \ldots, u(X) \sim_{1} u(10GBP) \tag{39}$$

Each decision maker has a required level of willingness, which can be seen as an inherent characteristic of the individual's personality: some individuals take actions when they are 50% or less confident about it, however some prefer to act in a more rigorous way and want to be totally convinced so that their required level of willingness is 100%. Consider the same example again, and suppose the individual's required level of willingness is 100%, and then the individual acts only if the decision satisfies all of the multiple selves. Thus, if the market price is $7, the individual will not buy the good, since it is not a market price that convinces all of the multiple-selves, the ones who value the good below $7. To make the intuition clearer, consider the English idiom 'having second thoughts', which means feeling doubts about the decisions you have made or about to make. In our conceptual framework, the second thoughts are the thoughts of the multiple selves who are not convinced about the decision such as paying $7 for the good. The multiple selves who value the good less than $7, will cause individual to feel doubt about his or her decision. If the required level of willingness is 100%, the individual does not like to have second thoughts, but for others, the confirmation of a certain majority of the multiple selves is sufficient.

We also, for simplicity, assume that for all types of tasks such as buying, selling, and choice, individuals have the same required level of willingness. However, extensions that assume different levels for different tasks are also possible. Consider an individual who has an unfortunate experience with buying a good in the past, becomes more meticulous as a result, and has a higher level of willingness for buying compared to other types of tasks such as selling and choice. Similarly, an individual might be an inexperienced seller and wishes to be totally convinced before setting his or her valuation for the good, therefore employing a higher level of willingness. This modelling scheme can also explain some real life situations: property advertisements tagged as ‘urgent sale’ signalling that the price is lower than the ‘normal’ price. In this case, we can explain the sellers’ situation with the decreased required level of willingness due to the urgent needs; therefore the price that is put on the advertisement is lower than the price that the owner would post in normal circumstances. We can extend the model by allowing the required level of willingness to depend on other factors as well.

Formally, we introduce the notations and definitions as follows. For simplicity, we assume that the smallest monetary increment is one. The first stage of the PCT gives an imprecision range between $EU_L(.)$ and $EU_H(.)$, for a good.

$$\{v_i \in V \subset \mathbb{Z}_+ \mid EU_L(.) \le u(v_i) \le EU_H(.)\} \forall i \in M \subset \mathbb{Z}_+ \tag{40}$$

The set M is the list of all multiple selves; v_i is the subjective valuation of the i^{th} multiple self which is an element of V, the collection of the subjective valuations of all multiple selves; and the elements are determined by the first stage of PCT. The elements of V are ordered in the following way: $v_i < v_j$ such that $i > j$.

In the new scheme for any good x and y, $x \succsim_w y$ if and only if $u(x,w) \ge u(y,w)$, where $u(.,w)$ is the baseline utility for a required level of willingness, w. The baseline utility of x is the utility of a baseline degenerate lottery (gives the same certain amount of monetary payoff for all states of the world) and when it is compared with x, x is preferred by $w \cdot n(M)$ of the multiple selves. It can be formally defined as:

$$\sum_{i=1}^{N} f\left(v_i, u^{-1}(x,w)\right) / n(M) = w \tag{41}$$

where $f\left(v_i, u^{-1}(x,w)\right)$ is a function which takes the value 1 when a self weakly prefers x over the baseline degenerate lottery and 0 otherwise.

$$f\left(v_i, u^{-1}(x,w)\right) = 1, \text{ if } u(v_i) - u(x,w) \ge 0 \tag{42}$$

$$f\left(v_i, u^{-1}(x,w)\right) = 0, \text{ if } u(v_i) - u(x,w) < 0 \tag{43}$$

For example, for two lotteries x and y, an individual ends up having the following range of expected utilities ($EU(.) = [EU_L(.), EU_H(.)]$) in the first stage of PCT:

$$EU^{-1}(x) = [3,6] \tag{44}$$

$$EU^{-1}\left(y\right)=[3,10] \tag{45}$$

Thus for x, there are four multiple selves, whereas for y, there are eight multiple selves:

$$M_X = \{1,2,3,4\} \tag{46}$$

$$M_y = \{1,2,3,4,5,6,7,8\} \tag{47}$$

The sets of subjective valuations for each good can be shown as:

$$V_x = \{3,4,5,6\} \tag{48}$$

$$V_y = \{3,4,5,6,7,8,9,10\} \tag{49}$$

Next, suppose the individual has a required level of willingness equal to 0.5. In order to predict whether x or y is more attractive for the individual, we have to find the baseline utilities for x and y for $w=0.5$. For x, it is 5 since when it is compared with 5, half of the multiple selves of the individual prefers x or 5. For y, it is 7 because multiple selves who value the good for at least 7 constitute half of the total number of multiple selves. Now we can state that the individual prefers y over x $\left(y \succ_{0.5} x\right)$ since $u\left(y,0.5\right) \geq u\left(x,0.5\right)$; $\left(7>5\right)$.

This scheme allows WTA and WTP to be different because at the same level of willingness the maximum buying price and minimum selling price are different, if the individual has imprecise preferences. Before demonstrating that, we need to redefine the WTP and WTA concepts for our framework: WTP is the maximum amount that $w \cdot n(M)$ of the multiple selves are willing to pay in exchange for the good; similarly, WTA is the minimum amount that $w \cdot n(M)$ of the multiple selves are willing to accept to give away the good. Consider the example in which the individual articulates the EU equivalent to the utility of \$1 to \$5 and individual has a required level of willingness $\left(w\right)$ equal to 4/5. Table 13 shows an analysis of the buying and selling decisions under this scheme; columns list the values inside the imprecision range, whereas multiple selves are listed in each row. The cells in the table show the responses of each multiple self for different WTA and WTP amounts in the imprecision range respectively, for example, the cell written in bold letters shows that the third self who thinks that the good is worth \$3 is willing to buy it for \$2 but is not willing to sell it for \$2.

Table 13. *Buying and selling responses of multiple selves for each value in imprecision range*

V_i: valuation of each self	WTP/WTA				
	$1	$2	$3	$4	$5
$1	YES/YES	NO/YES	NO/YES	NO/YES	NO/YES
$2	YES/NO	**YES/YES**	NO/YES	NO/YES	NO/YES
$3	YES/NO	YES/NO	YES/YES	NO/YES	NO/YES
$4	YES/NO	YES/NO	YES/NO	YES/YES	NO/YES
$5	YES/NO	YES/NO	YES/NO	YES/NO	YES/YES

The number of multiple selves, $n(M)$ =5 in the discrete case is listed in the first column. Since we assumed that the required level of willingness (w) equals 4/5, WTP is the sure amount of money which 4 of the multiple selves should be willing to pay for the good. In this case it is $2, because multiple selves who value the good at $2, 3, 4, and 5 are willing to pay $2 for the good. Only the most parsimonious self who values the good at $1 is not willing to pay $2 for the good, since $u(1\,\text{GBP}) < u(2\,\text{GBP})$. Similarly, WTA is the amount of money that should get confirmation from four of the multiple selves to give away the good, which is $4 in this example.

4.4 Explaining the Anomalies

4.4.1 Valuation Gap

Standard economic theory predicts that the two measures, WTP and WTA, should be equal when the income effects are negligible (Hanemann, 1991). However, for the last four decades a considerable amount of experimental literature reported that WTA is significantly higher than WTP (Horowitz and McConnel, 2002; Sayman and Onculer, 2005; Hammit and Tuncel, 2013). The typical setting of the experiments is to separate the subject pool into sellers and buyers and to ask for WTA and WTP, respectively, under an incentive compatible design such as the BDM and the second price auction, etc. The sellers are endowed with the good whereas buyers are not. The gap is important because if it does exist it means that Coase Theorem—that no matter who owns the property rights first, the parties will reach to a Pareto Optimum outcome after a series of transactions, assuming that the transaction costs are negligible—fails to hold. This theorem has important implications for environmental damage cases and constitutes the basis of the legal system related to these issues. Furthermore, if an individual’s subjective valuations depend on possession status, the preferences are reference dependent, upending standard economic theory.

To incorporate and explain the observed anomalies researchers developed so called non-standard models such as PT and its variants and RDUT. These non-standard models of preferences try to explain the endowment effect with loss aversion concepts that can be summarised as 'losses loom larger than gains'. Loss-averse sellers perceive giving away the good as a 'loss' and ask for more compensation.

However, recent findings on valuation gap suggest loss aversion might not be the explanation for the observed behaviour, or, at least, not the only one. These studies mostly focus on the problem from the Discovered Preference Hypothesis, i.e., people have well-defined stable preferences but they need to learn and discover them (Plott, 1996). The findings of this line of literature suggest that since the experimental mechanisms are not day-to-day procedures that subjects come across, they might find them difficult to understand and therefore the observed behaviour might not reflect 'true' preferences. Subjects need to understand the mechanism and find out that telling the true subjective valuations is the optimal response for them. Evidence coming from repeated setting experiments (List, 2004b, 2003; Loomes et al., 2003; Shogren et al., 2001) supports this claim as disparity declines with trading experience. Most recently, Plott and Zeiler (2005) conducted experiments which include more comprehensive training mechanisms, and they found that there is no disparity when mugs are traded in the experiments, however, for the lottery tickets the gap seems to be persistent and significant. There is also implicit evidence from Plott and Zeiler (2005) and Isoni et al. (2011) who find that the endowment effect is observed only for the lottery tickets, but not for ordinary market goods such as mugs and candies. Their result is important because after they implement procedures to minimise subject misconceptions and misunderstandings, persistent disparity in lottery tickets but not in ordinary market goods cannot be explained by loss aversion, so there is something special about the lottery tickets, which must be their uncertain nature.

PCT anticipates these results: it states that due to the individuals' vague perception of the numerical objective probabilities, they end up having a range of expected utilities, and then evaluate the desirability of the bounds in a reference dependent way, calculating the weighted average of the range by their intrinsic pessimism level. They weight the worst case by their pessimism parameter and assign the remaining weight to the best case.

Consider the previous lottery example:

$$M:\left(x_1=\$0,\pi_1=0.7;x_2=\$100,\pi_2=0.3\right) \tag{50}$$

Suppose, an individual perceives the probabilities 0.7 and 0.3 by mapping them into the following ranges: $[0.6,0.8]$ and $[0.2,0.4]$, respectively. This leads to the expected utility range of 0.2 and 0.4 as calculated in (32) and (33). Table 14 demonstrates the final wealth levels in buying and selling tasks.

Table 14. *Final wealth levels*

Task Type	Initial wealth	Worst possible final wealth	Best possible final wealth
Buying	M	$M+u^{-1}(0.2)$-WTP	$M+ u^{-1}(0.4)$-WTP
Selling	M	M+WTA- $u^{-1}(0.4)$	M+WTA- $u^{-1}(0.2)$

Looking at Table 14, it is easy to see that under PCT, WTA, and WTP are not necessarily equal if individuals exhibit imprecision and pessimism, i.e., $\alpha>0.5$, and WTA is higher than WTP. In order to see this, consider the following equations arising from the definition of WTP and WTA, respectively:

$$u(M)=\alpha\cdot[u(M+u^{-1}\left(0.2\right)-WTP])+(1-\alpha)\cdot[u(M+\ u^{-1}\left(0.4\right)-WTP)] \quad (51)$$

$$u(M)=\alpha\cdot[u(M+WTA-u^{-1}(0.4)]+(1-\alpha)\cdot u[(M+WTA-\ u^{-1}(0.2))] \quad (52)$$

For both tasks, the individual's initial wealth is *M* dollars. In buying the individual pays WTP and gets the good, which has an expected utility between 0.2 and 0.4. In buying, the best-case scenario is to pay WTP and get the good which has an expected utility of 0.4, whereas in the worst-case scenario individual pays WTP but the good has an expected utility of 0.2. As mentioned, the individual forms beliefs and calculates the weighted average of this range by assigning the weight α for the worst case and $1-\alpha$ for the best case. In selling, the individual gives away the good for WTA. In the best-case scenario the expected utility is 0.2 and in the worst-case scenario expected utility is 0.4. Similarly, the individual attaches weight α to the worst case and $1-\alpha$ to the best case. Therefore, in selling the upper bound of the imprecision range formed in the first stage of PCT corresponds to the worst case, whereas in buying it corresponds to the best case. To see this, consider the following inequalities which always hold: $M+WTA-u^{-1}\left(0.4\right)<M+WTA-u^{-1}\left(0.2\right)$ $M-WTP+u^{-1}\left(0.2\right)<M-WTP+u^{-1}\left(0.4\right)$. Note that WTA and WTP are monetary amounts so the only natural condition imposed is that they are non-negative.

In order to make the calculations easy, without loss of generality, we assume the individual is risk neutral so equations (44) and (45) become:

$$M=\alpha\cdot(M+0.2-WTP)+(1-\alpha)\cdot(M+\ 0.4-WTP) \quad (53)$$

$$M=\alpha\cdot(M+WTA-0.4)+(1-\alpha)\cdot(M+WTA-\ 0.2) \quad (54)$$

After arranging the equations for WTA and WTP:

$$WTP = 0.4 - 0.2\alpha \tag{55}$$

$$WTA = 0.2\alpha + 0.2 \tag{56}$$

PCT predicts WTA to be higher than WTP if the following condition holds:

$$0.2\alpha + 0.2 > 0.4 - 0.2\alpha \rightarrow \alpha > 1/2 \tag{57}$$

Therefore, an individual who decides according to PCT will state higher WTA than WTP if the individual exhibits imprecision, $\beta > 0$, and forms pessimistic beliefs, $\alpha > 0.5$. To sum up, the WTA-WTP disparity is the product of the pessimism under imprecision.

This also has intuitive appeal from the perspective of economic bubbles. If individuals are not good enough in evaluating outcomes and probabilities, they will end up with a range of expected utilities for goods, and thus a range of admissible subjective values. When the economic environment makes them optimistic, buyers overvalue assets causing market prices to increase and create bubbles. When the economic environment signals pessimism, the continuously overvalued assets are not as appealing to buyers, causing a sharp decrease in prices, which leads the bubble bursting.

4.4.2 Preference Reversals

Preference reversals (PR) were first documented in experimental studies by Lichtenstein and Slovic (1971) and Lindman (1971). The early literature was sceptical about the existence of this anomaly and claimed that it was an artefact of experimental design features, and thus tested its existence under various alterations of the experimental design, all of which ended up confirming the robustness of the phenomenon (Machina, 1992; Roth, 1988). Another group of researchers focused on investigating the issues such as whether PR might be a result of subjects' misunderstanding and/or insufficient incentives (Grether and Plott, 1979b; Pommerehne et al., 1982; Reilly, 1982).

In addition, there are also studies which criticise the preference reversal experiments from a theoretical perspective (Holt, 1986; Karni and Safra, 1987; Segal, 1988): The common argument of these studies is that if the individuals have non-expected utility preferences, violating either the independence axiom of EUT and/or the reduction of compound lotteries principle, the experimental procedures such as BDM and the random lottery incentive system could be biased, which might generate PR. In other words, if individuals have non-standard preferences, then the choice and the valuation tasks are no longer separable. In this case an individual might value one lottery more than another, but choose the less-preferred lottery at the same time. Therefore, observed valuations might not be the true certainty equivalents of the lotteries and the experimenter observes a spurious PR. However this line of criticism has been falsified in a series of experimental papers using modified mechanisms which are immune to those points, such as reduction of compound lotteries and violation of independence axiom research (Cubitt et al., 2004; Tversky et al., 1990). After these sceptical studies, the PR phenomenon is seen as replicable and robust, thus the focus of the succeeding literature has concentrated on the possible explanations and the factors affecting the phenomenon (Loomes, 1990).

There still remains a considerable interest in trying to find a satisfying explanation for PR, which can be summarised as three strands of explanations: Regret Theory, Reference-Dependent Theory, and Constructed Preference Theory.

Regret Theory, reviewed in Chapter 1, provides an explanation by incorporating the violation of the transitivity axiom. Loomes and Sugden ((1983) formulated the PR as three acts, $-bet, P-bet, and M, which are listed in Table 15 where *x*, *y*, and *m* are monetary consequences with following ordering: *x*>*y*>*m*.

Table 15. *Formulating preference reversals over three acts*

Acts	State 1	State 2	State 3
$-bet	x	0	0
P-bet	y	y	0
M	m	m	m

In line with the PR literature, $-bet gives a higher prize than P-bet with a lower winning probability and M is a degenerate lottery which gives $*m* with certainty. If individuals exhibit regret aversion then the intransitive cycle occurs in a specific direction under the Regret Theory preference functional. Thus, P-bet is preferred over $-bet, M over P-bet; and $-bet over M. To see how this explains PR, consider interpreting the valuation of P-bet and $-bet as two binary choice questions such as the valuation of $-bet is a choice between $-bet and M whereas the valuation of P-bet is a choice between P-bet and M. For example for $-bet, this interpretation can be understood as ‘which one is worth more, $-bet or m?’ The experimental tests for Regret Theory’s explanation of PR confirm the cycles predicted by Regret Theory (Loomes et al., 1991). However, Starmer and Sugden (1998) provided evidence which raises doubts about the account of Regret Theory on these intransitive cycles. In other words, tests of Regret Theory pointed out a new type of choice anomaly, however it does not seem to be the right explanation for it (Starmer, 2008).

The second explanation for PR is provided by the reference dependent models such as Subjective Expected Utility Theory (Sugden, 2003), and Prospect Theory and its variants. The common feature of these models is that preferences are defined over gains and losses relative to an initial reference point, and losses are weighted more than the gains in utility terms (loss aversion).

Subjective Expected Utility Theory is similar to the EUT in terms of having linear probabilities and being defined over acts, but each state of the world is seen as gains and losses and therefore it can accommodate loss aversion. It predicts PR when valuation tasks are the elicitation of selling prices. In selling, individuals are endowed with the lottery tickets and therefore the reference acts are the corresponding bets. Since $-bet has a higher winning prize and in selling it is perceived as a probabilistic loss if the sale occurs, individuals exhibiting loss aversion might choose P-bet but value $-bet higher.

Other reference-dependent models such as Prospect Theory and its variants have a similar approach to explaining PR, which is centred on notions of reference dependency and the asymmetric treatment of gains of losses. The third type of explanation for PR belongs to the psychology literature, which sees preference reversals as evidence against the central assumption of economics: individuals behave according to their stable preferences. Instead, the third type of explanation focuses on the decision processes and the factors affecting it, such as the stimuli P-bet and $-bet and the task type such as buying, selling, and choice. According to this line of explanation, individuals might reveal or state different rankings and ordering depending on task type as each task might invoke different heuristics and therefore alter the decision process and its outcome (see Lichtenstein and Slovic (2006) for further discussion).

A relatively recent explanation is the preference imprecision, also the focus of this study, proposed by MacCrimmon and Smith (1986). They conjectured that most of the individuals cannot come up with a precise valuation of the bets, but they can form a range of values as their potential responses for certainty equivalence questions. Moreover, they proposed that the range for the $-bet is wider than the P-bet because there is a wide range of potential responses for the $-bet which does not violate first-order stochastic dominance. Therefore, it is more likely to observe a higher valuation for $-bet than the P-bet.

In order to understand their idea, consider the two binary outcome bets: P-bet offering \$10 with a probability of 0.8 and \$-bet offering \$32 with a probability of 0.25. For both lotteries the losing payoff is zero. The individual will be sure that the certainty equivalent for the P-bet lies somewhere between \$0 and \$10 whereas for the \$-bet it is between \$0 and \$32. These ranges correspond with the potential response ranges for two bets, but for some individuals it will be easy to narrow these ranges further and others will confidently state a single valuation for the bets. However, most individuals might not be capable of doing so. For example, an individual might think that the certainty equivalents are between \$1 and \$4 for P-bet and \$1 and \$8 dollars for \$-bet. MacCrimmon and Smith (1986) call these ranges ‘imprecision ranges’ or imprecise equivalences from which an individual cannot confidently state a single value. In other words, individuals find it difficult to state a single amount from this range to reflect their true preferences, because they cannot articulate their preferences precisely. It is also assumed that the width and the location of the range are subjectively determined.

Based on MacCrimmon and Smith (1986), Butler and Loomes (2007) conducted an experiment in which they elicited imprecision intervals by an incremental choice method: subjects were asked a series of binary choice questions in which the first option is either P-bet or the \$-bet depending on the task, and the second option is a degenerate lottery which gives a sure sum of money. They used a P-bet offering of AUD 24 with a probability of 0.7; and a \$-bet offering of AUD 80 with a probability of 0.25. Half of the subjects are given ‘iterating up’ treatment and other half ‘iterating down’ treatment. In the first treatment, the second option starts with AUD 1, and iterates up in each question by AUD 1. In the second treatment, suppose the first option is P-bet, therefore the second option starts with AUD 24 and iterates down by AUD 1 at each question. If the first option is a \$-bet, it starts iterating down from AUD 80. For each question, subjects are also asked to select one of the four phrases signifying strength of preference such as ‘I definitely prefer Lottery A’, ‘I think I prefer Lottery A, but I'm not sure’, ‘I think I prefer Lottery B, but I'm not sure’ and ‘I definitely prefer Lottery B’. Therefore, the range between the switching points 1 to 2 and 3 to 4 might give some idea about the imprecision range conjectured by MacCrimmon and Smith (1986). Overall, the summary statistics reported by Butler and Loomes (2007) seems to support MacCrimmon and Smith’s conjectures: the imprecision range for the P-bet is between AUD 8 and AUD 13.98 whereas for the \$-bet it is between AUD 13.30 and AUD 32.11 for the iterating-down treatment. For the second treatment the imprecision range of P-bet is between AUD 13.73 and AUD 19.42, whereas for the \$-bet it is between AUD 14.96 and AUD 35.02.

Preference reversals can arise with our model, and can be illustrated using the CRRA utility function for $u(.)$

$$u(z) = z^a \tag{58}$$

For $a < 0$, the function is concave, implying risk aversion. For simplicity, we focus on P- and \$-bets that give either a positive payoff or zero: P-bet $= (x, p; 0)$ and \$-bet $= (y, q; 0)$ with $y > x > 0$ and $1 > p > q > 0$ where p and q are the winning probabilities, and x and y are the winning prizes of the P-bet and \$-bet, respectively. Following Schmidt et al. (2008), we normalise the expected value (EV) of the P-bet by setting its payoff equal to $1/p$; r is the EV of the \$-bet as a ratio to the EV of the P-bet. Therefore the winning prize of the \$-bet equals r/p.

If the individual prefers the P-bet over the \$-bet in the choice task, we can write:

$$EU(M + P - bet) \geq EU(M + \$ - bet) \tag{59}$$

Additionally, we assume that the initial wealth of the individual, M, is zero. In the first stage of PCT, an individual calculates the imprecision bounds for the two lotteries:

$$EU_L(P - bet) = (p - \beta) \cdot x^a + (1 - p + \beta) \cdot 0^a \tag{60}$$

$$EU_H(P - bet) = (p + \beta) \cdot x^a + (1 - p - \beta) \cdot 0^a \tag{61}$$

$$EU_L(\$ - bet) = (q - \beta) \cdot y^a + (1 - q + \beta) \cdot 0^a \tag{62}$$

$$EU_H(\$ - bet) = (q + \beta) \cdot y^a + (1 - q - \beta) \cdot 0^a \tag{63}$$

Naturally, when calculating the lower bound of the expected utilities for the two lottery tickets, individuals take the lower bound of the imprecisely perceived winning probabilities, $(p - \beta)$ and $(q - \beta)$, into account, whereas in calculating the upper bound of the expected utilities, individuals use the upper bound of the perceived winning probabilities, $(p + \beta)$ and $(q + \beta)$. The remaining probabilities are assigned to the second event, which pays out nothing if it occurs. In the second stage, the individual weights the worst final-level case by the pessimism parameter, α, and the best case with $(1 - \alpha)$. Since it is a choice task, the lower bound of the imprecision range corresponds to the worst case: $(M + EU_L(.) < (M + EU_U(.))$ where M is the initial wealth and is zero. Therefore we can rewrite the binary choice problem as follows:

$$\alpha[(p-\beta)x^a + (1-\alpha)[(p+\beta)x^a] \geq \alpha[(q-\beta)y^a] + (1-\alpha)[p+\beta]y^a \qquad (64)$$

As mentioned before in standard PR, individuals pick the P-bet implying that it leads to a higher final wealth utility than the \$-bet. Thus the critical value for α in determining whether the P-bet is preferred to the \$-bet is:

$$\alpha^* = [y^a(q+\beta) - x^a(p+\beta)] / 2\beta(y^a - x^a) \qquad (65)$$

If the actual α is greater than α^*, the individual chooses the P-bet; if it is less, the individual chooses the \$-bet.

When we come to valuations as in Table 12 and 14, we see that the lower bound of the imprecision range calculated in the first stage does not correspond to the worst case because it does not lead the individual to the worst-case final wealth level. The valuation problem can be written as:

$$u(M + WTA_{P-bet} - \mathrm{EU}_L^{-1}(P-bet)) > u(M + WTA_{P-bet} - \mathrm{EU}_H^{-1}(P-bet)) \quad (66)$$

$$u(M + WTA_{\$-bet} - \mathrm{EU}_L^{-1}(\$-bet)) > u(M + WTA_{\$-bet} - \mathrm{EU}_H^{-1}(\$-bet)) \quad (67)$$

which implies that an individual with an assumed initial wealth, M, of zero, pays WTA for the bet and gives it away. The left-hand side is always higher than the right-hand side since WTA is non-negative and $\mathrm{EU}_H(.)$ is greater than $\mathrm{EU}_L(.)$ by definition. Thus, the lower bound of the expected utility range calculated in the first stage of PCT corresponds to the worst-case final wealth for selling, unlike for buying and the choice task. Next, the individual calculates the weighted average of the bounds by multiplying the worst-case utility level by the pessimism parameter, α and the best case by $(1-\alpha)$. WTA is defined as the amount of money that keeps the individual at the same wealth level before the transaction:

$$\alpha \cdot u(M + WTA_{P-bet} - \mathrm{EU}_H^{-1}(P-bet)) + (1-\alpha) \cdot u(M + WTA_{P-bet} - \mathrm{EU}_L^{-1}(P-bet)) = u(M) \quad (68)$$

$$\alpha \cdot u(M + WTA_{\$-bet} - \mathrm{EU}_H^{-1}(\$-bet)) + (1-\alpha) \cdot u(M + WTA_{\$-bet} - \mathrm{EU}_L^{-1}(\$-bet)) = u(M) \quad (69)$$

We further simplify by separating the utility of WTA amounts and the lottery tickets, and plug in the expressions for $EU_L(.)$ and $EU_H(.)$, which leads to the WTA amounts for the two bets:

$$u^{-1}(WTA_{P-bet}) = \alpha[(p+\beta)x^a] + (1-\alpha)[(p-\beta)x^a] \tag{70}$$

Similarly,

$$u^{-1}(WTA_{\$-bet}) = \alpha[(q+\beta)y^a] + (1-\alpha)[(q-\beta)y^a] \tag{71}$$

The critical value for α is:

$$\alpha^{**} = [x^a(p-\beta) - y^a(q-\beta)] / 2\beta(y^a - x^a) \tag{72}$$

If α is greater α^{**} the individual values the \$-bet more than the P-bet; if it is lower, the P-bet is valued more than the \$-bet.

We explore the parameters of PCT in three cases: risk neutral, risk averse, and a risk loving. Consider Figure 7, where r is set to 1; P-bet $=(1.25, 0.8; 0)$, \$-bet $=(5, 0.2; 0)$ [17]

The dashed line shows the α^{**} boundary and the solid line is the α^{*} boundary. Above the dashed line, the \$-bet is valued more and above the solid grey line the P-bet is chosen; the region between the two lines is called the consistency range where the chosen bet is valued more. For a risk-neutral individual, in the case of imprecision ($\beta > 0$), a standard preference reversal occurs if $a > 0.5$; when it is less than 0.5, the model predicts a non-standard preference reversal. One prominent and natural difference between the risk-loving and risk-averse individual is that in the consistency range a risk-averse individual chooses the P-bet and values it more; whereas the risk-loving individual chooses the \$-bet and values it more. It is a natural conclusion since the P-bet would be more attractive for a risk-averse individual. Overall, in the case of imprecision ($\beta > 0$), a sufficiently high level of pessimism results in a standard preference reversal while optimism implies a non-standard preference reversal.

[17] For the imprecision level, we use $\beta(\psi, p) = \psi \cdot p(1-p)$, although there is no particular reason behind choosing this except that it is simple and satisfies the assumptions of the theory. We normalise the expected value (EV) of the P-bet by setting its payoff equal to $1/p$; r is the EV of the \$-bet as a ratio of the EV of the P-bet. Therefore the winning prize of the \$-bet equals r/p.

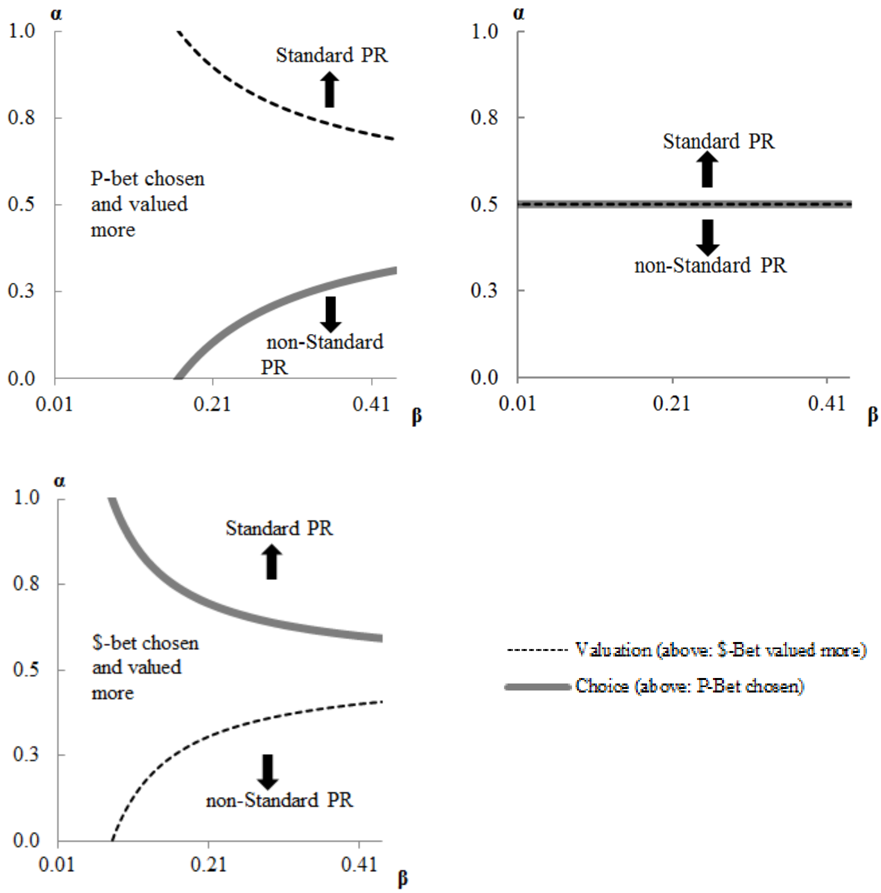


Figure 7. Preference reversals and parameters of Preference Cloud Theory (Starting from top left *a* equals 0.7 (risk averse), 1 (risk neutral) and 1.3 (risk loving) to reflect different levels of the curvature of the CRRA utility function.)

Next we consider the case in which the winning probabilities remain the same, but the winning prize of the $-bet varies.

The dashed lines show the valuation bounds and the solid lines show the choice bounds for three levels of *r* (0.8,1,1.2); these are coloured light grey, dark grey, and black, respectively. For a risk-averse individual in Figure 8, the consistency range shrinks as *r* increases up to a certain level. The parameter values to induce standard and non-standard preference reversals converge to the risk-neutrality baseline case. However, above this critical level of *r*, the consistency range favours the $-bet and it expands as *r* increases. In other words, since *r* implies the relative attractiveness of the $-bet, as it increases up to a certain level it makes the $-bet more attractive than the P-bet for a risk-averse individual. Even if we increase the relative attractiveness of the $-bet to extreme values, the model predicts that both standard and non-standard preference reversals can be observed.

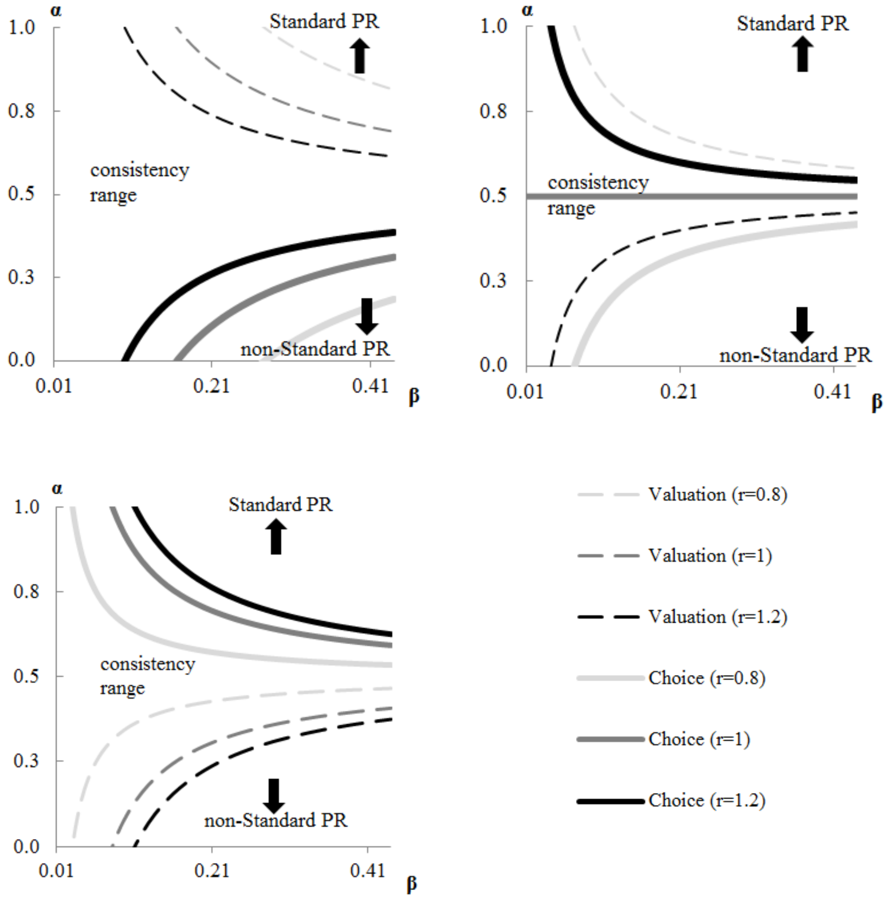


Figure 8. Increase in relative attractiveness of the $-bet (starting from top left, *a* equals 0.7 (risk averse), 1 risk neutral, and 1.3 (risk loving) to reflect different levels of the curvature of the CRRA utility function.)

For the risk neutrality case in Figure 8, as the relative attractiveness of the $-bet in terms of EV is increased or decreased, consistency range expands. The difference is that, as r increases above 1, the individual chooses $-bet and values it more inside the consistency range. This pattern resembles the risk-loving case. On the other hand, as r is decreased further below 1, P-bet is chosen and valued more inside the consistency range, which resembles the risk-aversion case.

For the risk-loving case in Figure 8, as the relative attractiveness of the $-bet increases the consistency range expands further. Overall, the regions which allow for standard and non-standard preference reversals seem to shrink as the relative attractiveness of the \$-bet is increased. The required level of pessimism to observe standard preference reversals increases as we increase further; this can be seen in the shrinking region of standard preference reversals and interpreted as the tendency to exhibit reversal decreases as the difference between the lotteries becomes more prominent. Therefore, individuals with even greater imprecision and thus less probabilistic sophistication will behave consistently in terms of their choices and valuations as we increase the attractiveness of the \$-bet.

4.4.3 Allais Paradox

As introduced in Chapter 1, the Allais Paradox is the first challenge proposed to EUT in which individuals violate the independence axiom. The inconsistent patterns pointed out in the Allais Paradox have led to the development of the alternative models reviewed in Chapter 1. In order to see the differences between the EUT and the alternatives, it will be helpful to use the probability triangle and demonstrate the Allais type of bets on the triangle. These bets are characterised as three outcome lotteries where the outcomes are x_1, x_2 and x_3, which have the following order in terms of magnitude: $x_1 < x_2 < x_3$. The corresponding probabilities of these outcomes are a vector of probabilities: $(p_1, p_2, 1-p_1-p_2)$. For the original version of the Allais problem the outcomes x_1, x_2 and x_3 are \$0, \$1M, and \$5M. The probabilities for the four bets (S_1, R_1, S_2, R_2) are shown in Table 16 below:

Table 16. *Bets in Allais Paradox*

Outcomes (x_i)	S_1	R_1	S_2	R_2
\$0	-	0.01	0.89	0.90
\$1M	1.00	0.89	0.11	-
\$5M	-	0.10	-	0.10

As introduced in Chapter 1, in an Allais type of problem, individuals respond to two binary choice questions: in the first question they make a choice between S_1 and R_1 whereas in the second question they choose either S_2 or R_2. According to EUT, individuals should either choose S or R types of lotteries in both questions, however the observed tendency is to choose S_1 in the first question and R_2 in the second one. It is inconsistent with EUT, because the second set of lotteries is formed by subtracting the common question from S_1 and R_1. As in EUT the probabilities enter into the calculation in a linear manner, this subtraction should not alter a change in the ranking of the lotteries. Figure 9 demonstrates the problem in a probability triangle where the vertical axis shows the probability of best consequence, whereas the horizontal axis measures the probability of the worst consequence. Therefore the remaining probability corresponds to the consequence, which is $1M.

The bets that are located on the triangle boundaries assign positive probabilities only for two consequences out of four. Since S_1 gives $1M with a probability of 1, it is centred in the corner where the probabilities of other consequences are zero ($p_1 = p_3 = 0$). In addition, since S_2 has positive probabilities for the consequences such as 0 and $1M, it lies on the horizontal axis. Similarly, R_2 does not assign a positive probability for winning $1M therefore it is on the hypotenuse, which depicts the probability of winning $1M. The interior of the triangle includes the bets that assign positive probability to all three consequences; in this case it is R_1. The crucial point on Figure 9 is that the lines joining the two pairs (R_1-S_1 and R_2-S_2) are parallel.

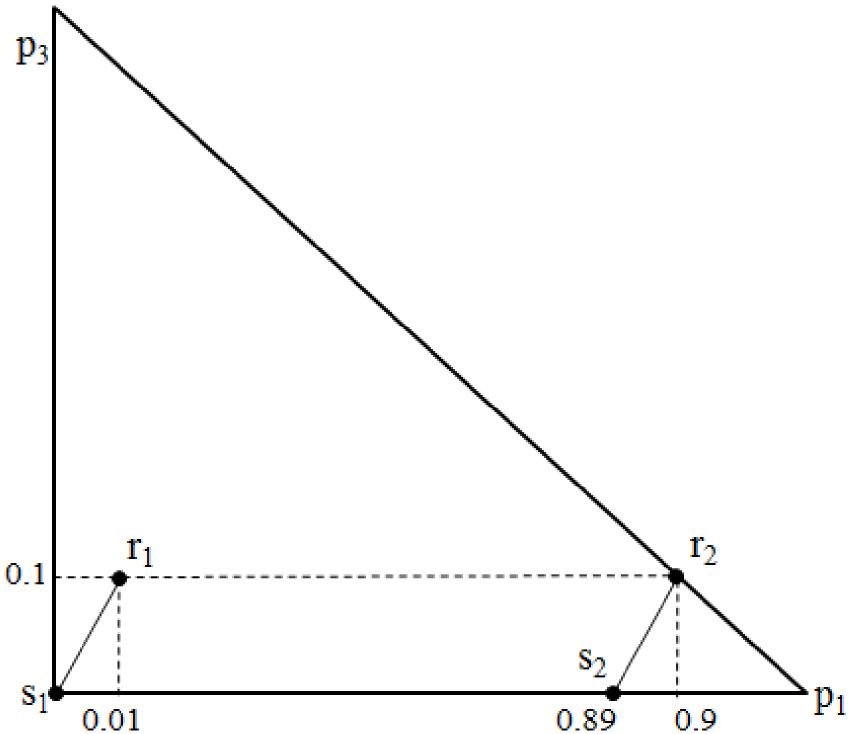


Figure 9. Probability triangle and Allais bets

We can demonstrate the preferences on the triangle with indifference curves. They are parallel lines because probabilities are treated linearly in expected utility calculations. Moreover, they are increasing in terms of desirability towards the northwest of the triangle since the best outcome is located on the vertical axis and the worst outcome is on the horizontal axis. Figure 10 shows an example of indifference curves drawn according to EUT.

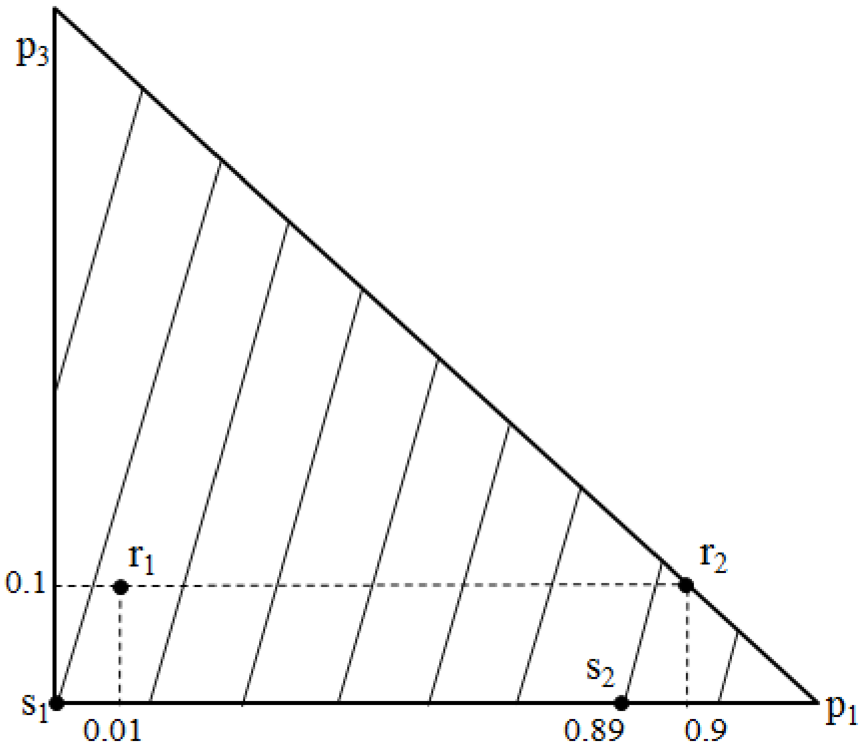


Figure 10. Expected Utility Theory and indifference curves

Under EUT, the slope of the indifference curves implies the risk attitude of the individuals: the steeper the slope, the more risk averse the individual is, as shown in Figure 11.

The solid line in the figure implies relatively more risk aversion compared to the dashed line: *x* on the figure gives $1M with certainty, whereas *y* and *z* are the risky prospects that assign positive probability to the worst ($0) and the best consequences ($1M), but zero for the middle-ranked consequence ($1M). Furthermore, *y* assigns a higher probability to $5M than *z*. Therefore the solid line belongs to an individual who demands a higher probability of getting $5M to be indifferent between the risky prospect and $1M with certainty.

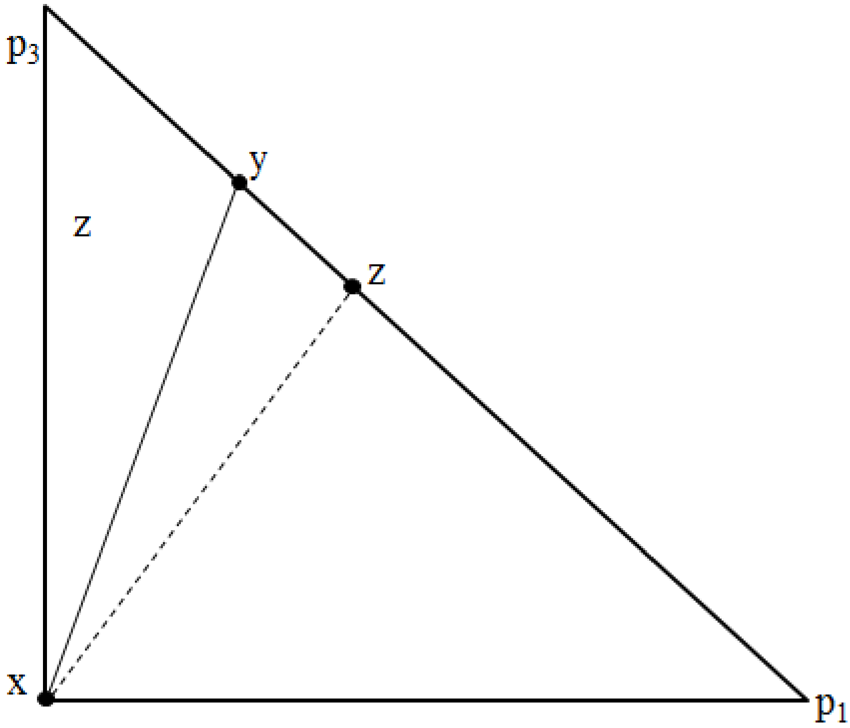


Figure 11. Risk Attitudes on Probability Triangle

So under EUT, throughout the triangle the individual maintains the risk attitude by having the parallel indifference curve covering the triangle. Since the lines connecting the pairs are also parallel (Figure 9), according to EUT the individual should pick either S- or R-type lotteries in both questions to maintain consistency. However the actual behaviour observed in the literature contradicts the prediction of EUT. Figure 12 demonstrates the observed behaviour: the individual choosing S_1 in the first question signals an indifference curve similar to c_1, which means that the indifference curve that passes through R_1 lies somewhere below c_1, which is in the less desirable region. On the other hand, if the individual chooses R_2 in the second question it means that the indifference curve passes through S_2 and lies somewhere below c_2. It is easy to see that c_1 and c_2 are not parallel which means that individual acts as though less risk averse while making a choice between S_2 and R_2 as compared to when making the choice between S_1 and R_1. This behaviour is inconsistent with EUT, because it implies that the risk attitude of the individual does not remain the same across the choices between two pairs. This pattern of unstable risk attitudes is hypothetised as indifference curves being fanning out from the bottom-left corner of the triangle.

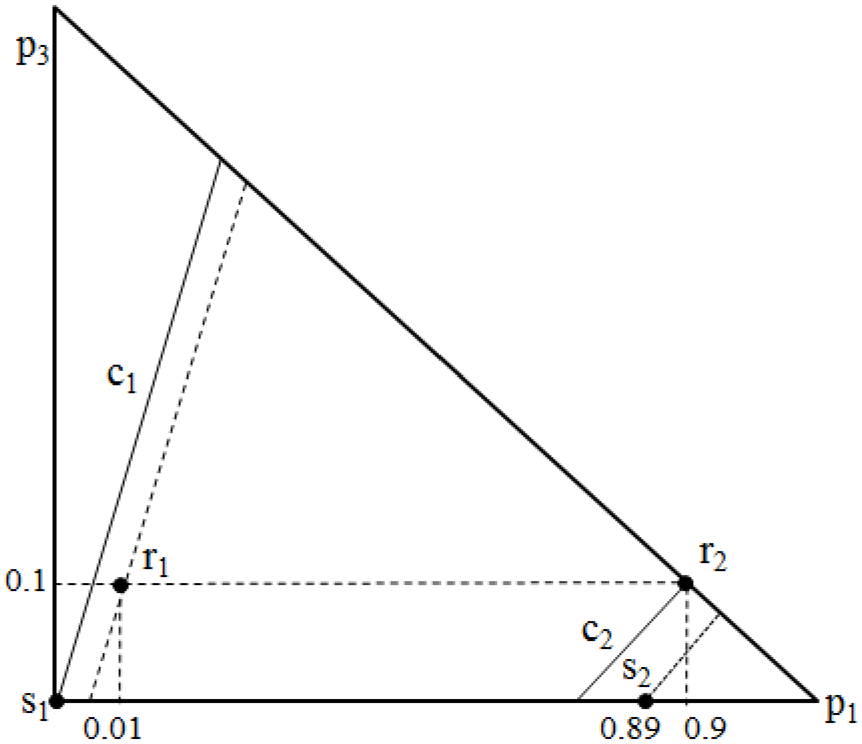


Figure 12. Observed behaviour in Allais Paradox

To maintain transitivity it is assumed that the starting point of fanning out is located outside the triangle as shown in Figure 13.

Figure 13 shows the typical linear but fanning out indifference curves under the Weighted Utility Theory developed by Chew and MacCrimmon (1979). There are also different patterns produced by alternative theories, which allow for Allais behaviour. Figure 14 shows the indifference curves of Rank-Dependent Utility Theory with a concave probability weighting function.

The curves are steepest in the bottom-right corner where the probability of the middle-ranked outcome ($1M) equals one. They get flatter as we move along the horizontal and vertical axes and finally become parallel close to the hypotenuse where the probability of the middle-ranked outcome equals zero. Overall, alternative theories treat the probabilities in a nonlinear manner, which then relaxes the linearity and/or parellelism of the indifference curves (see Camerer (1989) for a detailed analysis).

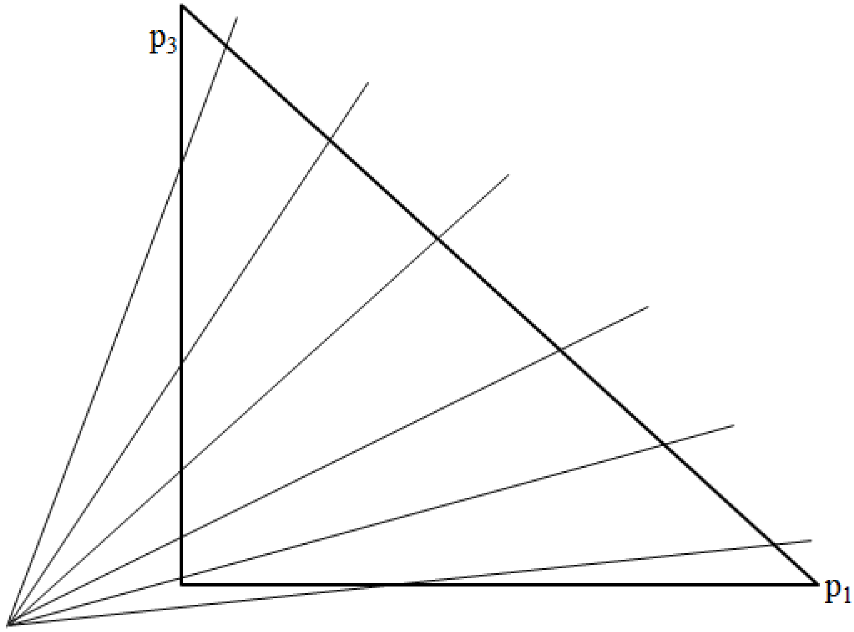


Figure 13. Fanning-out hypothesis

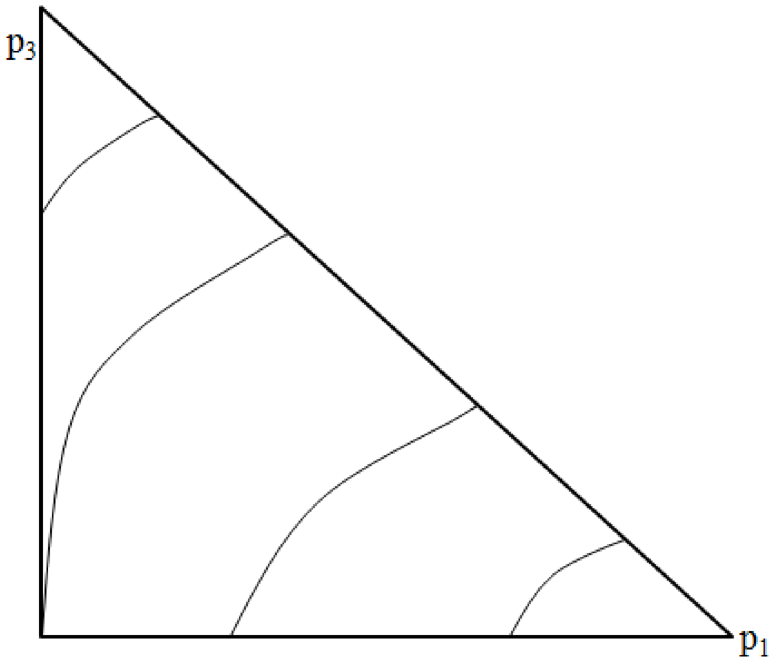


Figure 14. Indifference curves of Rank-Dependent Utility Theory

Besides the theoretical advances in the literature to explain Allais Paradox, there are also studies that empirically question and test its robustness. Studies in this line of literature are defenders of EUT that claim that the violations can be explained by misunderstandings and inattentiveness (Allais, 1990; Amihud, 1979a, 1979b; Morgenstern, 1979). In an experimental study, Savage (1954) modifies the representation of the lotteries in order to highlight the similarity of the bets in two questions, as shown in Table 17.

Table 17. *Savage's representation of the Allais bets*

	1	2-11	12-100
A	1000	1000	1000
B	0	5000	1000
A'	1000	1000	0
B'	0	5000	0

The last three columns include the different way of presenting the probabilities associated with the three outcomes. For example, suppose a subject chooses B in the first question and the random number drawn equals 9, then the subject wins 5000, since it is between 2 and 11. This representation facilitates understanding the similarity between the first and the last two lotteries shown in the table: discarding the common consequence of winning 1000 if the random number is between 12 and 100 from A and B produces A' and B'. Although this modification in the presentation of the lotteries decreases the inconsistencies from 60% to 40%, they do not disappear (Incekara-Hafalir and Stecher, 2012). Conlisk (1989) also focuses on the presentation of the lottery tickets and finds that the inconsistencies decrease from 50% to 28%. In addition to the subject misunderstandings, Harrison (1994) criticises the hypothetical nature of the surveys that document the inconsistencies and suggests that it would be premature to discard EUT based on them. Burke et al. (1996) takes the critics of Harrison into account and use real monetary payoffs in an experimental study which again reduces the inconsistent preference statements but does not eliminate them completely (see Camerer (1989) for another example with real payoffs). Finally, in a more recent and comprehensive study, Harman and Gonzales (2015) find that the inconsistent statements disappear with experience, which can be seen as a support for the preference imprecision argument that is reviewed in detail in Chapter 2. As subjects gain experience they will be more precise about their probability judgments and exhibit lower imprecision.

In order to see how PCT incorporates the Allais Paradox, the same approach is used as was used to explain PR in Section 4.4.2 using CRRA utility function for $u(.)$:

$$u(z) = z^a \tag{73}$$

For $a < 0$, the function is concave, implying risk aversion. Here, use $k\sigma$ model for the first stage of PCT and α pessimism specification for the second stage to explain the Allais Paradox in this section. It is also possible to use the original version of PCT to explain the preference reversals but the original version is applicable for only the two-outcome lotteries. In order to extend it for the lotteries that have more than two outcomes one can use rank-dependent probability transformation technique as in Cumulative Prospect Theory and Rank-Dependent Utility Theory. The bets that are used in this section are same as the ones listed in Table 17 above. Remember that under the $k\sigma$ model, an individual forms the imprecision range that has a width of $2k \cdot u(\cdot\sigma)$ in the first stage of the PCT and the standard expected utility of the bet is at the centre of this range. Thus for any lottery X the bounds are calculated in the first stage as:

$$EU_L(X) = EU(X) - k \cdot u(\sigma) \tag{74}$$

$$EU_H(X) = EU(X) + k \cdot u(\sigma) \tag{75}$$

For the second stage of the theory, assume that individuals weight the worst case for final wealth level by α, the pessimism parameter. Since the tasks under Allais problems are simple choice tasks, we can take the lower bound of the imprecision range as the worst case and the upper bound as the best case. Remember that in preference reversals or valuation gap problems, for selling, the upper bound corresponds to the worst case because it is associated to the worst-case scenario in terms of final wealth (see Section 4.2 for a detailed discussion).

Thus an individual calculates the expected utility of a bet X under PCT by calculating the weighted average of the bounds as:

$$EU_{PCT}(X) = \alpha \cdot [EU(X) - k \cdot u(\sigma)] + (1-\alpha) \cdot [EU(X) + k \cdot u(\sigma)] \tag{76}$$

If an individual prefers S_1 to R_1 in the first task and R_2 to S_2 in the second task, this can be represented by the following inequalities:

$$EU_{PCT}(S_1) > EU_{PCT}(R_1) \tag{77}$$

$$EU_{PCT}(R_2) > EU_{PCT}(S_2) \tag{78}$$

These inequalities simply say that individual prefers S_1 over R_1 because the first one gives higher satisfaction to the individual calculated in accordance with the PCT. Similarly the second inequality implies that the chosen bet in the second question (R_2) gives higher utility than the other one (S_2) according to PCT.

In order to find the critical values for the parameters of PCT that allow for this kind of behaviour, we need to plug in the expressions for $EU_{PCT}(S_1)$, $EU_{PCT}(R_1)$, $EU_{PCT}(S_1)$ and $EU_{PCT}(R_1)$:

$$EU_{PCT}(S_1) = 1^a \tag{79}$$

$$EU_{PCT}(R_1) = \alpha \cdot [0.89 \cdot 1^a + 0.1 \cdot 5^a - k \cdot 1.21^a] + (1-\alpha) \cdot [0.89 \cdot 1^a + 0.1 \cdot 5^a + k \cdot 1.21^a] \tag{80}$$

$$EU_{PCT}(S_2) = \alpha \cdot [0.11 \cdot 1^a - k \cdot 0.31^a] + (1-\alpha) \cdot [0.11 \cdot 1^a + k \cdot 0.31^a] \tag{81}$$

$$EU_{PCT}(R_2) = \alpha \cdot [0.1 \cdot 5^a - k \cdot 1.5^a] + (1-\alpha) \cdot [0.1 \cdot 5^a + k \cdot 1.5^a] \tag{82}$$

Since S_1 gives \$1M with certainty, the standard deviation is zero, which then reduces to the standard expected utility formulation. For the other three lotteries, the standard deviations are 1.21, 0.31, and 1.5, respectively. All of the payoffs are simplified, and the common multiplier is suppressed. For the benchmark case, set a to 0.5, which determines the curvature of the utility function. Next, solve the inequalities (70) and (71) for α to find the critical values. For this analysis, it is useful to graph the combinations of parameters (k and α) that allow for Allais Paradox as shown in Figure 15.

The vertical axis measures α values whereas the horizontal axis lists values for k. the solid curve shows the critical α values for the first task where the individual has to make a choice between S_1 and $R_{1,}$ and, above this curve, S_1 is chosen over R_1. Second, the dashed curve shows the critical values for α in the second task where the individual has to make a choice between S_2 and $R_{2,}$ and below this curve, R_2 is chosen over S_2. Thus, below the solid curve the individual prefers R_1 and R_2 in both tasks whereas above the dashed line the individual prefers S_1 and S_2 in both tasks. These regions include the combination of parameters, α and k which result in consistent behaviour with EUT. On the other hand, the region between these two curves includes the parameter combinations that allow for the paradoxical behaviour: the individual prefers S_1 in the first task and R_2 in the second task. Overall, as the level of imprecision increases, the critical α value that allows for the Allais Paradox decreases to 0.5.

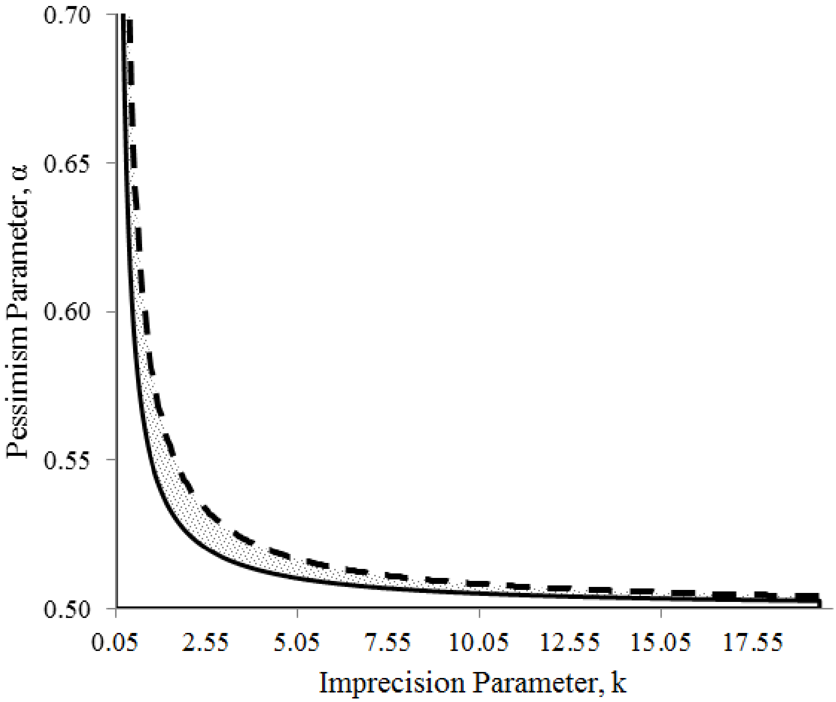


Figure 15. Allais Paradox and PCT parameters. (Solid line shows the critical α values in the first task, whereas the dashed line shows the ones in the second task. Above the solid curve, S_1 is chosen over R_1, whereas above the dashed curve, S_2 is chosen over R_2).

For the benchmark case, I set the parameter a to 0.5; decreasing this parameter moves the two curves towards southeast of the origin and expands the region, which includes the parameters allowing the Allais Paradox.

4.5 Conclusion

There is a theory similar to the original version of PCT but it is for decision under ambiguity: α-MaxMin model of decision under ambiguity. It asserts that under ambiguity individuals form multiple priors and select one of them depending on the pessimism/optimism parameter. However, α-MaxMin becomes EUT when the probabilities are known, therefore for decisions under risk it reduces to EUT (Wakker, 2010): the individual forms multiple priors when the probabilities are unknown (consider the two ambiguous Ellsberg urns); on the other hand, it asserts that, as in EUT, individuals can perceive the numerical, objective probabilities perfectly when they are given as information.

The disadvantage with α-MaxMin is that for some cases its predictions contradict the notion of monotonicity. Consider two ambiguous lotteries, and three states of the world: A=(1,0,0) and B=(1,1,0). An individual who decides according to α-MaxMin will end up having the same expected utilities for the two lotteries, but B is obviously better. The reason is that the set of multiple priors under α-MaxMin is assumed to include all of the possible probability distributions over state space. Therefore, the problem of α-MaxMin is that it does not provide a method or formulation for how individuals form these priors. Instead, it is assumed to be the same for all individuals: a set of priors is the set of all possible probability distributions. But PCT tells us how individuals form multiple priors: by imprecision parameter, β. When we extend PCT for the risky prospects with more than three outcome by using rank dependent probability weighting, there is no violation of dominance or monotonicity.

Another close companion of PCT is RDUT in terms of including factors such as optimism/pessimism; however this theory cannot explain anomalies such as PR and valuation gap. The reason is that it predicts the same expected utility for the same good in different tasks such as choice, buying, and selling. The major problem with RDUT is that it lacks a plausible behavioural foundation: the rank dependent cumulative probability transformation—ranking the outcomes and converting the probabilities into decision weights in a cumulative way—is a complicated task for the ordinary person whose cognitive capabilities are indeed questioned by the literature proposing these alternative models to EUT. While these models are questioning the cognitive capabilities of the individuals, it seems paradoxical to model their behaviour with a more complicated manner, i.e., by asserting that they can do complicated calculations such as rank-dependent cumulative probability transformation. One way to make RDUT explain preference reversals and a valuation gap is to add loss aversion, but then it becomes 3rd Generation Prospect Theory which includes both rank dependency and loss aversion. However, it cannot offer plausible parameter values that can capture the strong reversals and non-standard reversals that are reported in Butler and Loomes (2007).

Another theory that we should pay attention to is Regret Theory, which can also explain preference reversals, however the theory itself depends on the state-wise comparison of the two options (e.g., P-bet and $-bet), and individuals develop disutility of regret for the states in which the option that is not chosen has higher utility. Moreover, it also failed other tests (Starmer and

Sugden, 1998). The disadvantage with Regret Theory is that the utility of an option not only depends on its consequences, but also on the available options. The extension of Regret Theory for the cases that include more than two outcomes is not straightforward.

Most importantly, besides the disadvantages stated above, none of these theories can predict that individuals might have imprecise preferences; therefore they do not take into account the evidence recently emerging in the literature. In Chapter 2, I reviewed the stochastic preference approach as a possible incorporation of the imprecision in existing theories, but the results suggest that even with stochastic specifications, existing theories cannot explain a significant portion of the observed behaviour. PCT accomplishes incorporating the imprecision in preferences by the first step in which the individuals' vague perception of the numerical probabilities plays a central role. This vagueness of perception causes individuals to have a range of expected utility, the imprecision range. For obvious reasons, both the experimental settings and the real life situations demand a single amount from individuals: for example, you cannot pay for goods in terms of intervals. Thus, the nature of the experiments and of the real world forces individuals to withdraw a single amount from the imprecision range formed at the first step.

The second step of PCT describes how individuals reduce the range to a single amount, which is modelled by incorporating Hurwicz's α. We rationalise it in the following way: since the individual does not have prior information about the probability distribution of the imprecision range, i.e., does not know which value is the true expected utility, an individual has to form beliefs. Belief formation depends on the individual's degree of optimism or pessimism. According to the pessimism/optimism level, the individual calculates the weighted average of this range and considers that single amount as a criterion for decisions. To sum up, PCT offers a final product that is a single precise amount, as the other theories do, but also it provides the imprecision range as a product embedded in the first step of PCT.

Moreover, an alternative that we suggest for the second stage of the PCT, the multiple selves framework, provides a meaningful preference representation for the values stated as the imprecision range by incorporating the level of willingness in preference relations. The standard way of representing the preferences sees the values inside the imprecision range as equally desirable, but this view is problematic for monotonicity. Thus, PCT provides insights and a more meaningful picture about the imprecision range observed in emerging literature, but also explains the anomalies of standard economic theory.

There is also a probabilistic choice model proposed by Blavatsky (2009), which accommodates preference imprecision by taking EUT and embedding it in some particular stochastic specification. However, PCT incorporates the preference imprecision in a much simpler way by providing a preference functional. Finally, I do not rule out extensions such as modifying PCT with a stochastic component, as in stochastic preferences literature. The pessimism parameter, α , can be assumed to be randomly drawn for each task as in a random preference approach. Moreover, an error term such as white noise can be added to the deterministic part of PCT, which is similar to the approach that Hey and Orme (1994) used. Another possible extension is to assume the pessimism parameter is dependent on several factors such as the ratio of past winning for the individual and/or the moving average of past winning, etc. It is plausible to assume that pessimism itself depends on the good and the bad outcomes that an individual experienced. That past experiences determine individual's beliefs about future outcomes is just a simple and natural extension and can be easily incorporated in PCT.

Another point where PCT has an advantage over other theories is that none of the theories can explain the findings of what is called 'preference paradox' in the psychophysics literature, that is, the bidding pattern is identical no matter how we present the probabilities: numerically, verbally, and/or graphically (see Section 4.2 for the related studies). How can RDUT—or any other theory in which the expected utility calculation is done by the precisely perceived probability—explain the similar bidding patterns between the qualitative and quantitative representations of the probabilities? Suppose you are told that it is 'less likely' you will get \$10 or that you will get \$10 with a 30% probability. Given that, in those experiments, subjects are not given any information about what 'less likely' means to the experimenter, they are expected to derive a subjective meaning from the phrases on their own. None of the existing theories can explain this phenomenon. The only way to explain this phenomenon is to assume that individuals perceive the numerical objective probabilities in a vague way, similar to the way that they perceive the verbal expressions. PCT accomplishes this by imprecision parameter, β.

Another issue that needs to be discussed is which criteria we should assess the theories on. Some theories might make similar predictions but they might provide different underlying stories for the observed behaviour. Predictive power is not the only criteria to assess a theory on; the 'true' insights are also an important criterion.

Economics is interested in developing homeomorphic models, not paramorphic models. The reason is that economists demand the parameters and assumptions to have psychologically plausible stories (Wakker, 2010).

Paramorphic models correctly describe the data and make perfect predictions, but they are not concerned with the ‘true’ underlying decision process. Because the aim of these models is to predict market outcomes, they are not concerned with how people actually make decisions, and do not reflect the true underlying decision process. As Friedman (1953) states, market models can make correct predictions even if their assumptions about consumers do not match actual consumers’ behaviour.

On the other hand, homeomorphic models not only match their predictions with the data but also describe how individuals really think. Economists are interested in homeomorphic models, because the aim of economics as a science is not just predicting, but also, designing economic policies and market schemes. The effectiveness and success of such policies and market schemes depend on the extent to which we can understand the underlying decision mechanism of individuals. Relying on erroneous but seemingly true assumptions and models while designing economic policies is like barking up the wrong tree; it can be a winning strategy by chance, but in the long run will reveal its weaknesses.

References

Adamowicz, W.L., Bhardwaj, V., Macnab, B., 1993. Experiments on the difference between willingness to pay and willingness to accept. Land Economics 69, 416–427.

Allais, M., 1990. Allais Paradox, in: Eatwell, J., Milgate, M., Newman, P. (Eds.), Utility and Probability. Macmillian Press, Hong Kong, pp. 3–9.

Allais, M., 1979. The foundations of a positive theory of choice involving risk and a criticism of the postulates and axioms of the American School (1952). Springer.

Allais, M., 1953. Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine. Econometrica 21, 503–546. doi:10.2307/1907921

Amihud, Y., 1979a. Critical examination of the new foundation of utility, in: Expected Utility Hypotheses and the Allais Paradox. Springer, pp. 149–160.

Amihud, Y., 1979b. A reply to Allais, in: Expected Utility Hypotheses and the Allais Paradox. Springer, pp. 185–190.

Ballinger, T.P., Wilcox, N.T., 1997. Decisions, error and heterogeneity. The Economic Journal 107, 1090–1105.

Banerjee, P., Shogren, J.F., 2014. Bidding behavior given point and interval values in a second-price auction. Journal of Economic Behavior & Organization 97, 126–137.

Baucells, M., Heukamp, F.H., 2006. Stochastic dominance and cumulative prospect theory. Management Science 52, 1409–1423.

Bayrak, O.K., Hey, J.D., 2015. Preference Cloud Theory: Imprecise Preferences and Preference Reversals (SSRN Scholarly Paper No. ID 2633392). Social Science Research Network, Rochester, NY.

Bayrak, O.K., Kriström, B., 2016. Is there a valuation gap? The case of interval valuations. Economics Bulletin 36, 218–236.

Becker, G.M., DeGroot, M.H., Marschak, J., 1963. Stochastic models of choice behavior. Behavioral science 8, 41–55.

Bell, D.E., 1982. Regret in decision making under uncertainty. Operations research 30, 961–981.

Belyaev, Y.K., Kriström, B., 2015. Analysis of survey data containing rounded censoring intervals. Информатика и её применения 9, 2–16.
Billard, L., Diday, E., 2007. Symbolic Data Analysis: Conceptual Statistics and Data Mining (Wiley Series in Computational Statistics). John Wiley & Sons.
Bisantz, A.M., Marsiglio, S.S., Munch, J., 2005. Displaying uncertainty: Investigating the effects of display format and specificity. Human Factors: The Journal of the Human Factors and Ergonomics Society 47, 777–796.
Blavatskyy, P.R., 2007. Stochastic expected utility theory. Journal of Risk and Uncertainty 34, 259–286.
Block, H.D., Marschak, J., others, 1960. Random orderings and stochastic theories of responses. Contributions to probability and statistics 2, 97–132.
Bohm, P., Lindén, J., Sonnegård, J., 1997. Eliciting Reservation Prices: Becker-DeGroot-Marschak Mechanisms vs. Markets. The Economic Journal 107, 1079–1089.
Boyce, R.R., Brown, T.C., McClelland, G.H., Peterson, G.L., Schulze, W.D., 1992. An Experimental Examination of Intrinsic Values as a Source of the WTA-WTP Disparity. The American Economic Review 82, 1366–1373.
Braga, J., Starmer, C., 2005. Preference anomalies, preference elicitation and the discovered preference hypothesis. Environmental and Resource Economics 32, 55–89.
Brookshire, D.S., Coursey, D.L., 1987. Measuring the value of a public good: an empirical comparison of elicitation procedures. The American Economic Review 554–566.
Brown, T.C., Champ, P.A., Bishop, R.C., McCollum, D.W., 1996. Which Response Format Reveals the Truth about Donations to a Public Good? Land Economics 72, 152–166. doi:10.2307/3146963
Budescu, D.V., Wallsten, T.S., 1995. Processing linguistic probabilities: General principles and empirical evidence. Psychology of Learning and Motivation 32, 275–318.
Budescu, D.V., Wallsten, T.S., 1990. Dyadic decisions with numerical and verbal probabilities. Organizational Behavior and Human Decision Processes 46, 240–263.
Budescu, D.V., Weinberg, S., Wallsten, T.S., 1988. Decisions based on numerically and verbally expressed uncertainties. Journal of Experimental Psychology: Human Perception and Performance 14, 281.
Burke, M.S., Carter, J.R., Gominiak, R.D., Ohl, D.F., 1996. An experimental note on the allais paradox and monetary incentives. Empirical Economics 21, 617–632.

Butler, D.J., Loomes, G.C., 2007. Imprecision as an Account of the Preference Reversal Phenomenon. American Economic Review 97, 277–297. doi:10.1257/aer.97.1.277
Butler, D., Loomes, G., 2011. Imprecision as an account of violations of independence and betweenness. Journal of Economic Behavior & Organization 80, 511–522. doi:10.1016/j.jebo.2011.05.008
Butler, D., Loomes, G., 1988. Decision difficulty and imprecise preferences. Acta Psychologica 68, 183–196. doi:10.1016/0001-6918(88)90054-6
Camerer, C.F., 1989. An experimental test of several generalized utility theories. Journal of Risk and uncertainty 2, 61–104.
Camerer, C.F., Hogarth, R.M., 1999. The effects of financial incentives in experiments: A review and capital-labor-production framework. Journal of risk and uncertainty 19, 7–42.
Chateauneuf, A., Cohen, M., 1994. Risk seeking with diminishing marginal utility in a non-expected utility model. Journal of Risk and Uncertainty 9, 77–91.
Chew, S.H., MacCrimmon, K.R., 1979. Alpha-nu choice theory: A generalization of expected utility theory. University of British Columbia.
Cohen, M., Jaffray, J.-Y., Said, T., 1987. Experimental comparison of individual behavior under risk and under uncertainty for gains and for losses. Organizational behavior and human decision processes 39, 1–22.
Conlisk, J., 1989. Three variants on the Allais example. The American Economic Review 392–407.
Coombs, C.H., Bezembinder, T.G., Goode, F.M., 1967. Testing expectation theories of decision making without measuring utility or subjective probability. Journal of Mathematical Psychology 4, 72–103.
Coursey, D.L., Hovis, J.L., Schulze, W.D., 1987. The disparity between willingness to accept and willingness to pay measures of value. The Quarterly Journal of Economics 679–690.
Cubitt, R.P., Munro, A., Starmer, C., 2004. Testing explanations of preference reversal*. The Economic Journal 114, 709–726. doi:10.1111/j.1468-0297.2004.00238.x
Cubitt, R.P., Navarro-Martinez, D., Starmer, C., 2015. On preference imprecision. J Risk Uncertain 50, 1–34. doi:10.1007/s11166-015-9207-6
Davies, G.B., Satchell, S.E., others, 2004. Continuous cumulative prospect theory and individual asset allocation. Faculty of Economics, University of Cambridge.
Diecidue, E., Wakker, P.P., 2001. On the Intuition of Rank-Dependent Utility. Journal of Risk and Uncertainty 23, 281–298. doi:10.1023/A:1011877808366

Dubourg, Jones-Lee, Loomes, G., 1997. Imprecise Preferences and Survey Design in Contingent Valuation. Economica 64, 681–702. doi:10.1111/1468-0335.00106

Dubourg, W.R., Jones-Lee, M.W., Loomes, G., 1994. Imprecise preferences and the WTP-WTA disparity. J Risk Uncertainty 9, 115–133. doi:10.1007/BF01064181

Fechner, G.T., 1966. Elements of psychophysics (New York: Holt, Rinehart and Winston). Inc.

Friedman, M., 1953. Methodology of Positive Economics. University of Chicago Press, Chicago.

Gal, D., 2006. A psychological law of inertia and the illusion of loss aversion. Judgment and Decision Making 1, 23–32.

Georgescu-Roegen, N., 1958. Threshold in Choice and the Theory of Demand. Econometrica: Journal of the Econometric Society 157–168.

Gescheider, G.A., 2013. Psychophysics: the fundamentals. Psychology Press.

Gregory, R., Lichtenstein, S., Brown, T.C., Peterson, G.L., Slovic, P., 1995. How precise are monetary representations of environmental improvements? Land Economics 462–473.

Grether, D.M., Plott, C.R., 1979a. Economic theory of choice and the preference reversal phenomenon. The American Economic Review 69, 623–638.

Grether, D.M., Plott, C.R., 1979b. Economic theory of choice and the preference reversal phenomenon. The American Economic Review 623–638.

Gul, F., 1991. A theory of disappointment aversion. Econometrica: Journal of the Econometric Society 667–686.

Hadar, J., Russell, W.R., 1969. Rules for ordering uncertain prospects. The American Economic Review 59, 25–34.

Hagen, O., 1979. Towards a positive theory of preferences under risk, in: Expected Utility Hypotheses and the Allais Paradox. Springer, pp. 271–302.

Hammack, J., Brown, G.M., 1974. Waterfowl and wetlands: Toward bioeconomic analysis. Resources for the Future Washington, DC.

Hanemann, W.M., 1991. Willingness to Pay and Willingness to Accept: How Much Can They Differ? The American Economic Review 81, 635–647.

Harless, D.W., Camerer, C.F., 1994. The predictive utility of generalized expected utility theories. Econometrica: Journal of the Econometric Society 1251–1289.

Harman, J.L., Gonzalez, C., 2015. Allais from experience: Choice consistency, rare events, and common consequences in repeated decisions. Journal of Behavioral Decision Making 28, 369–381.

Harrison, G.W., 1994. Expected utility theory and the experimentalists. Springer.

Helson, H., 1964. Adaptation-level theory: an experimental and systematic approach to behavior.
Hey, J.D., 2005. Why we should not be silent about noise. Experimental Economics 8, 325–345.
Hey, J.D., Orme, C., 1994. Investigating Generalizations of Expected Utility Theory Using Experimental Data. Econometrica 62, 1291–1326. doi:10.2307/2951750
Holt, C.A., 1986. Preference reversals and the independence axiom. The American Economic Review 508–515.
Horowitz, J.K., McConnell, K.E., 2002. A review of WTA/WTP studies. Journal of Environmental Economics and Management 44, 426–447.
Hsee, C.K., 1998. Less is better: when low-value options are valued more highly than high-value options. J. Behav. Decis. Making 11, 107–121. doi:10.1002/(SICI)1099-0771(199806)11:2<107::AID-BDM292>3.0.CO;2-Y
Hsee, C.K., 1996. The evaluability hypothesis: An explanation for preference reversals between joint and separate evaluations of alternatives. Organizational behavior and human decision processes 67, 247–257.
Incekara-Hafalir, E., Stecher, J.D., 2012. An experimental test of theories of behavior in Allais-type tasks. Unpublished manuscript.
Irwin, J.R., 1994. Buying/selling price preference reversals: Preference for environmental changes in buying versus selling modes. Organizational Behavior and Human Decision Processes 60, 431–457.
Isoni, A., Loomes, G., Sugden, R., 2011. The Willingness to PayWillingness to Accept Gap, the Endowment Effect, Subject Misconceptions, and Experimental Procedures for Eliciting Valuations: Comment. The American Economic Review 101, 991–1011.
Kaas, K.P., Ruprecht, H., 2006. Are the Vickrey Auction and the BDM Mechanism Really Incentive Compatible? - Empirical Results and Optimal Bidding Strategies in Cases of Uncertain Willingness-to-pay. Schmalenbach Business Review (sbr) 58, 37–55.
Kahneman, D., Knetsch, J.L., Thaler, R.H., 1990. Experimental tests of the endowment effect and the Coase theorem. Journal of political Economy 1325–1348.
Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. Econometrica: Journal of the Econometric Society 263–291.
Karni, E., Safra, Z., 1987. “ Preference reversal” and the observability of preferences by experimental methods. Econometrica: Journal of the Econometric Society 675–685.
Knetsch, J.L., Sinden, J.A., 1984. Willingness to Pay and Compensation Demanded: Experimental Evidence of an Unexpected Disparity in Measures of Value. The Quarterly Journal of Economics 99, 507–521. doi:10.2307/1885962

Knetsch, J.L., Tang, F.-F., Thaler, R.H., 2001. The endowment effect and repeated market trials: Is the Vickrey auction demand revealing? Experimental Economics 4, 257–269.
Lichtenstein, S., Slovic, P., 2006. The construction of preference. Cambridge University Press.
Lichtenstein, S., Slovic, P., 1971. Reversals of preference between bids and choices in gambling decisions. Journal of experimental psychology 89, 46.
Lindman, H.R., 1971. Inconsistent preferences among gambles. Journal of Experimental Psychology 89, 390.
List, J.A., 2004a. Neoclassical theory versus prospect theory: Evidence from the marketplace. Econometrica 72, 615–625.
List, J.A., 2004b. Neoclassical Theory versus Prospect Theory: Evidence from the Marketplace. Econometrica 72, 615–625.
List, J.A., 2003. Does Market Experience Eliminate Market Anomalies? The Quarterly Journal of Economics 118, 41–71. doi:10.1162/00335530360535144
Loomes, G., 2005. Modelling the Stochastic Component of Behaviour in Experiments: Some Issues for the Interpretation of Data. Exp Econ 8, 301–323. doi:10.1007/s10683-005-5372-9
Loomes, G., 1990. Preference reversal: Explanations, evidence and implications. Annals of Operations Research 23, 65–90.
Loomes, G., 1988. Different experimental procedures for obtaining valuations of risky actions: Implications for utility theory. Theory and Decision 25, 1–23.
Loomes, G., Moffatt, P.G., Sugden, R., 2002. A microeconometric test of alternative stochastic theories of risky choice. Journal of risk and Uncertainty 24, 103–130.
Loomes, G., others, 1997. Imprecise preferences and survey design in contingent valuation. Economica 64, 681–702.
Loomes, G., Starmer, C., Sugden, R., 2003. Do Anomalies Disappear in Repeated Markets?*. The Economic Journal 113, C153–C166.
Loomes, G., Starmer, C., Sugden, R., 1991. Observing Violations of Transitivity by Experimental Methods. Econometrica 59, 425–439. doi:10.2307/2938263
Loomes, G., Sugden, R., 1998. Testing different stochastic specifications of risky choice. Economica 581–598.
Loomes, G., Sugden, R., 1995. Incorporating a stochastic element into decision theories. European Economic Review 39, 641–648.
Loomes, G., Sugden, R., 1983. A rationale for preference reversal. The American Economic Review 428–432.
Loomes, G., Sugden, R., 1982. Regret theory: An alternative theory of rational choice under uncertainty. The Economic Journal 805–824.
Loomis, J., Brown, T., Lucero, B., Peterson, G., 1997. Evaluating the Validity of the Dichotomous Choice Question Format in Contingent Valuation.

Environmental and Resource Economics 10, 109–123. doi:10.1023/A:1026403916622
Luce, R.D., 1959. Individual choice behavior: A theoretical analysis. John Wiley & Sons, New York.
Luce, R.D., 1958. A probabilistic theory of utility. Econometrica: Journal of the Econometric Society 193–224.
Luce, R.D., 1956. Semiorders and a Theory of Utility Discrimination. Econometrica 24, 178–191. doi:10.2307/1905751
Luce, R.D., Suppes, P., 1965. Preference, utility, and subjective probability. Wiley.
MacCrimmon, K., Smith, M., 1986. Imprecise equivalences: Preference reversals in money and probability. University of British Columbia working paper 1211, 311–312.
Machina, M.J., 1992. Choice under uncertainty: Problems solved and unsolved, in: Foundations of Insurance Economics. Springer, pp. 49–82.
Mahieu, P.-A., Riera, P., Kriström, B., Brännlund, R., Giergiczny, M., 2014. Exploring the determinants of uncertainty in contingent valuation surveys. Journal of Environmental Economics and Policy 3, 186–200.
McCollum, D.W., Miller, S.M., 1994. Alaska Voters: Their Wildlife Viewing Trip Characteristics and Economics. Alaska Department of Fish and Game, Division of Wildlife Conservation, Anchorage.
Mitchell, R.C., Carson, R.T., 1989. Using surveys to value public goods: the contingent valuation method. Resources for the Future.
Morgan, M.G., Henrion, M., Small, M., 1992. Uncertainty: a guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge university press.
Morgenstern, O., 1979. Some reflections on utility. Springer.
Morgenstern, O., 1976. The collaboration between Oskar Morgenstern and John von Neumann on the theory of games. Journal of Economic Literature 14, 805–816.
Morrison, G.C., 1998. Understanding the disparity between WTP and WTA: endowment effect, substitutability, or imprecise preferences? Economics Letters 59, 189–194.
Morrison, G.C., 1998. Understanding the disparity between WTP and WTA: endowment effect, substitutability, or imprecise preferences? Economics Letters 59, 189–194. doi:10.1016/S0165-1765(98)00049-4
Morrison, G.C., 1997. Willingness to pay and willingness to accept: some evidence of an endowment effect. Applied Economics 29, 411–417.
Mosteller, F., Nogee, P., 2006. An experimental measurement of utility, in: Selected Papers of Frederick Mosteller. Springer, pp. 179–219.
Neilson, W.S., 2008. Impulsive Actions and Agonizing Decisions. Citeseer.
Nowlis, S.M., Simonson, I., 1997. Attribute-task compatibility as a determinant of consumer preference reversals. Journal of marketing research 205–218.

Peters, E., Slovic, P., Gregory, R., 2003. The role of affect in the WTA/WTP disparity. Journal of Behavioral Decision Making 16, 309–330.
Plott, C.R., 1996. Rational individual behavior in markets and social choice processes: the discovered preference hypothesis. McMillian.
Plott, C.R., Zeiler, K., 2005. The Willingness to Pay-Willingness to Accept Gap, the "Endowment Effect," Subject Misconceptions, and Experimental Procedures for Eliciting Valuations. The American Economic Review 95, 530–545.
Pommerehne, W.W., Schneider, F., Zweifel, P., 1982. Economic theory of choice and the preference reversal phenomenon: A reexamination. The American Economic Review 569–574.
Prelec, D., 1998. The probability weighting function. Econometrica 497–527.
Preston, M.G., Baratta, P., 1948. An experimental study of the auction-value of an uncertain outcome. The American journal of psychology 61, 183–193.
Price, C.R., Sheremeta, R.M., 2011. Endowment effects in contests. Economics Letters 111, 217–219. doi:10.1016/j.econlet.2011.02.003
Quandt, R.E., 1956. A probabilistic theory of consumer behavior. The Quarterly Journal of Economics 507–536.
Quiggin, J., 1994. Regret theory with general choice sets. Journal of Risk and Uncertainty 8, 153–165.
Quiggin, J., 1982. A theory of anticipated utility. Journal of Economic Behavior & Organization 3, 323–343. doi:10.1016/0167-2681(82)90008-7
Ready, R.C., Whitehead, J.C., Blomquist, G.C., 1995. Contingent Valuation When Respondents Are Ambivalent. Journal of Environmental Economics and Management 29, 181–196. doi:10.1006/jeem.1995.1040
Reaves, D.W., Kramer, R.A., Holmes, T.P., 1999. Does Question Format Matter? Valuing an Endangered Species. Environmental and Resource Economics 14, 365–383. doi:10.1023/A:1008320621720
Reilly, R.J., 1982. Preference reversal: Further evidence and some suggested modifications in experimental design. The American Economic Review 576–584.
Roth, A.E., 1988. Laboratory experimentation in economics: A methodological overview. The Economic Journal 98, 974–1031.
Savage, L.J., 1972. The Foundations of Statistics, 2nd Revised ed. edition. ed. Dover Publications, New York.
Savage, L.J., 1954. The foundations of statistics. Wiley, New York.
Sayman, S., Öncüler, A., 2005. Effects of study design characteristics on the WTA–WTP disparity: A meta analytical framework. Journal of Economic Psychology 26, 289–312. doi:10.1016/j.joep.2004.07.002
Schmidt, U., Starmer, C., Sugden, R., 2008. Third-generation prospect theory. Journal of Risk and Uncertainty 36, 203–223.

Schmidt, U., Zank, H., 2008. Risk aversion in cumulative prospect theory. Management Science 54, 208–216.
Segal, U., 1988. Does the preference reversal phenomenon necessarily contradict the independence axiom? The American Economic Review 233–236.
Shah, A.K., Oppenheimer, D.M., 2008. Heuristics made easy: an effort-reduction framework. Psychological bulletin 134, 207.
Shogren, J.F., Cho, S., Koo, C., List, J., Park, C., Polo, P., Wilhelmi, R., 2001. Auction mechanisms and the measurement of WTP and WTA. Resource and Energy Economics 23, 97–109.
Shogren, J.F., Shin, S.Y., Hayes, D.J., Kliebenstein, J.B., 1994. Resolving differences in willingness to pay and willingness to accept. The American Economic Review 255–270.
Sopher, B., Gigliotti, G., 1993. Intransitive cycles: Rational Choice or random error? An answer based on estimation of error rates with experimental data. Theory and decision 35, 311–336.
Starmer, C., 2008. preference reversals, in: Durlauf, S.N., Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics. Nature Publishing Group, Basingstoke, pp. 608–611.
Starmer, C., 2000. Developments in Non-expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk. Journal of Economic Literature 38, 332–382. doi:10.1257/jel.38.2.332
Starmer, C., Sugden, R., 1998. Testing alternative explanations of cyclical choices. Economica 65, 347–361.
Starmer, C., Sugden, R., 1989. Violations of the independence axion in common ratio problems: An experimental test of some competing hypotheses. Annals of Operations Research 19, 79–102.
Sugden, R., 2003. Reference-dependent subjective expected utility. Journal of economic theory 111, 172–191.
Sugden, R., 1993. An axiomatic foundation for regret theory. Journal of Economic Theory 60, 159–180.
Thaler, R., 1980. Toward a positive theory of consumer choice. Journal of Economic Behavior & Organization 1, 39–60.
Trepel, C., Fox, C.R., Poldrack, R.A., 2005. Prospect theory on the brain? Toward a cognitive neuroscience of decision under risk. Cognitive Brain Research 23, 34–50.
Tunçel, T., Hammitt, J.K., 2014. A new meta-analysis on the WTP/WTA disparity. Journal of Environmental Economics and Management 68, 175–187. doi:10.1016/j.jeem.2014.06.001
Tversky, A., Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and uncertainty 5, 297–323.
Tversky, A., Slovic, P., Kahneman, D., 1990. The causes of preference reversal. The American Economic Review 204–217.

Von Neumann, J., Morgenstern, O., 1944. Theory of games and economic behavior. Princeton university press.
Wakker, P.P., 2010. Prospect theory: For risk and ambiguity. Cambridge University Press.
Wilcox, N.T., 2008. Stochastic models for binary discrete choice under risk: A critical primer and econometric comparison. Risk aversion in experiments 12, 197–292.
Wu, G., Gonzalez, R., 1996. Curvature of the probability weighting function. Management science 42, 1676–1690.
Wu, G., Zhang, J., Abdellaoui, M., 2005. Testing prospect theories using probability tradeoff consistency. Journal of Risk and Uncertainty 30, 107–131.
Yaari, M.E., 1987. The dual theory of choice under risk. Econometrica: Journal of the Econometric Society 95–115.
Zimmer, A.C., 1984. A model for the interpretation of verbal predictions. International Journal of Man-Machine Studies 20, 121–134.
Zimmer, A.C., 1983. Verbal vs. numerical processing of subjective probabilities. Decision making under uncertainty

Appendix

Appendix includes the instructions used in the experiment presented in Chapter 3. These are the instructions used in the interval treatment for the Buyer-Seller uncertainty and buyers group, the instructions for the rest of the treatments and groups can be easily reproduced by making obvious modifications.

Instructions for Buyer-Seller Uncertainty Group in Intervals Treatment

This is an experiment in individual decision-making. Our purpose is to study technical issues involved in decision-making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn some money and/or other things. What you earn will depend on the decisions you make and some chance. The responses of others do not affect your payoff. It is for your interest to answer truthfully since there is no right or wrong answer in this experiment.

Important Rules

We ask that you do not communicate with other people during the experiment. Please refrain from verbally reacting to events that occur during the experiment. If you have any questions, write your questions on the paper titled as ‘Write Your Question Here’ which is placed on your desk. After writing your question raise your hand, experimenter will come to your desk and will write the answer on the same sheet.

Important Notice about How to Retrieve Rewards

Please write your ID on each task sheet when you receive them. Keep your ID card because you will retrieve your rewards by showing this card to person in charge with payoff distribution after the experiment. Note that the experimenter will not be able to link any specific participant name to a participant identification number. Therefore the experimenter will not know subject payoffs by individual. The person that does this experiment and the person that you will get your earnings from is different. So anonymity of the responses is maintained.

Tasks

You have 4 tasks to complete. In each task there are different goods. So there will be totally 4 goods. After all the tasks are completed, one of the tasks will be selected and played for real. The selection will be done like this: experimenter will write the task numbers on papers and will put them in different envelopes and place the envelopes in an opaque bag and select one of the tasks randomly. All of these will be done in front of you. So each task has equal chance of being selected. It is for your interest to see all of these tasks as real and make your decisions according to it, because each one of them have equal chance to be selected and played for real.

General Instructions for the Tasks

You all are given 100 SEK and the good. You will state your offer and after that, experimenter will determine whether you are buyer or seller by a random mechanism. The mechanism works as follows: right after you wrote your offer in each task, experimenter will collect your response sheet. Next, experimenter will select one of four tasks to run for real randomly. After that, experimenter will write “buyer” on a piece of paper and “seller” on another piece of paper. Experimenter will place these two papers in different envelopes and put the envelopes in an opaque bag. From that bag, experimenter will pick one of the envelopes which will determine whether you are buyer or seller. After you’re determined as buyer or seller, experimenter will announce the randomly selected market price. The random mechanism for market price selection works as follows: The market price will be determined randomly by using the 30 Ping-Pong balls. On each Ping-Pong ball; there is a number written on. The numbers are between 1 and 30 SEK. There are totally 30 Ping-Pong balls, so the market price can be any number between 1 SEK and 30 SEK. Experimenter will select one of the Ping-Pong balls and that will be the market price. Notice that each ball has equal chance of being selected so the market price can be any number between 1 and 30. All of these will be done in front of you. You are free to inspect the material that is used in random mechanisms after the experiment.

Depending on whether you are buyer or seller the outcome will be determined like this:

1. Seller: If the random process determines you as seller, it means that you own the good. If your offer is higher than the market price you will not sell the good. But if your offer is equal or lower than the market price, you will sell the good and <u>get the amount of money equal to the market price</u>. The important point here is that you will get the market price not your offer. The comparison between your offer and market price will determine whether you will sell the good or not.
2. Buyer: If the random process determines you as buyer, it means that you do not own the good. If your offer is below the market price you will not buy the good. But if your offer is equal or higher than the market price, you will buy the good and <u>pay the market price</u>. The important point here is that you will pay the market price not your offer. The comparison between your offer and market price will determine whether you will buy the good or not.

Notice the following two things:

1. Your decision can have no effect on the market price actually used because the market price will be selected at random.
2. It is in your interest to indicate your true preferences.

The Experimental Steps:

1. First you will write your offer**.** After you write your offer experimenter will collect the response sheets.
2. Experimenter will select the 1 out of 4 tasks, randomly.
3. Than experimenter will determine whether you are buyer or seller, randomly. (As explained above).
4. After that experimenter will announce the market price which is selected randomly between 1 and 30 SEK by using the Ping**-**Pong balls.

Remember, there are no advantages to strategic behavior. Your best strategy is to determine your personal value for the item and record that value as your offer. There is not necessarily a "correct" value. Personal values can differ from individual to individual.

Example: Suppose wrote 1000 as your offer on the response sheet. And suppose by the random mechanism you happen to be buyer. Next, experimenter will announce the market price which is selected randomly. In this case you happen to be a buyer so if the market price equals 1000 or lower than 1000 you will buy the good and <u>pay the market price, not your offered amount</u>. Suppose market price is 900, so you buy the good and pay 900 for the good.

If the market price is higher than your offer such as 1100, you will not buy the good, you keep your money. If the market price is 1000 then you will buy the good and pay 1000.

Example: Suppose you wrote 1000 as your offer on the response sheet. And suppose by the random mechanism you happen to be seller. That means that you own the good. Next experimenter will announce the market price which is selected randomly. In this example you happen to be a seller so if the market price equals or higher than 1000 you will sell your good and <u>get the amount of money which equals market price by giving away your good.</u> Suppose market price is 1200, so you sell the good and get 1200 in return.

If the market price is lower than your offer such as 800, you will not sell the good, you will keep your good. If the market price is 1000, you will sell the good and get 1000 in return.

You will see in the answer sheet that there are two boxes to enter your offer:

$$\boxed{\qquad\qquad} - \boxed{\qquad\qquad}$$

If your offer a single amount then write the same number inside the two boxes such as:

$$\boxed{1000} - \boxed{1000}$$

If you cannot provide a single amount such as 1000-1020 than you can write a range such as:

$$\boxed{1000} - \boxed{1020}$$

If you wrote a range and you happen to be a seller by the random process (that means you own the good) you will sell the good if the market price falls inside or above the range you specified. In this example if the market price is between 1000 and 1020 or above 1020 you will sell the good and get the market price. So it means you will sell the good if the market price is higher than 1000.

Example: Suppose you wrote 1000-1020 and random mechanism determined you as seller. After that, experimenter selected 900 randomly as market price. It means that you will not sell the good and you will keep the good. Because it is lower than your specified range.

What if the random market price happens to be 1010? You sell the good because it is inside the range you specified and get 1010 in return.

What if the random market price happens to be 1021? You do sell the good because it is higher than your specified range. So you will sell the good and get 1021 in return.

If you wrote a range and you happen to be a buyer by the random process (that means you do not own the good) you will get the good if the market price falls inside or below the range you specified. In this example if the market price is between 1000 and 1020 and below 1000 you will buy the good and pay the market price. So it means you will buy the good if the market price is lower than 1020.

Example: Suppose you wrote 1000-1020 and random mechanism determined you as buyer. After that, experimenter selected 900 randomly as market price. It means that you will buy the good and pay 900. Because it is lower than your specified range.

What if the random market price happens to be 1010? You buy the good because it is inside the range you specified and pay 1010 and get the good.

What if the random market price happens to be 1021? You do not buy the good because it is higher than your specified range. So you keep your money.

Notice that at the beginning of each task you have 100 SEK. As it is mentioned before, only one of the tasks will be selected randomly and will be played for real.

Guidelines

What Is Your 'Best Strategy'?

Remember there is no right or wrong answer but it is for your advantage to be honest and answer truthfully.

What Happens If I State A Lower Amount Than My True Value?

Let's assume that your true value is 1000 SEK, however you wrote a smaller amount on your sheet, let's say 950 SEK. Next experimenter will determine whether you are buyer or seller, randomly. Suppose you are assigned as seller. After that, let's say, experimenter announces the market price as 970 SEK. Since your offer (950 SEK) is lower than the market price (970 SEK), you will sell the good and get 970 SEK. Remember your true value was 1000

SEK. So by stating a lower value than your true value, you give away the good for a lower amount (970) than your true value (1000). If you had told the truth by stating 1000 SEK, you could have kept the good and will not sell it for 970 SEK.

What Happens If I State A Higher Amount Than My True Value?

Let’s assume that your true value is 1000 SEK, however you wrote a higher amount on your sheet, let’s say 1100 SEK. Next experimenter will determine whether you are buyer or seller, randomly. Suppose you are assigned as buyer. After that the experimenter announces the market price and it happened to be 1050 SEK. Since the market price (1050 SEK) is lower than your stated offer (1100 SEK), you buy the good and pay 1050 SEK. Remember your true value was 1000 SEK but you have stated a higher amount (1100 SEK). You get the good by paying 1050 SEK; however the good is worth only 1000 SEK to you. So if you pay a higher amount than your true value, you lose out.

Notice that it is your interest to state your true value. Since being a buyer and seller has equal chance of being selected you have to consider the two possible cases. As explained above, experimenter will prepare two envelopes: one of them has the word ‘seller’ and the other one has the word ‘buyer’. Since there are two envelopes they have equal chance of being selected from the opaque bag.

Being a seller means that you will be given the good in the task, so you own the good (plus the 100 SEK). Your offer will be compared to market price. This comparison will determine whether you sell the good or keep the good.

Being a buyer means that you do not own the good (only 100 SEK). Your offer will be compared to market price. This comparison will determine whether you buy the good or not.

HYPOTHETICAL TASK

The Aim of the task is to train you and make you understand the procedures

Write you ID here:________

Training Task 1: In this task the good is a candy. Now state your offer for that good. The market price will be selected from the range of 1-30 SEK. Each amount in this range has equal chance to be selected. The price increments are 1 SEK. Therefore there are 1,2,3,4,….30 SEK in this range.

If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

$\boxed{\quad\quad} - \boxed{\quad\quad}$

After everyone completes answering their offer, experimenter will announce whether you are buyer or seller, which is selected randomly by using the two envelopes. After that experimenter will select the market price randomly by using 30 Ping-Pong balls which are numbered from 1 to 30.

TASK 1

Write you ID here:________

In this task the good is Maribou Premium (86 %Cocoa) Chocolate.
Now state your offer for this chocolate:
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

$\boxed{}$ – $\boxed{}$

TASK 2

Write you ID here:________

In this task the good is 3 cans of Coke. Below are the 5 types of Coke. Now create your own pack by selecting three. You are free to mix and match.
For example if you want all of them to be Coca Cola Zero then write 3 inside the box next to Coca Cola Zero. If you want 2 Zero and 1 Cherry write 2 next to Coca Cola Zero and 1 Coca Cola Cherry.

Amount	Types
	Coca Cola Light
	Coca Cola Zero
	Coca Cola Regular
	Coca Cola Cherry
	Coca Cola Vanilla

Now state your offer for this package includes 3 cans of Coke that you specified above.
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

[ ] – [ ]

TASK 3

Write you ID here:________

Below is the list of Maribou chocolates with different flavors. Mark your favorite one with “X” inside the box next to it.

	Vit Choklad med Smak av vanilj (white chocolate)
	Jordgubb (Strawberry)
	Mörk Choklad (Dark)
	Mjölk Choklad (Milk)
	With Oreo
	Helnöt (Hazelnut)
	M Peanut
	Frukt & Mandel (Fruit and Almond)
	Digestive
	Daim

Now state your offer for your favourite Maribou chocolate that you specified above.

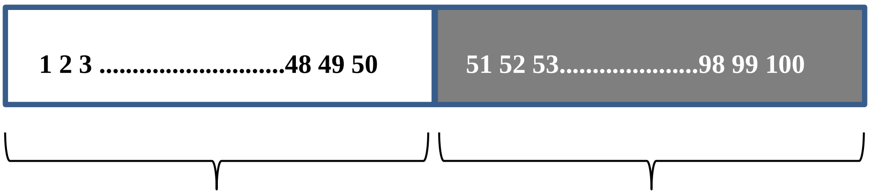
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

☐ – ☐

TASK 4

Write you ID here:________

In this task the good is a lottery ticket gives 30 SEK with 0.5 chance and 0 SEK with 0.5 chance. There is a bag which includes 100 Ping-Pong balls. Each ball is numbered from 1 to 100. At the end experimenter will select a ball randomly from the bag in front of you. If the number on the ball is 50 or below; lottery gives 30 SEK, if the number is 51 and higher it gives nothing. As you can see there is 50:50 chance of winning and losing. Because there are equal numbers of balls (50) that can make you win and equal number of balls (50) that can make you lose. Each ball has equal chance of being selected. Experimenter will select a ball from an opaque bag. You can inspect the material that is used after the experiment.



50 balls for winning **50 balls for loosing**

Now state your offer for the lottery ticket:

If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

–

Instructions for Buyers Group in Intervals Treatment

This is an experiment in individual decision-making. Our purpose is to study technical issues involved in decision-making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn some money and/or other things. What you earn will depend on the decisions you make and some chance. The responses of others do not affect your payoff. It is for your interest to answer truthfully since there is no right or wrong answer in this experiment.

Important Rules

We ask that you do not communicate with other people during the experiment. Please refrain from verbally reacting to events that occur during the experiment. If you have any questions, write your questions on the paper titled as 'Write Your Question Here' which is placed on your desk. After writing your question raise your hand, experimenter will come to your desk and will write the answer on the same sheet.

Important Notice about How to Retrieve Rewards

Please write your ID on each task sheet when you receive them. Keep your ID card because you will retrieve your rewards by showing this card to Mr. Brian Danley after the experiment. Note that the experimenter will not be able to link any specific participant name to a participant identification number. Therefore the experimenter will not know subject payoffs by individual. The person that does this experiment and the person that you will get your earnings from is different. So anonymity of the responses is maintained.

Tasks

You have 4 tasks to complete. In each task there are different goods. So there will be totally 4 goods. After all the tasks are completed, one of the tasks will be selected and played for real. The selection will be done like this: experimenter will write the task numbers on papers and will put them in different envelopes and place the envelopes in an opaque bag and select one of the tasks randomly. All of these will be done in front of you. So each task has equal chance of being selected. It is for your interest to see all of these tasks as real and make your decisions according to it, because each one of them have equal chance to be selected and played for real.

General Instructions for the Tasks

You all are given 100 SEK. You will state your offer and after that, experimenter will announce the randomly selected market price from a specified range (1 SEK to 30 SEK). If your offer is below the market price you will not buy the good. But if your offer is higher than the market price, you will buy the good and pay the market price. The important point here is that you will pay the market price not your offer. The comparison between your offer and market price will determine whether you will buy the good or not.

Notice the following two things:

1. Your decision can have no effect on the market price actually used because the market price will be selected at random.
2. It is in your interest to indicate your true preferences.

As you will see, your best strategy is to determine the maximum you would be willing to pay for the item and offer that amount. It will not be to your advantage to offer more than this maximum, and it will not be to your advantage to offer less. Simply determine the maximum you would be willing to pay and make that amount as your offer.

The market price will be determined randomly by using the Ping-Pong balls. Experimenter will select one of the Ping-Pong balls and that will be the market price. On each Ping-Pong ball; there is a number written on. There are totally 30 Ping-Pong balls, so the market price can be any number between 1 SEK and 30 SEK. Your offer will be compared to the market price. If it is higher than the market price you will buy the good and pay the market price. As you can see the market price will be completely unrelated to your offer and to the offers of all other persons in the room.

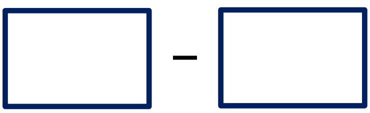
Example: if you offer 1,000 and the market price is happen to be 950, you have the high offer. You buy the item but pay only 950.

If your offer is less than the market price then you do not buy the item. Instead, you keep your money.

Example: if you offer 1,000 and the market price is happen to be 1,020; you do not have the high offer. Therefore, you do not buy the item. You keep your money.

Remember, there are no advantages to strategic behavior. Your best strategy is to determine your personal value for the item and record that value as your offer. There is not necessarily a "correct" value. Personal values can differ from individual to individual.

You will see in the answer sheet that there are two boxes to enter your offer:



If your offer a single amount then write the same number inside the two boxes such as:

1000 – 1000

If you cannot provide a single amount such as 1000-1020 than you can write a range such as:

1000 – 1020

If you wrote a range: you will get the good if the market price falls inside or below the range you specified. In this example if the market price is inside 1000 and 1020 and below 1000 you will buy the good and pay the market price. So it means you will buy the good if the market price is lower than 1020.
Example: Suppose you wrote 1000-1020 and experimenter selected 900 randomly as market price. It means that you will buy the good and pay 900. Because it is lower that your specified range.
What if the random market price happens to be 1010? You buy the good because it is inside the range you specified and pay 1010 and get the good.
What if the random market price happens to be 1021? You do not buy the good because it is higher than your specified range. So you keep your money.
Notice that at the beginning of each task you have 100 SEK. As it is mentioned before, only one of the tasks will be selected randomly and will be played for real.

Guidelines

Guidance to Find Your Offer
After you see the good, start thinking about the smallest monetary unit such as 1 SEK. Ask yourself:
-Do I want to pay 1 SEK for this good?
If your answer is ‘YES’, try to think about a higher amount such as 2 SEK.
-Do I want to pay 2 SEK for this good?
If the answer is ‘YES’, try to think about a higher amount such as 3 SEK.

Increase until you reach an amount that makes you indifferent between getting the good and keeping that amount of your money.

EXAMPLE: Suppose we reached 800 SEK by this reasoning and ask yourself: Would I pay 800 SEK for the good? Yes. Would I pay 900 SEK for the good? No, not that much. Then decrease a little bit. Would I pay 895 SEK for the good? No, not that much. What about 892 SEK? Well, I don't care whether I end up with 892 SEK or the good. Then that is the maximum I'd be willing to pay for the good. You are indifferent between getting the good for 892 SEK and keeping your money. You will record that number on your information sheet.

If you are indifferent between more than one value, you can state a range of values. For example if you think: "paying 892 SEK, 893 SEK and 894 SEK does not matter".

Then you can write 892 SEK – 894 SEK inside the two boxes.

What Is Your 'Best Strategy'?

Remember there is no right or wrong answer but it is for your advantage to be honest and answer truthfully.

What happens if I state a lower amount than my true value?

For example, suppose you think that you would pay a maximum of 1000 SEK for the good, however you wrote a smaller amount in your record sheet, let's say 950 SEK. The experimenter announces the market price as 970 SEK. Since your offer (950 SEK) is lower than the market price (970 SEK), you will not get the good. Remember your true value was 1000 SEK. So by stating a lower value than your true value, you miss the opportunity to get the good that is worth 1000 SEK for you. If you had told the truth by stating 1000 SEK, you could have got the good by paying only 970 SEK.

What happens if I state a higher amount than my true value?

Let's assume that your true value is 1000 SEK and you wrote 1100 SEK on your sheet. The experimenter announces the market price and it happened to be 1050 SEK. Since the market price (1050 SEK) is lower than your stated offer (1100 SEK), you buy the good and pay 1050 SEK. Remember your true maximum offer was 1000 SEK but you have stated a higher amount (1100 SEK). You get the good by paying 1050 SEK; however the good is worth only 1000 SEK to you. So if you pay a higher amount than you are willing to pay for the good. You lose out.

HYPOTHETICAL TASK

The Aim of the task is to train you and make you understand the procedures

Write you ID here:________

Training Task 1: In this task the good is a candy. Now state your offer for that good. The market price will be selected from the range of 1-30 SEK. Each amount in this range has equal chance to be selected. The price increments are 1 SEK. Therefore there are 1,2,3,4,….30 SEK in this range.

If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

–

After everyone completes answering their offer, the experimenter will select the market price.

TASK 1

Write you ID here:________

In this task the good is Maribou Premium (86 %Cocoa) Chocolate
Now state your offer for this chocolate:
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

$\square$ – $\square$

TASK 2

Write you ID here:________

In this task the good is 3 cans of Coke. Below are the 5 types of Coke. Now create your own pack by selecting three. You are free to mix and match.

For example if you want all of them to be Coca Cola Zero then write 3 inside the box next to Coca Cola Zero. If you want 2 Zero and 1 Cherry write 2 next to Coca Cola Zero and 1 Coca Cola Cherry.

Amount	Types
	Coca Cola Light
	Coca Cola Zero
	Coca Cola Regular
	Coca Cola Cherry
	Coca Cola Vanilla

Now state your offer for this package includes 3 cans of Coke.
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

–

TASK 3

Write you ID here:________

Below is the list of Maribou chocolates with different flavors. Order them from your most preferred one to the least preferred one. For example: write 1 next to the type of Maribou that you like most.

	Vit Choklad med Smak av vanilj (white chocolate)
	Jordgubb (Strawberry)
	Mörk Choklad (Dark)
	Mjölk Choklad (Milk)
	With Oreo
	Helnöt (Hazelnut)
	M Peanut
	Frukt & Mandel (Fruit and Almond)
	Digestive
	Daim

Now state your offer for your favourite Maribou chocolate that you stated above.

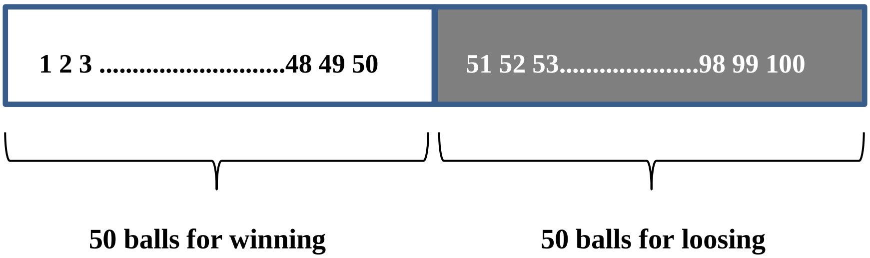
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

☐ – ☐

TASK 4

Write you ID here:________

In this task the good is a lottery ticket that gives 30 SEK with 0.5 chance and 0 SEK with 0.5 chance. There is a bag which includes 100 Ping-Pong balls. Each ball is numbered from 1 to 100. At the end experimenter will select a ball randomly from the bag in front of you. If the number on the ball is 50 or below; lottery gives 30 SEK, if the number is 51 and higher it gives nothing. As you can see there is 50:50 chance of winning and losing. Because there are equal numbers of balls (50) that can make you win and equal number of balls (50) that can make you lose. Each ball has equal chance of being selected. Experimenter will select a ball from an opaque bag. You can inspect the material that is used after the experiment.



Now state your offer for the lottery ticket:
If your offer is a single amount write the same amount in both of the boxes below. However If you cannot provide a single amount, you can enter a range of values. Therefore write the lower bound on the box left and the upper bound on the box right.

–