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# Selling real assets: The impact of idiosyncratic project risk in an auction environment

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**Abstract**

Consider a seller auctioning a real asset among n agents. Each agent contemplates a specific investment project and the asset is crucial for its activation. Project cash flows and their volatility are private information. A first-price auction is considered and the asset is granted in exchange for a payment to be paid at the investment time. Here we determine the optimal bid function and show that the auction is efficient. The asset is assigned to the project characterised by the highest volatility in the associated cash flows. Interestingly, the bid does not depend on the time at which the project is actually executed or on the changes in post-auction cash flows. We also address concerns about the distribution of the project value among the parties and show that i) the winner always holds the largest share of the ex-post project value when projects are characterized by sufficiently high cash flow volatility and ii) negative systematic risk reduces, ceteris paribus, the share accruing to the seller. Finally, we show that cash flow volatility has an ambiguous effect on losses due to the presence of information asymmetry.

KEYWORDS: FIRST-PRICE AUCTIONS, PROCUREMENT, IDIOSYNCRATIC RISK, ADVERSE SELECTION, MORAL HAZARD, CONTINUOUS-TIME MODELS.
JEL CLASSIFICATION: C61, D44, D82.

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# 1 Introduction

In this paper we study a first-price auction for a real asset whose control gives the option to initiate several potential investment projects. There are several sound examples. Consider, for instance, corporate restructuring of distressed state-owned and/or private companies, which may involve a change of ownership and/or ownership structure and also a significant reorganisation of the company's operations.[1] Another example is concession to private agents of natural assets owned by a government. These may include land that, once transferred, may be allocated by the private agent to alternative uses such as agriculture or real estate, or forests and mines which may be exploited on the basis of different management plans.[2] A third example is technological innovation with different potential commercial uses, the value of which may be magnified by granting the right to develop it to another agent.[3]

Governments and private companies owning a specific real asset may want to auction the right to use it simply owing to a need to generate revenue and/or to lack of the managerial and/or technological ability necessary for managing it at its best.

Auctioning a real asset is a challenging task, however. Each potential use must in fact be carefully evaluated so that the asset is assigned to the project that, once developed, magnifies its value. This already demanding task becomes even more complex when crucial information concerning the current and future economic prospects associated with use of the asset is asymmetrically distributed. This asymmetry may concern information about the buyer type, such as their capability to develop the project, the evolution of the process governing the project value or the expected value and/or volatility of the project's rate of return.

Two main issues immediately emerge: evaluation of the asset in light of the potential projects that may be developed once its use is granted and the timing of actual exercise of any embedded investment option.[4] As the value of the project depends on investment timing, the two issues are surely related, but a trade-off between revenue maximisation and investment timing can potentially arise whenever the seller has different preferences concerning the exercise of the investment option held by the selected buyer.

In dealing with these issues, the use of contingent payments has attracted considerable attention in the literature on auctions.[5] Studies worth mentioning in this regard include those by DeMarzo et al. (2005) and Board (2007). By comparing the seller's revenue when bids are in cash (i.e. independent of future events) with those accruing when bids are securities whose value is contingent on the future change in the asset's value, DeMarzo et al. (2005) show that steeper securities yield higher revenues for the seller.[6] Board (2007) examines the seller's optimal payment scheme when auctioning real options from a mechanism design perspective and shows that the optimal mechanism that maximises revenue is composed of an up-front fee and a contingent payment to be made at the time of the investment.[7] Notably, this latter payment does not depend on the value of the project, but only on the private information of the buyer.[8]

In this paper, we examine the implications of using a contingent payment in terms of bidding strategy and ex-post party payoffs in the presence of information asymmetry about i) the state of the process illustrating the investment project's cash flows and ii) the volatility of the project's cash flows.

The novel aspect of this paper is its focus on the volatility of project cash flows as an element of ex ante information asymmetry. This is of interest since using a contingent payment when bidding increases the strike price of the embedded (real) investment option. This in turn means delayed execution. Similarly, as volatility increases, investment in expected terms is delayed. Furthermore, i) the payment is dependent on the realised project's value and ii) the impact of volatility on the project value is ambiguous and depends on the extent to which the project cash flows are characterised by systematic risk.[9] Hence, in the light of the potential conflict between project value and investment timing considerations, investigating the impact on the bidding process and ex-post party payoffs of a privately known volatility level is definitely worth attention.

We examine this by developing our analysis in a continuous-time, first-price auction framework. We consider i) an agent owning a specific asset and auctioning the right to use it and ii) $n$ potential buyers, each contemplating a specific investment project. The value of each project is stochastic and here we characterise its specificity through the volatility of the associated cash flows. Information about the current project's cash flows is known to all agents, while future cash flows and their volatility are private information of the potential buyer.

We derive the winner's bid in closed form. We show that the auction is efficient and assigns the asset to the bidder contemplating the project with the most volatile cash flows. This implies that the winner is, in expected terms, the agent i) investing and, consequently, ii) paying the seller later than everyone else in the pool.

We show that, in line with findings in Board (2007), the bid does not depend on either the time at which the project is actually executed or on the change in post-auction cash flows. Instead, a novel and interesting result concerns the magnitude of the (winning) bid and auction participation. We show in fact that they both depend on the evaluation of the project cash flow at the time of auction. As one can immediately see, these properties have important implications. First, the seller does not need any specific information for holding the auction, i.e. she[10] does not need to be informed about current and future project cash flows. Second, even if informed about the actual realisations, not knowing the actual investment time threshold kills any incentive for renegotiating the contract. Last, the seller may, by setting a cap on the highest acceptable bid, trade revenue off against investment timing.

We address concerns concerning awarding of the asset and the distribution of the project value among the parties in an uncertain economic environment. We show that the winner always holds the largest share of the project value when projects are characterized by sufficiently high volatility in the cash flows. In addition, we find that negative systematic risk reduces, ceteris paribus, the share accruing to the seller. We then compare our findings with the case of a (hypothetical) fully informed seller. We observe that, if the investment projects are characterised by a positive systematic risk component, an informed seller would always opt for the project with the lowest possible volatility level. Hence, since by auctioning the asset the opposite would occur, a notable distortion can be associated with the auction process. Finally, by comparing auctioning the right to use the asset with the case of a fully informed seller, we identify the value loss due to the information failure. We observe that an increase in the level of volatility has an ambiguous effect on the magnitude of losses.

Last, it is worth mentioning that our paper is closely related to the literature examining optimal contracts in a principal agent setting in the presence of private information concerning both the state of the process governing the project value and some of the project's features. DeMarzo and Sannikov (2006), for instance, consider a continuous-time financial contracting model where the state variable is the current cash flow of the project and the agent may decide to divert part of this cash flow for personal gain. The moral hazard problem emerges as the principal does not observe the cash flow. Sung (2005) and Sannikov (2007) examine, in a continuous-time setting, a dynamic agency problem in the presence of both moral hazard and adverse selection. In Sung (2005), an optimal managerial compensation scheme must be set by a principal having imperfect knowledge about the agent's ability to control the project outcome. In Sannikov (2007), an optimal dynamic financing contract is designed in the presence of adverse selection (the agent knows the initial quality of the project) and moral hazard (the agent privately observes the stochastic project cash flows and can manipulate them using hidden savings). Cvitanic and Zhang (2007) develop a continuous-time model where the private information concerns the drift of the underlying process governing the project pay-offs and not the realisations of the process itself. Bergemann and Strack (2015) study a revenue-maximising mechanism for repeatedly selling a non-durable good in a continuous time setting. Each agent's valuation is private information and changes over time. When contracting, each agent privately observes his initial type, i.e. the initial state of the valuation process, the drift or the volatility of the process. In the revenue-maximising mechanism, high initial types are favoured.[11] Kruse and Strack (2015) restate the moral hazard problem of DeMarzo and Sannikov as an adverse selection problem and show how the principal can induce truth telling about the state of the process by setting appropriate transfers that do not depend on private information of the agent. In the same vein, Arve and Zwart (2014) deal with the optimal choice of the supplier in procurement auctions for new technologies when the auctioneer does not observe either the initial value of the investment cost or its change over time.

The remainder of the paper is organised as follows. In section 2 we present the basic set-up for our model. Section 3 identifies the payoff associated with the use of the awarded asset and characterises the auction frame and economic environment. In Section 4 we solve the bidding game and discuss the properties of the solution. In Section 5 we present and discuss the implications of our findings for ex-post project value and relative distribution between the parties. Section 6 presents some conclusions. Appendix A1-A7 contain proofs omitted from the text.

## 2 The basic set-up

Consider a risk-neutral agent owning a real asset, control of which gives the opportunity to activate $n > 1$ potential investment projects. The asset is a close complement to each investment project, so that projects cannot be developed without it. We assume that each potential investment project is irreversible and has an infinite life time. Furthermore, the specificity of each investment project passes through the associated cash flows. In particular, we assume that, once the investment decision has been undertaken, each project $i$ generates a cash flow stream $x_i(t)$ which evolves according to the following diffusion:

$$dx_i(t)/x_i(t) = (r - \delta_i)dt + \sigma_i d\omega_i(t), \text{ with } x_i(0) = x_i > 0, \text{ for } i = 1, ..., n \tag{1}$$

where $r$ is the constant risk-free interest rate, $\delta_i > 0$ is the "*rate-of-return shortfall*" (i.e. a sort of rate of dividend yield),[12] $\sigma_i$ is the constant instantaneous volatility, and $\omega_i(t)$ is a standard Wiener process under a risk-neutral measure.[13]

In order to focus on the impact that project cash flow volatility may have on the allocation of the main asset, we introduce two simplifying assumptions. First, we refrain from considering the presence of a drift for the cash flows, and second, we assume that the project returns are equally correlated to the expected return on the market portfolio. The first assumption can be justified considering that in many real projects the rate of expected change in the cash flows does not depend on the volatility of the underlying asset (see Davis, 2002). The second assumption implies that, even if all project returns are perfectly correlated with respect to their systematic risk, the associated market beta values, $\beta_i = \rho \cdot (\sigma_i/\sigma_m)$, may differ.[14]

Then, by invoking the single-beta version of the Capital Asset Pricing Model,[15] we are able to write $\delta_i$ as:

$$\delta_i = r + \lambda\rho\sigma_i \tag{2.1}$$

where $\lambda$ is the market price of risk, $\rho$ measures the correlation between the return of the project $i$ and the expected return on the market portfolio, $r_m$, and:

$$d\omega_i(t) = \lambda\rho dt + d\psi_i(t) \tag{2.2}$$

where $d\psi_i(t)$ is the increment of a standard Wiener process with $E_0\left[d\psi_i(t)\right] = 0$, $E_0\left[d\psi_i(t)^2\right] = dt$.

In Eq. (2.1) the "rate-of-return shortfall", $\delta_i$, results from adjusting the risk-free interest rate for the systematic risk component,[16] i.e. $\lambda\rho\sigma_i$. The rate responds to change in cash flows volatility and, depending on the sign of $\rho$, the rate can be increasing or decreasing in $\sigma_i$ ($\rho > 0$ and $\rho < 0$, respectively). Eq. (2.2) accounts for the evolution over time in both the systematic and idiosyncratic risk components of the investment project $i$, i.e. $\lambda\rho t$ and $\psi_i(t)$, respectively.

Last, assuming that, for the sake of simplicity, none of the projects requires payment of investment costs to be activated,[17] the current value of the generic investment project $i$ is equal to the expected present value of any future associated cash flow, i.e.:

$$U_i(x_i;\sigma_i) = E_0[\int_0^\infty \exp(-rt)\cdot x_i(t)] = x_i/\delta_i, \text{ for } i = 1, ..., n \tag{3}$$

where $E_0[.]$ is the expectation taken at time $t = 0$ with respect to Eq. (2) and for $x_i(0) = x_i$.[18]

# 3 The investment problem

Suppose now that the asset owner considers auctioning the right to use her asset to a specific risk-neutral bidder (firm) in exchange for a contingent payment to be made at the time of the investment. For the sake of simplicity, we assume that for each project $i$ there is only one potential firm $(i)$ able to undertake it.

## 3.1 Information and auction format

We assume that each bidder has private information on both the cash flow stream $x_i(t)$ and its volatility $\sigma_i$. This means that, at every $t > 0$, the realisations of the process $x_i(t)$ are observed only by bidder $i$. It is, however, public knowledge that $\sigma_i$ is drawn from a common prior cumulative distribution $F(\sigma)$ with continuously differentiable density $f(\sigma)$ defined on a positive support $\Sigma = [\sigma^l, \sigma^h] \subseteq R_+$. Furthermore, we assume that agents' information about $\sigma_i$ and $\psi_i(t)$ is independently distributed among projects.[19] In addition, the asset owner and all $n$ bidders know the initial project cash flow, i.e. $x_i(0) = x_i$. These values may be viewed as the estimates, provided by some independent experts, of the initial cash flow level associated with each project. For convenience, we sort these initial values as $x_1 \geq x_2 \geq ... \geq x_n$.

At $t = 0$, the asset owner establishes a sealed-bid auction where the bidders competitively bid by offering a fixed payment, $p_i$ (or a flow of periodic payments, $w_i$, such that $p_i = w_i/r$). Since the probability distribution of $x_i(t)$ and its future realisations are private information, we exclude contingent payments as a function of the realised cash flow $x_i(t)$. More specifically, we consider only time-contingent payments: i.e. after the auction, the control of the asset is transferred to the winner in exchange for $p_i$ paid at the time of the investment. Then as the winning bidder's cash flow changes over time, he can decide when to exercise the embedded investment call option where the bid plays the role of the strike price.

Our framework is consistent with several potential situations. Consider, for instance, pure equity auctions where it is difficult for investors to verify the actual periodic profits of the firm from which they are buying stock[20] or auctions for contracts granting the right to exploitation of natural resources where payments (i.e. royalties) are set on the basis of estimates of top-line revenues.[21] Furthermore, one may include delivery-contingent contracts for real estate agents who are compensated when they are able to locate (verifiable) suitable buyers[22] or Pre-Commercial Procurement (PCP) where a public buyer contracts for R&D of new innovative goods before they are commercially available.[23]

Finally, at no loss for what may concern our results, we exclude the presence of ownership transfer costs.[24]

### 3.2 The ex-post value of the asset

After the auction, the winning bidder, by gaining full control over the asset, must decide his timing of investment by solving the following problem:[25]

$$V_i(x_i;\sigma_i) = \max_{T_i} E_0[\exp(-rT_i)][(x_i^*/\delta_i) - p_i], \tag{4}$$

where $T_i = \inf\{t \geq 0 \mid x_i(t) = x_i^*\}$ is the bidder's optimal investment time and $x_i^*$ is the cash flow level triggering investment.

The actual investment cost for the bidder in problem (4) is represented by the payment $p_i$ to be paid at $T_i$. We assume that $0 \leq p_i \leq x_i^*/\delta_i$. This implies that, in expected terms, the present value of cash flows accruing from the project, $x_i^*/\delta_i$, covers the initial outflow, $p_i$. Hence, the ex-post value of the project, once invested at $T_i$, is given by $V_i(x_i^*;\sigma_i) = (x_i^*/\delta_i) - p_i \geq 0$.[26]

Problem (4) can be rearranged as follows:

$$V(x_i;\sigma_i) = \begin{cases} (x_i/x_i^*)^{\gamma_i}[(x_i^*/\delta_i) - p_i] & \text{for } x_i < x_i^* \\ (x_i/\delta_i) - p_i & \text{for } x_i \geq x_i^* \end{cases} \tag{5}$$

where $\gamma_i$ is the positive root of $\Psi(\gamma_i) = (\sigma_i^2/2)\gamma_i(\gamma_i - 1) + (r - \delta_i)\gamma_i - r = 0$.[27] As can be easily shown, $\partial\gamma_i/\partial\sigma_i < 0$, $\partial\gamma_i/\partial r > 0$ and $\partial\gamma_i/\partial\rho_i > 0$.[28]

By standard arguments, the optimal investment threshold and project value function are given by:

$$x_i^* = [1 + 1/(\gamma_i - 1)]\delta_i p_i, \tag{6}$$

$$V(x_i, p_i; \sigma_i) = \Gamma(x_i; \sigma_i) p_i{}^{1-\gamma_i}, \tag{7}$$

where $\Gamma(x_i; \sigma_i) = \{(x_i/\delta_i)[1 - (1/\gamma_i)]\}^{\gamma_i}/(\gamma_i - 1)$.

As can be easily seen, the investment threshold is increasing in the level of volatility, i.e. $\partial x_i^*/\partial \sigma_i > 0$. This is a well-known result in the literature on investment under uncertainty. It basically implies that the higher the uncertainty characterising the project pay-off, the later, in expected terms, the project will be undertaken. In other words, the bidder tries to limit any potential downside loss by waiting until the option is sufficiently "*in the money*".

# 4 The auction

In this section we solve the bidding game presented above. On the basis of our set-up, agents i) observe the initial project cash flows $\{x_i,\ i = 1, ..n\}$, ii) have rational expectations about $\psi_i(t)$, and iii) have private information about $\sigma_i$ and $\psi_i(t)$. We can then proceed to the analysis of the underlying game adopting a standard independent private value auction framework.

## 4.1 Equilibrium strategy

Each agent $i$ sets his optimal bidding strategy, $p_i$, by maximising the following function:

$$W(x_i; p_i) = V(x_i; p_i) \cdot \Pr(\text{of win}/p_i) + 0 \cdot (1 - \Pr(\text{of win}/p_i)) \tag{8}$$

where $\Pr(\text{of win}/p_i)$ is the probability of winning the auction conditional on the reported bid $p_i$. Thus, at $t = 0$, with probability $\Pr(\text{of win}/p_i)$, the agent $i$ wins and gets the value associated with the asset, i.e. the value of the embedded investment project, $V(x_i; p_i)$. In contrast, with probability $(1 - \Pr(\text{of win}/p_i))$, the agent does not win and gets 0.

Since $\gamma_i$ is monotonic in $\sigma_i$, bidders may be equivalently characterised in terms of $\gamma_i = \gamma(\sigma_i)$. It follows that, as $d\gamma_i/d\sigma_i < 0$, $G(\gamma_i) = 1 - F(\sigma_i)$, $G(\underline{\gamma}) = 0$ and $G(\overline{\gamma}) = 1$ where $\underline{\gamma} = \gamma(\sigma^h)$ and $\overline{\gamma} = \gamma(\sigma^l)$.

The solution of the bidding game is given in the following proposition:

**Proposition 1** *For any finite $n > 1$, there exists a Bayesian Nash equilibrium in symmetric and strictly increasing strategies $p(\sigma_i)$ for all $i \neq 1$, characterised by:*

*1.1) the bidding function:*

$$p(\sigma_i) = C \cdot \exp(\Phi(\gamma_i)) \cdot F(\sigma_i)^{\frac{n-1}{\gamma_i - 1}}, \quad \text{for } \sigma_i \in [\widehat{\sigma}, \sigma^h] \tag{9}$$

*where $C \geq x_1/r$ is an arbitrary constant, $\Phi(\gamma_i) = \int_{\underline{\gamma}}^{\gamma_i} [\ln(1-G(z))^{n-1}/(1-z)^2]dz < 0$, $p(\sigma^h) = C$ and the cut-off $\widehat{\sigma}$ solves the equation:*

$$p(\widehat{\sigma}) = x_1/r;$$

*1.2) the optimal investment trigger:*

$$x^*(\sigma_i) = [1 + 1/(\gamma_i - 1)]\delta_i p(\sigma_i) \tag{10}$$

*where*

$$x^*(\sigma^h) = [1 + 1/(\underline{\gamma} - 1)]\delta_i(\sigma^h)C \text{ and } x^*(\widehat{\sigma}) = [1 + 1/(\gamma(\widehat{\sigma}) - 1)]\delta(\widehat{\sigma})(x_1/r);$$

*while, for agent 1,*

*2.1) the bidding function is:*

$$p_1 = \max[p(\sigma_1), p(\widehat{\sigma})], \text{ for } \sigma_1 \in [\sigma^l, \sigma^h] \tag{9.1}$$

*2.2) the optimal investment trigger is:*

$$x^*(\sigma_1) = [1 + 1/(\gamma_i - 1)] \cdot \delta_1 \cdot [p(\widehat{\sigma}) + I_{(p(\sigma_1) > p(\widehat{\sigma}))} \cdot (p(\sigma_1) - p(\widehat{\sigma}))] \tag{10.1}$$

*where $I_{(p(\sigma_1) > p(\widehat{\sigma}))}$ is an indicator function which takes value 1 if $p(\sigma_1) > p(\widehat{\sigma})$ and 0 otherwise.*

**Proof.** See Appendix A.3. ■

By taking the derivative of $p(\sigma_i)$ with respect to $\sigma_i$, we can isolate one of the central findings of our model. It is easy to show in fact that $\partial p(\sigma_i)/\partial \sigma_i > 0$. Although the asset is awarded to the ex-ante most efficient agent, i.e. the agent making the highest bid, this corresponds to the bidder who may later undertake the project characterised by the highest volatility in the cash flows. It is also worth highlighting that, for any $C \geq x_1/r$, the bid function (Eq.( 9) and (9.1)) does not depend on the time at which the project is actually executed and on the changes in post-auction cash flows. These properties have some important implications. First, the seller does not need any specific information for holding the auction. Second, even though only the firms are informed about the change in $x_i(t)$ after $t = 0$, this information advantage does not yield any additional rents.[29] Third, even if the seller were able to observe the actual cash flows $x_i(t)$, as the investment timing is not known there would not be any incentive for renegotiating the contract.[30] Finally, the seller is able to set $C$ so that a reserve value can be established and used as a benchmark for assessing submitted bids and selecting participants. In the next section we discuss this issue.

Continuing with the properties of Eq. (9), note that participation in the auction is restricted to a specific set of agents. Only the agents likely to develop a project whose cash flows have a volatility $\sigma_i$ no lower than $\widehat{\sigma} > \sigma^l$ participate (see Appendix A.3). Intuitively, this occurs because the option-like nature of the contract allows the bidders to decide the time of investment. As bidders with more volatile projects benefit from delaying investment, this will stimulate more aggressive bids. In other words, the marginal disutility of an extra dollar of $p_i$ decreases as $\sigma_i$ increases.

Finally, since the degree of shading, $\exp(\Phi(\gamma_i)) \cdot F(\sigma_i)^{\frac{n-1}{\gamma_i - 1}} < 1$, decreases with the number of bidders, the level of competition has an important impact on bidding behaviour. In fact $\partial p_i/\partial n < 0$, which in turn implies that $\partial x_i^*/\partial n < 0$, i.e. delays in the project activation are less likely when the level of competition is high. This is consistent with our framework since, while squeezing agents' rents, open competition can induce the agents to anticipate their investment for balancing profit reduction.

In Figure 1 we illustrate our findings by drawing the bid function and the corresponding investment threshold as functions of $\sigma_i$ for a specific range of parameter values, i.e. $x_1 = 4.5$, $C = 100$, $\lambda = 0.30$, $r = 0.05$ and $\rho = \{-1, -0.5, 0, 0.5, -1\}$. The solid lines indicates the bids and the corresponding triggers within the admissible range $\Sigma(\hat{\sigma}) = [\hat{\sigma}, \sigma^h]$. The restriction on the range of admissible $\sigma_i$ depends on the correlation parameter $\rho$ and is set in order to ensure that $\delta_i > 0$.

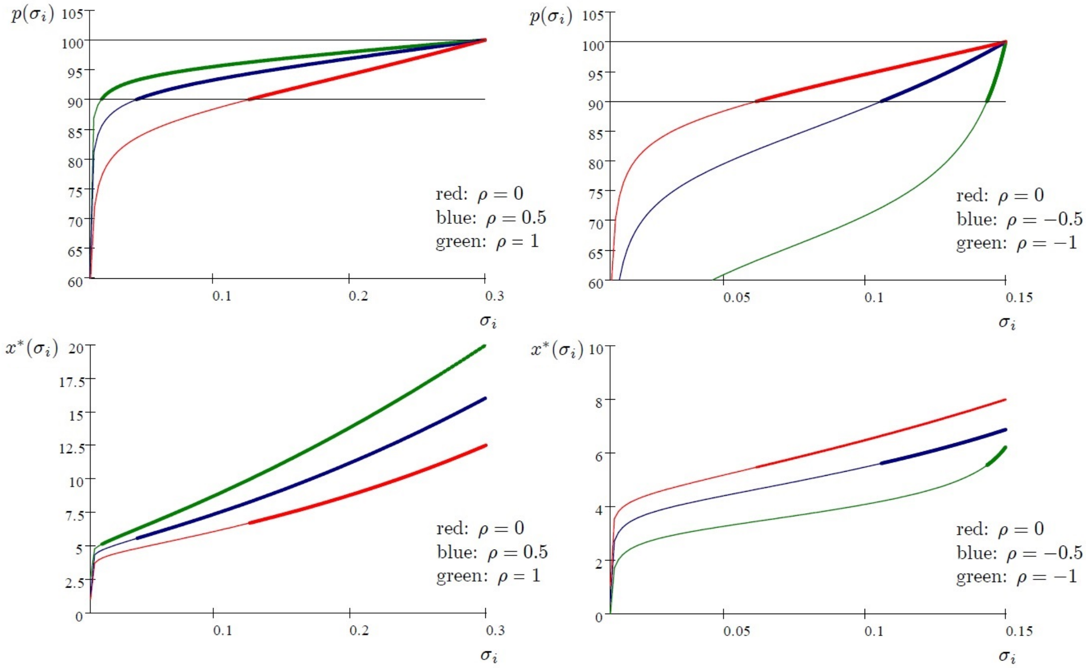


**Figure 1**: Bids and investment thresholds for $x_1 = 4.5,\ C = 100,\ \lambda = 0.30,\ r = 0.05$

## 4.2 Auction participation and bidding cap

By studying the equilibrium in Proposition 1, we observe three important aspects about auction participation. First, we observe that participation in the auction depends on the degree of potential competition. In particular, competition may restrict the participation only to the agents having very valuable projects which, in our frame, are the projects characterised by higher volatility in their cash flows. This conclusion finds support in Proposition 2:

**Proposition 2** *As $n$ increases, fewer agents actively participate in the auction, i.e.:*

$$\partial\widehat{\sigma}/\partial n = -\int_{\underline{\gamma}}^{\gamma(\widehat{\sigma})} \ln(1-G(z))/(1-z)^2 dz/[(n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] > 0 \qquad (11)$$

**Proof.** See Appendix A.4 ■

Second, as expected, the participation is negatively related to the rank of the initial cash flows, i.e.:

**Proposition 3** *An increase in agent $1'$s revenue reduces the number of agents that participate in* *the auction, i.e.:*

$$\partial\widehat{\sigma}/\partial x_1 = 1/r(n-1)[(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] > 0 \qquad (12)$$

**Proof.** See Appendix A.4 ■

Finally, it is interesting to study the impact that a change in $C$ has in terms of participation. This is given by Proposition 4.

**Proposition 4** *As $C$ increases, more agents will actively participate in the auction, i.e.:*

$$\partial\widehat{\sigma}/\partial C = -1/C(n-1)[(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] < 0 \qquad (13)$$

**Proof.** See Appendix A.4 ■

Hence, the exogenous parameter $C$ may be thought as capturing the actual target set by the seller in terms of participation. More specifically, $C$ may be considered as a cap set on the maximum level of allowed bids, or equivalently, by the relationship between presented bid, $p_i$, and the corresponding investment trigger, $x_i^*$, as a limit imposed to the maximum acceptable investment timing.

Note that, if this is the case, setting, for instance, a looser cap would have a twofold effect. In fact, it would increase the range of types participating in the auction and it would also increase the payment finally accruing to the seller. Nothing would change concerning the characteristics of the winning bid. The asset would, in fact, still be awarded, to the agent among the participants investing in the project with higher volatility in the cash flows. However, as a higher payment is due to the seller, the project, in expected terms, will clearly be delayed.

In other words, the level of discretion by the seller in deciding the range of risky projects permitted to participate in the auction magnifies the effect of uncertainty vis-a-vis the effect of competition. In this respect, each bidder taking account of the uncertainty about his project's cash flows and the level of competition strategically chooses a higher degree of flexibility that results in an increase in both the bid and the investment trigger.[31]

On the basis of these considerations, suppose that the seller sets $C$ by targeting a certain probability that the investment will be eventually undertaken. In particular, defining with $q(x_i; x^*)$ the probability that the process in Eq. (2) will eventually hit the threshold $x^*(\sigma_i)$, this is equal to (see Dixit, 1993):

$$q(x_i; x^*) = x_i/x^*(\sigma_i) = [1 - (1/\gamma_i)] \cdot (x_i/\delta(\sigma_i)p(\sigma_i)) = (1/p(\sigma_i))(x_i/\iota(\sigma_i)) \tag{14}$$

where $\iota(\sigma_i) = r + (1/2){\sigma_i}^2\gamma_i$.[32] Notice that the probability of investment is basically given by the ratio between the present value of the stream of $x_i$ computed at $t = 0$ using the adjusted discount rate $\iota(\sigma_i)$ and the price paid to the seller to be awarded the asset. Note also that, as expected, the probability of investment is unambiguously decreasing in $\sigma_i$, i.e. $dq/d\sigma_i = -(q/x^*)(dx^*/d\sigma_i) < 0$.

Now suppose that, with the information available at $t = 0$, the seller considers the project with the highest initial cash flows, i.e. $x_1$, and the potentially most risky project in the range $\Sigma = [\sigma^l, \sigma^h]$.[33] Hence, by Eq. (9), the corresponding cap is such that:

$$q^{\min}C = x_1/\iota(\sigma^h) \tag{15}$$

where $q^{\min}$ is the targeted (minimally acceptable) probability of investment.[34] By Eq. (15), consistently with our discussion above, $C$ is set such that the minimal expected payment the seller would receive is equal to the stream of $x_1$ discounted by the adjusted discount rate $\iota(\sigma^h)$.

In order to illustrate the impact of introducing a bid cap, in Figure 2 we plot the bid function and the corresponding investment thresholds for $x_1 = 4.5$, $\lambda = 0.30$, $r = 0.05$ and $\rho = 0.5, -0.5$. This is done for three potential levels of minimal probability of investment, $q^{\min}$, namely for 20%, 25% and 30%. We observe that, irrespective of the sign of $\rho$, bids are decreasing in the strictness of the cap. In contrast, investment, in expected terms, is anticipated. Last, in line with Proposition 4 but only evident for the scenario where $\rho = 0.5$ and $q^{\min} = 30\%$, the number of projects considered by the seller (on the solid thicker line) is decreasing with the strictness of the cap.

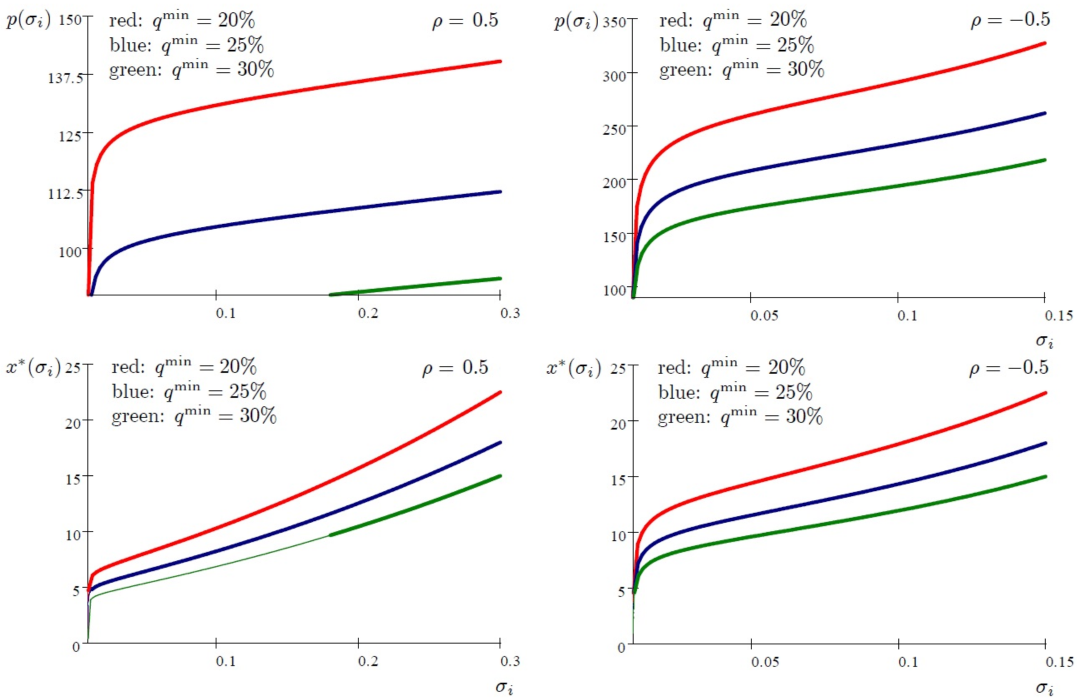


**Figure 2**: Bids and investment thresholds with cap for $x_1 = 4.5,\ \lambda = 0.30,\ r = 0.05$

We conclude this section by discussing the limit case where $C = x_1/r$. In this case, the seller basically awards the asset to agent 1 in exchange for the payment flow $p_1 = x_1/r$. Then, once offered $p_1$, consistently with his own type, $\sigma_1$, agent 1 will activate the project at:

$$x^*(\sigma_1) = [1 + 1/(\gamma_i - 1)]\delta_1(x_1/r)$$

# 5 Model implications

In this section we investigate the implications that selling the right to develop the asset may have on the distribution of the ex-post project value among the parties. We also investigate the role played by risk in the ex-post distribution. More specifically, in Section 5.1 we show how the value of the winner's project is shared between the seller and the winning bidder. In Section 5.2, using as a benchmark the ex-post value that could have been generated under a first-best scenario, we examine the losses arising in our auction frame. In Section 5.3 we study the impact that selecting riskier projects has on the parties' share. In all cases, we employ numerical examples to illustrate our findings.

## 5.1 Value shares

The ex-post values accruing to winner and seller are:

$$V(x_i; p_i) = (x_i/x_i^*)^{\gamma_i} p_i/(\gamma_i - 1) \tag{16.1}$$

$$R(x_i; p_i) = (x_i/x_i^*)^{\gamma_i} p_i = V(x_i; p_i)(\gamma_i - 1) \tag{16.2}$$

respectively, where Eq. (16.1) is obtained by substituting Eq. (6) into Eq. (5).

The ex-post social project's value, $S(x_i; p_i)$, is equal to the sum of the parties' payoffs, i.e.

$$S(x_i; p_i) = V(x_i; p_i) + R(x_i; p_i) = \gamma_i V(x_i; p_i) \tag{17}$$

It is easy to show that the project value shares accruing to the parties are

$$V(x_i; p_i)/S(x_i; p_i) = 1/\gamma_i \tag{17.1}$$

$$R(x_i; p_i)/S(x_i; p_i) = 1 - (1/\gamma_i) \tag{17.2}$$

Note that, since $\partial\gamma_i/\partial\sigma_i < 0$, $\partial\gamma_i/\partial r > 0$ and $\partial\gamma_i/\partial\rho > 0$, the share of the project value accruing to the winner is increasing in the volatility of its cash flow and decreasing in the risk-free interest rate and in the correlation of the project returns with the return on the market portfolio. Opposite considerations should be made when considering the seller. Concerning the impact of volatility, we notice that

**Proposition 5** *If $\gamma_i < 2$, the winner holds the largest share of the value of the project. Otherwise, the opposite occurs.*

This means that the winner is paid the largest share when projects are characterized by highly volatile cash flows. An interesting limit result is $\lim_{\sigma\to\infty}(1/\gamma_i) = 1$ which implies that the winner would be able to cash the entire value of the project. So, at least for what may concern the share, as the auction always awards the asset to the riskier project (see Proposition 1), the seller may be seen as losing. However, this is not necessarily the case as the social value, $S(x_i; p_i)$, totally generated is, in contrast, increasing in $\sigma_i$ (see Figure 4). Last, as $\partial\gamma_i/\partial\rho > 0$, negative systematic risk reduces, ceteris paribus, the share accruing to the seller.

In Figure 3 we illustrate these findings by plotting $V(x_i; p_i)$, $R(x_i; p_i)$ and $S(x_i; p_i)$ as a function of $\sigma_i$ for the scenarios $\rho = 0.5$ and $\rho = -0.5$. Other parameters are as above. In Figure 3 we also check for the effect of setting a cap on the acceptable bids. We consider three levels of probability of actual investment, namely $q^{\min} = 20\%$, 25% and 30%. We note that, ceteris paribus and irrespective of the sign of $\rho$, a higher social value, $S(x_i; p_i)$, can be generated in the presence of a stricter cap.

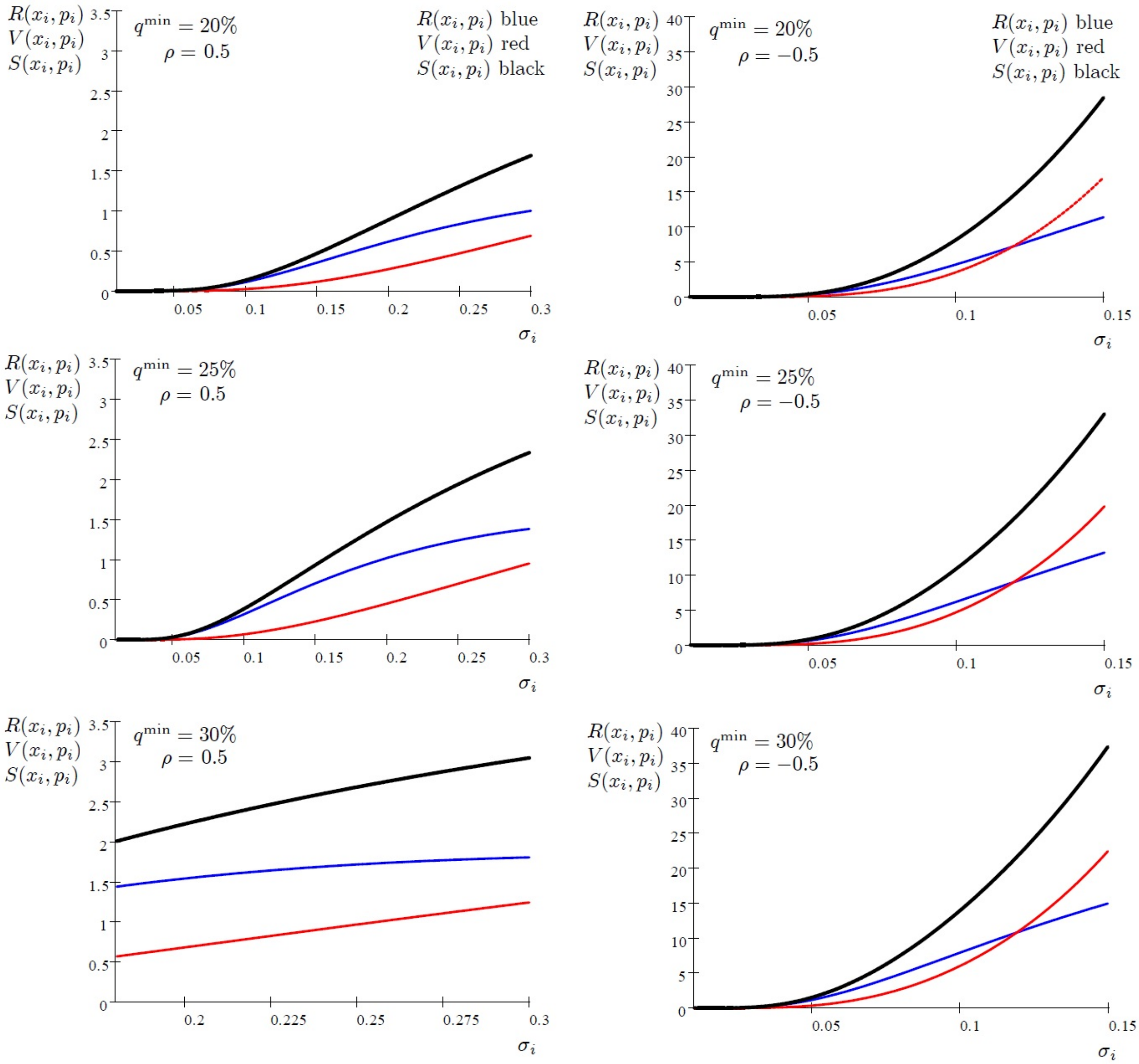


**Figure 3** : $V(x_i; p_i)$, $R(x_i; p_i)$ and $S(x_i; p_i)$ for $x_1 = 4.5$, $\lambda = 0.30$, $r = 0.05$

## 5.2 Social loss

The ex-post social loss due to the presence of an information failure is defined as the difference between the outcome, in terms of ex-post project value, resulting in a first-best scenario and that accruing when auctioning the asset, i.e.:

$$\begin{aligned} L(x_i;\sigma_i) &= U(x_i;\sigma_i) - S(x_i;p_i) \\ &= [1-(x_i/x_i^*)^{\gamma_i-1}]U(x_i;\sigma_i) > 0 \end{aligned} \tag{18}$$

From Eq. (18), the loss due to the information failure corresponds to the portion $[1-(x_i/x_i^*)^{\gamma_i-1}]$ of the first-best outcome. Note that $(x_i/x_i^*)^{\gamma_i-1} = V_{x_i}/\delta_i < 1$ where $V_{x_i} = \partial V(x_i;p_i)/\partial x_i$. We can then rearrange Eq. (18) as follows:

$$L(x_i;\sigma_i) = (\delta_i - V_{x_i})U(x_i;\sigma_i)/\delta_i > 0 \tag{18.1}$$

where $\delta_i - V_{x_i} > 0$.[35] From Eq. (18.1), the magnitude of losses can be linked to the difference between the rate-of-return shortfall, $\delta_i$, of the winning project and the marginal return, $V_{x_i}$, attached to the option to invest in the winning project at the time of award. It is worth stressing that, ceteris paribus, as $V_{x_i}$ is increasing in $x_i$, the seller may be able to reduce the ex-post social loss $L(x_i;\sigma_i)$ by choosing when the auction should be held.[36]

### 5.3 Are riskier projects better?

In a first-best scenario, the ex-post social project value would be equal to $U_i(x_i;\sigma_i)$. This value is affected by the cash flow volatility as follows:

$$\partial U_i(x_i;\sigma_i)/\partial\sigma_i = \begin{cases} < 0 & \text{for } \rho > 0, \\ \geq 0 & \text{for } \rho \leq 0. \end{cases} \tag{19}$$

This result leads to the following consideration:

**Remark 2:** *In a first-best scenario, having the possibility of choosing any of the available $n$ investment projects, the seller would choose the project with the highest expected present value, i.e. $\max[U_i(x_i;\sigma_i)]$ for all $i$. From Eq. (19) and provided that $x_i/\delta_i \geq x_1/\delta_1$, this is the project with cash flows characterised by i) the highest volatility for $\rho > 0$ or ii) the lowest volatility for $\rho \leq 0$.*

Hence, as the auction would always award the asset to the project with the highest volatility in the cash flows, the ranking identified in Remark 2 is fully violated when the systematic risk component is positive or, in other words, the project is positively correlated with the market portfolio.

Pushing the analysis further, it is interesting to examine, still using as a benchmark the first-best payoff $U(x_i;\sigma_i)$, how the ex-post social loss responds to changes in the volatility level. In order to do this, we first define the ratio:

$$\Delta^S(x_i;p_i) = S(x_i;p_i)/U(x_i;\sigma_i) = S(x_i;p_i)(\delta_i/x_i) < 1; \tag{20}$$

taking its derivative with respect to $\sigma_i$ yields the following result:

**Proposition 6** *An increase in $\sigma_i$ has an ambiguous effect on the ex-post social loss, i.e.:*

$$\begin{aligned}\partial\Delta^S(x_i;p_i)/\partial\sigma_i &= \Delta^S(x_i;p_i)\{[\ln(x_i/x_i^*) + (1/\gamma_i)](\partial\gamma_i/\partial\sigma_i) - (n-1)(f(\sigma_i)/F(\sigma_i)) + \\ &\quad -\lambda\rho(\gamma_i - 1)/\delta_i\} \qquad \textit{(20.1)}\end{aligned}$$

**Proof.** See Appendix A.7 ■

Three effects are in place. The first is the so-called "*asset substitution*" (see Shibata 2009, p. 916). If $|\ln(x_i/x_i^*)| < 1/\gamma_i$, then a riskier project reduces social losses; otherwise, an increase in $\sigma_i$ reduces the social value of the project. The second, definitely negative, effect depends on the information rents to be paid to the most efficient bidder. The third is the correlation between the project returns and the return on the market portfolio. In this respect, we note that, as expected, the relative term enters positively for $\rho < 0$.

Similar considerations can be made when considering the ratio between the ex-post value accruing to the winner and the first-best outcome:

$$\Delta^R(x_i;p_i) = R(x_i;p_i)/U(x_i;p_i) = [1 - (1/\gamma_i)]\Delta^S(x_i;p_i) < 1; \tag{21}$$

and its derivative with respect to $\sigma_i$, i.e.

$$\begin{aligned}\partial\Delta^R(x_i;p_i)/\partial\sigma_i &= [1 - (1/\gamma_i)]\Delta^S(x_i;p_i)[\ln(x_i/x_i^*) + 1/(\gamma_i - 1))(\partial\gamma_i/\partial\sigma_i) + \\ &\quad -(n-1)(f(\sigma_i)/F(\sigma_i)) - \lambda\rho(\gamma_i - 1)/\delta_i] \qquad (21.1)\end{aligned}$$

To illustrate how these three effects work, we plot in Figure 4 the social value accruing when auctioning the asset, $S(x_i;p_i)$, and the associated losses, $L(x_i;\sigma_i)$, as a function of $\sigma_i$ for the scenarios $\rho = 0.5$ and $\rho = -0.5$. We again consider three levels of probability of actual investment, namely $q^{\min} = 20\%$, $25\%$ and $30\%$. Other parameters are as above. We observe that, irrespective of the sign of $\rho$, $S(x_i; p_i)$ is increasing in the volatility of the winning project. However, since $U_i(x_i; \sigma_i)$ depends on the sign of $\rho$, the loss curve, $L(x_i; \sigma_i)$, takes a different shape. Note in fact that for a positive $\rho$, losses are decreasing in $\sigma_i$ while, driven by the term $\lambda\rho(\gamma_i - 1)/\delta_i$, they are increasing for the case of a negative systematic risk. We observe that, however, the rate of increase is decreasing in $\sigma_i$. We also observe that, irrespective of the sign of $\rho$, losses are lower when a stricter cap is imposed on bids. This positive effect is exclusively due to the higher social value that, ceteris paribus, can be generated in the presence of a stricter cap.

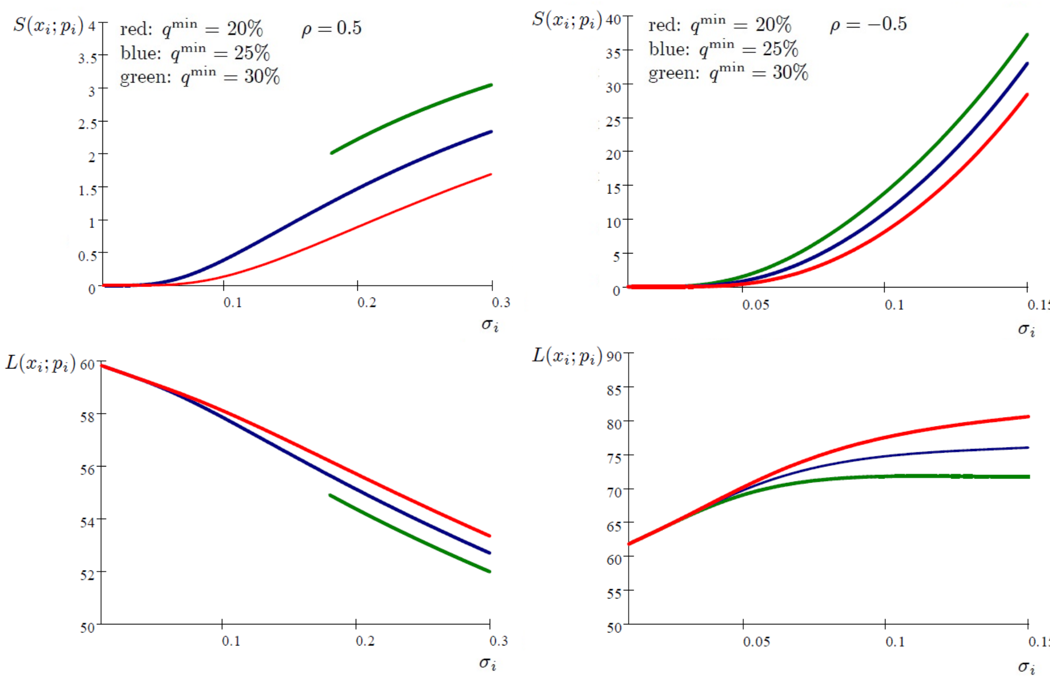


**Figure 4**: Social value and losses for $x_1 = 4.5,\ \lambda = 0.30,\ r = 0.05$

## 6 Conclusions

In several economic situations, the right to use a real asset is essential for activation of an investment project. In this paper, we consider a seller who auctions such an asset among n agents. Each agent contemplates a potential investment project and has private information about the associated cash flows and their volatility. The asset is granted in exchange for a payment to be made at the time of investment and is awarded to the bidder making the highest bid. We show that the auction is efficient and assigns the asset to the agent contemplating the investment project characterised by the highest volatility in the associated cash flows. The winner is then the agent i) investing and, consequently, ii) paying the seller later than anyone else in the project pool. The optimal bid function has interesting properties, namely, the bid does not depend on: i) the time at which the project is actually executed and ii) the change in post-auction cash flows. We also examine the distribution of the ex-post project value among the parties and show that i) the winner always holds the largest share of the project value when projects are characterized by sufficiently high volatility in the cash flows and that ii) negative systematic risk reduces, ceteris paribus, the share accruing to the seller. We then evaluate, using the case of a fully informed seller as a benchmark, the impact that information issues have in a dynamic and uncertain environment. We show that when project returns and return on the market portfolio are positively correlated, a fully informed seller would always grant the asset to the agent considering a project with the lowest volatility in cash flows. This is in evident contradiction of the auction outcome, by which the asset would be granted to the project with the lowest volatility in cash flows. Last, when comparing first-best and auction outcomes from a societal perspective, we show that an increase in the level of volatility has an ambiguous effect on the magnitude of losses due to the presence of information asymmetry.

# A Appendix A1-A7

## A.1 Project cash flow and its diffusion

Assume that the stream $x_i(t)$ follows geometric Brownian motion:

$$dx_i(t)/x_i(t) = \mu_i dt + \sigma_i d\psi_i(t)$$

where $\mu_i$ is the drift rate, $\sigma_i$ is the constant instantaneous volatility, and $\psi_i(t)$ is a standard Wiener process. Under the assumption of a complete capital market, a traded security (or a portfolio) $y_i(t)$ capable of hedging the risk of the process $\psi_i(t)$ exists. Assume that $y_i(t)$ follows a stochastic differential equation of the form $dy_i(t)/y_i(t) = \eta_i dt + \xi_i d\psi_i(t)$. Given the assumption of complete markets, the process $y_i(t)$ can be written as (Harrison and Pliska, 1981):

$$\begin{aligned} dy_i(t)/y_i(t) &= rdt - rdt + \eta_i dt + \xi_i d\psi_i(t) \\ &= rdt + \xi_i d\omega_i(t), \end{aligned} \tag{A.1.1}$$

where $r$ is the riskless interest rate, $(\eta_i - r)/\xi_i$ is the market price of the risk class $\psi_i(t)$ and $d\omega_i(t) = (1/\xi_i)(\eta_i - r)dt + d\psi_i(t)$. Under the measure $\omega_i(t)$, the process $x_i(t)$ can be written as:

$$\begin{aligned} dx_i(t)/x_i(t) &= \mu_i dt + \sigma_i d\psi_i(t) \\ &= [\mu_i - (\sigma_i/\xi_i)(\eta_i - r)]dt + \sigma_i d\omega_i(t) \\ &= (r - \delta_i)dt + \sigma_i d\omega_i(t), \end{aligned} \tag{A.1.2}$$

where $\delta_i = r + (\sigma_i/\xi_i)(\eta_i - r) - \mu_i$. Note that $r + (\sigma_i/\xi_i)(\eta_i - r)$ represents the project's expected rate of return, i.e. $(E_{x_i}(dx_i)/dt)/x_i = \delta_i + \mu_i$. In order to obtain Eq. (2), it suffices to set $\mu_i = 0$ and $(\eta_i - r)/\xi_i = \lambda\rho_i$, where $\lambda = (r_m - r)/\sigma_m$ is the market price of risk with $r_m$ and $\sigma_m$ indicating expected return and volatility of the market portfolio, respectively, and $\rho_i = cov(dx_i/x_i, r_m)/\sigma_i\sigma_m$ measures the correlation of the asset $x_i$ with the market portfolio. Finally, a simple algebra yields:

$$\begin{aligned} \rho_i &= cov(dx_i/x_i, r_m)/\sigma_i\sigma_m = cov((r - \delta_i)dt + \sigma_i d\omega_i(t), r_m)/\sigma_i\sigma_m \\ &= cov(\sigma_i d\omega_i(t), r_m)/\sigma_i\sigma_m = cov(d\psi_i(t), r_m)/\sigma_m. \end{aligned} \tag{A.1.3}$$

## A.2 Some comparative statics

From $\Psi(\gamma_i) = 0$ we obtain:

$$\partial\gamma_i/\partial\sigma_i = \gamma_i[\lambda\rho - \sigma_i(\gamma_i - 1)]/Y < 0 \tag{A.2.1}$$

$$\partial\gamma_i/\partial r = 1/Y > 0 \tag{A.2.2}$$

$$\partial\gamma_i/\partial q = \sigma_i\gamma_i/Y > 0 \tag{A.2.3}$$

where $q = \lambda\rho$ and $Y = (1/2)\sigma_i^2(2\gamma_i - 1) + (r - \delta_i)$.

Note in fact that:

$$\lambda\rho < \sigma_i(\gamma_i - 1) \text{ and } Y > 0$$

## A.3 Proof of Proposition 1

Agent $i'$s expected payoff from bidding $p_i$ is given by:

$$W(x_i; p_i) = V(x_i; p_i) \cdot \Pr(\text{of win}/p_i) + 0 \cdot (1 - \Pr(\text{of win}/p_i)). \tag{A.3.1}$$

Now, consider the agent $i'$s bidding behaviour. Assume that all other agents use a strictly monotonically increasing bid function $p(\sigma_j)$, i.e. $p(\sigma_j) : [\sigma^l, \sigma^h] \to [p(\sigma^l), p(\sigma^h)]\ \forall j \neq i$. Since, by assumption, $p(\sigma_i)$ is monotonous in $[\sigma^l, \sigma^h]$, the probability of winning by bidding $p(\sigma_i)$ is $\Pr(p(\sigma_i) > p(\sigma_j) \mid \forall j \neq i) = \Pr(\sigma_j < p^{-1}(p(\sigma_i)) \mid \forall j \neq i) = F(\sigma_i)^{n-1}$. It follows that agent $i$ chooses reporting $\widetilde{\sigma}_i$ by solving the following problem:

$$\begin{aligned} W(\sigma_i, \widetilde{\sigma}_i) &= \max_{\widetilde{\sigma}_i} V(x_i; p(\widetilde{\sigma}_i)) \Pr(\text{of win}/p(\widetilde{\sigma}_i)) = \max_{\widetilde{\sigma}_i} V(x_i; p(\widetilde{\sigma}_i)) \Pr(p(\widetilde{\sigma}_i) > \max_{j \neq i} p_j) \\ &= \max_{\widetilde{\sigma}_i} V(x_i; p(\widetilde{\sigma}_i)) F(\widetilde{\sigma}_i)^{n-1}, \end{aligned} \tag{A.3.2}$$

where $F(\widetilde{\sigma}_i)^{n-1}$ is the probability that all other bidders have a $\widetilde{\sigma}_i$ lower than that of the winner.

Note that bidders may be equivalently characterised in terms of $\gamma_i$. It follows that, as $d\gamma_i/d\sigma_i < 0$, $G(\gamma_i) = 1 - F(\sigma_i)$, $G(\underline{\gamma}) = 0$ and $G(\overline{\gamma}) = 1$ where $\underline{\gamma} = \gamma(\sigma^h)$ and $\overline{\gamma} = \gamma(\sigma^l)$. Hence, maximising the objective (A.3.2) with respect to $\widetilde{\sigma}_i$ and imposing the truth-telling condition $\widetilde{\sigma}_i = \sigma_i$ yields the necessary condition:

$$\partial W(\sigma_i, \widetilde{\sigma}_i)/\partial\widetilde{\sigma}_i|_{\widetilde{\sigma}_i = \sigma_i} = \partial W(\gamma_i, \widetilde{\gamma}_i)/\partial\widetilde{\gamma}_i|_{\widetilde{\gamma}_i = \gamma_i} \cdot \partial\gamma_i/\partial\sigma_i| = 0, \tag{A.3.3}$$

where $\widetilde{\gamma}_i = \gamma(\widetilde{\sigma}_i)$ and $W(\gamma_i, \widetilde{\gamma}_i) = V(x_i; p(\widetilde{\gamma}_i))(1 - G(\widetilde{\gamma}_i))^{n-1}$.

This is equivalent to imposing:

$$\begin{aligned}
\partial W(\gamma_i, \widetilde{\gamma}_i)/\partial\widetilde{\gamma}_i|_{\widetilde{\gamma}_i=\gamma_i} &= \Gamma(x_i; \sigma_i)(1-\gamma_i)p(\gamma_i)^{-\gamma_i}\ \partial p(\gamma_i)/\partial\widetilde{\gamma}_i|_{\widetilde{\gamma}_i=\gamma_i}(1 - G(\widetilde{\gamma}_i))^{n-1} + \\
&\quad -\Gamma(x_i; \sigma_i)p(\gamma_i)^{1-\gamma_i}(n-1)[g(\gamma_i)/(1 - G(\gamma_i))](1 - G(\widetilde{\gamma}_i))^{n-1} \\
&= W(\gamma_i)[(1-\gamma_i)(\partial p(\gamma_i)/\partial\widetilde{\gamma}_i|_{\widetilde{\gamma}_i=\gamma_i}/p(\gamma_i)) - (n-1)[g(\gamma_i)/(1 - G(\gamma_i))]] = 0. \qquad \text{(A.3.4)}
\end{aligned}$$

By Eq. (A.3.4), the maximisation problem can be reduced to the following first-order linear differential equation:

$$\partial p(\gamma_i)/\partial\gamma_i - (n-1)[g(\gamma_i)/(1 - G(\gamma_i))][p(\gamma_i)/(1-\gamma_i)] = 0. \qquad \text{(A.3.5)}$$

The solution to the differential equation (A.3.5) is given by:

$$\begin{aligned}
p(\gamma_i) &= C \cdot \exp((n-1)\int_{\underline{\gamma}}^{\gamma_i}\{[g(z)/(1 - G(z))]/(1-z)\}dz) \\
&= C \cdot \exp(-(n-1)\,|\ln(1 - G(z))/(1-z)|_{\underline{\gamma}}^{\gamma_i} + \Phi(\gamma_i)) \\
&= C \cdot (1 - G(\gamma_i))^{\frac{n-1}{\gamma_i - 1}} \cdot \exp(\Phi(\gamma_i)), \qquad \text{(A.3.6)}
\end{aligned}$$

where $\Phi(\gamma_i) = \int_{\underline{\gamma}}^{\gamma_i}[\ln(1 - G(z))^{n-1}/(1-z)^2]dz$ and $C$ is an arbitrary constant.

Rearranging in terms of $\sigma_i$, we have:

$$\begin{aligned}
p_i &= p(\sigma_i) = C \cdot \exp(\Phi(\gamma_i)) \cdot F(\sigma_i)^{\frac{n-1}{\gamma_i - 1}} \qquad \text{(A.3.7)} \\
x_i^* &= x^*(\sigma_i) = [1 + 1/(\gamma_i - 1)]p(\gamma_i)\delta_i = [(\sigma_i^2/2)\gamma_i + r]p_i \qquad \text{(A.3.8)}
\end{aligned}$$

where

$$V(x_i; p_i) = \Gamma(x_i; \sigma_i) \cdot [\exp((1-\gamma_i)\int_{\underline{\gamma}}^{\gamma_i}\ln(1 - G(z))/(1-z)^2 dz/(1 - G(\gamma_i))]^{n-1} \cdot C^{1-\gamma_i} = \Gamma(x_i; \sigma_i)p_i^{1-\gamma_i} \qquad \text{(A.3.9)}$$

where $\Gamma(x_i; \sigma_i) = \{(x_i/\delta_i)[1 - (1/\gamma_i)]\}^{\gamma_i}/(\gamma_i - 1)$ or, equivalently,

$$V(x_i; p_i) = (x_i/x_i^*)^{\gamma_i}p_i/(\gamma_i - 1) \qquad \text{(A.3.9a)}$$

By evaluating the extremes, we have:

$$p(\sigma^h) = C, \quad p(\sigma^l) = 0, \qquad \text{(A.3.7a-A.3.7b)}$$

$$x^*(\sigma^h) = [1 + 1/(\underline{\gamma} - 1)]\delta_i(\sigma^h)C \text{ and } x^*(\sigma^l) = 0. \qquad \text{(A.3.8a-A.3.8b)}$$

Note that each agent identifies two potential bids contingent to the exercise time, i.e. $p(\sigma_i)$ and $x_i(0)/r$. Note that $x_i(0)/r$ is the only alternative bid that an auctioneer not able to verify the actual cash flows $x_i(t)$ may accept. In order to maximise the probability of winning, the bidder should report the highest value between the two potential bids, i.e. $p_i = \max[p(\sigma_i), x_i(0)/r]$. However, since, by assumption, the initial values $x_i$ are publicly known, each agent knows that i) agent 1 will report $p_1 = \max[p(\sigma_1), x_1/r]$ and ii) $x_1 \geq x_2 \geq ..... \geq x_n$. Hence, agent $i$ participates in the auction if, and only if, $p(\sigma_i) \geq x_1/r$ for any $i \neq 1$. It follows that actual participation in the auction is restricted to agent types in the range $\sigma_i \geq \widehat{\sigma}$ where the cut-off type $\widehat{\sigma}$ is determined by solving the following equation:

$$p(\widehat{\sigma}) = x_1/r \tag{A.3.7c}$$

On the basis of these considerations, note that for $C = x_1/r$, the auctioneer is basically awarding the asset to agent 1 in exchange for the payment flow $p_1 = x_1/r$.

It is easy to show that both the payment, $p_i$, and the investment trigger, $x_i^*$, are monotonically increasing in $\sigma_i$. Concerning the payment, note in fact that:

$$\begin{aligned}
\partial p_i/\partial\sigma_i &= C \cdot [\partial(1-G(\gamma_i))^{\frac{n-1}{\gamma_i-1}}/\partial\gamma_i + (1-G(\gamma_i))^{\frac{n-1}{\gamma_i-1}} \cdot \partial\Phi(\gamma_i)/\partial\gamma_i] \exp(\Phi(\gamma_i))(\partial\gamma_i/\partial\sigma_i) \\
&= -(n-1)[g(\gamma_i)/(1-G(\gamma_i))]p_i(\partial\gamma_i/\partial\sigma_i)/(\gamma_i-1) \\
&= (n-1)(f(\sigma_i)/F(\sigma_i))p_i/(\gamma_i-1) > 0, \qquad \text{(A.3.10)}
\end{aligned}$$

Now, taking the derivative of $x_i^*$ with respect to $\sigma_i$, we have:

$$\begin{gathered}
\partial x_i^*/\partial\sigma_i = [\sigma_i\gamma_i + (1/2)\sigma_i^2(\partial\gamma_i/\partial\sigma_i)]p_i + [(\sigma_i^2/2)\gamma_i + r](\partial p_i/\partial\sigma_i) \\
= \{(\sigma_i^2/2)\gamma_i(\sigma_i\gamma_i - \lambda\rho) + \\
+(n-1)(f(\sigma_i)/F(\sigma_i))[(\sigma_i^2/2)\gamma_i + r]/(\gamma_i-1)\}/[(\sigma_i^2/2)\gamma_i + r]\}x_i^* > 0 \qquad \text{(A.3.11)}
\end{gathered}$$

Last, by taking the derivative of Eq. (A.3.7), with respect to $n$, we get:

$$\begin{aligned}
\partial p(\sigma_i)/\partial n &= C \cdot \exp(\Phi(\gamma_i)) \cdot \{(\partial\Phi(\gamma_i)/\partial n)F(\sigma_i)^{\frac{n-1}{\gamma_i-1}} + \ln F(\sigma_i)[F(\sigma_i)^{\frac{n-1}{\gamma_i-1}}/(\gamma_i-1)]\} \\
&= p(\sigma_i)[\partial\Phi(\gamma_i)/\partial n + \ln F(\sigma_i)/(\gamma_i-1)] \\
&= p(\sigma_i)[\int_{\underline{\gamma}}^{\gamma_i} \ln(1-G(z))/(1-z)^2 dz - \ln(1-G(\gamma_i))/(1-\gamma_i)] < 0 \qquad \text{(A.3.12)}
\end{aligned}$$

It immediately follows that $\lim_{n\to\infty} p(\sigma_i) = 0$.

Last, the ex-ante value functions are:

$$W(x_i;\sigma_i) \equiv E_{\sigma_{-i}}\left[V(x_i;p(\sigma_i))\right] = \Gamma(x_i;\sigma_i)(Ce^{\Phi(\gamma_i)})^{1-\gamma_i},\ \text{for all } i \neq 1 \tag{A.3.13}$$

and

$$W(x_1;\sigma_1) = E_{\sigma_{-i}}\left[V(x_1;p(\widehat{\sigma}))\right] + I_{(p(\sigma_1)>p(\widehat{\sigma}))} \cdot (E_{\sigma_{-i}}\left[V(x_1;p(\sigma_1))\right] - E_{\sigma_{-i}}\left[V(x_1;p(\widehat{\sigma}))\right]),\ \text{for all } i = 1 \tag{A.3.14}$$

where $I_{(p(\sigma_1)>p(\widehat{\sigma}))}$ is an indicator function which takes value 1 if $p(\sigma_1) > p(\widehat{\sigma})$ and 0 otherwise.

## A.4 Proof of Proposition 2

Furthermore, differentiating on both sides of Eq. (A.3.7c) with respect to $n$ gives:

$$\partial p(\widehat{\sigma})/\partial n = 0 \tag{A.4.1}$$

Expanding the RHS of Eq. (A.3.1) yields

$$\begin{aligned}\partial p(\widehat{\sigma})/\partial n &= p(\widehat{\sigma})[\partial\Phi(\gamma(\widehat{\sigma}))/\partial n + (n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial n)/(\gamma(\widehat{\sigma})-1) + \\ &\quad -(n-1)\ln F(\widehat{\sigma})(\partial\gamma(\widehat{\sigma})/\partial n)/(\gamma(\widehat{\sigma})-1)^2]\end{aligned}$$

where

$$\begin{aligned}\partial\Phi(\gamma(\widehat{\sigma}))/\partial n &= \int_{\underline{\gamma}}^{\gamma(\widehat{\sigma})} \ln(1-G(z))/(1-z)^2 dz + (n-1)\ln(1-G(\gamma(\widehat{\sigma})))(\partial\gamma(\widehat{\sigma})/\partial n)/(\gamma(\widehat{\sigma})-1)^2 \\ &= \int_{\underline{\gamma}}^{\gamma(\widehat{\sigma})} \ln(1-G(z))/(1-z)^2 dz + (n-1)\ln F(\widehat{\sigma})(\partial\gamma(\widehat{\sigma})/\partial n)/(\gamma(\widehat{\sigma})-1)^2\end{aligned}$$

Hence, Eq. (A.4.1) reduces to:

$$\int_{\underline{\gamma}}^{\gamma(\widehat{\sigma})} \ln(1-G(z))/(1-z)^2 dz + (n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial n)/(\gamma(\widehat{\sigma})-1) = 0$$

and it is easy to show that:

$$\partial\widehat{\sigma}/\partial n = -\int_{\underline{\gamma}}^{\gamma(\widehat{\sigma})} \ln(1-G(z))/(1-z)^2 dz/[(n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] > 0 \tag{A.4.2}$$

## A.5 Proof of Proposition 3

By differentiating on both sides of Eq. (A.3.7c) with respect to $C$, we get:

$$\partial p(\widehat{\sigma})/\partial C = 0 \tag{A.5.1}$$

Expanding the RHS of Eq. (A.5.1) yields:

$$\begin{aligned}\partial p(\widehat{\sigma})/\partial C &= p(\widehat{\sigma})[(1/C) + \partial \Phi(\gamma(\widehat{\sigma}))/\partial C + (n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial C)/(\gamma(\widehat{\sigma})-1) + \\ &\quad -(n-1)\ln F(\widehat{\sigma})(\partial\gamma(\widehat{\sigma})/\partial C)/(\gamma(\widehat{\sigma})-1)^2]\end{aligned}$$

where

$$\partial\Phi(\gamma(\widehat{\sigma}))/\partial C = (n-1)\ln(1-G(\gamma(\widehat{\sigma})))(\partial\gamma(\widehat{\sigma})/\partial C)/(\gamma(\widehat{\sigma})-1)^2$$

Hence, Eq. (A.5.1) reduces to:

$$(1/C) + (n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial C)/(\gamma(\widehat{\sigma})-1) = 0$$

and it is easy to show that:

$$\partial\widehat{\sigma}/\partial C = -1/[C(n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] < 0 \tag{A.5.2}$$

Furthermore, by differentiating on both sides of Eq. (A.3.7c) with respect to $x_1$, we get:

$$\partial p(\widehat{\sigma})/\partial x_1 = 1/r \tag{A.5.3}$$

Expanding the RHS of Eq. (A.5.3) yields:

$$\begin{aligned}\partial p(\widehat{\sigma})/\partial x_1 &= p(\widehat{\sigma})[\partial \Phi(\gamma(\widehat{\sigma}))/\partial x_1 + (n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial x_1)/(\gamma(\widehat{\sigma})-1) + \\ &\quad -(n-1)\ln F(\widehat{\sigma})(\partial\gamma(\widehat{\sigma})/\partial x_1)/(\gamma(\widehat{\sigma})-1)^2]\end{aligned}$$

where

$$\partial\Phi(\gamma(\widehat{\sigma}))/\partial x_1 = (n-1)\ln(1-G(\gamma(\widehat{\sigma})))(\partial\gamma(\widehat{\sigma})/\partial x_1)/(\gamma(\widehat{\sigma})-1)^2$$

Hence, Eq. (A.5.3) reduces to:

$$(n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))(\partial\widehat{\sigma}/\partial x_1)/(\gamma(\widehat{\sigma})-1) = 1/r$$

and it is easy to show that:

$$\partial\widehat{\sigma}/\partial x_1 = 1/[r(n-1)(f(\widehat{\sigma})/F(\widehat{\sigma}))/(\gamma(\widehat{\sigma})-1)] > 0 \tag{A.5.4}$$

## A.6 Setting $C$

Let us set a minimal acceptable probability level $q^{\min}$ and then consider the highest risk profile, $\sigma^h$, and the highest potential current payoff $x_1$. Plugging these element into Eq. (14) and rearranging, we obtain:

$$C = (1/q^{\min})\{x_1/[(1/2)\sigma^{h2}\underline{\gamma} + r]\} \tag{A.6.1}$$

where the second term represents a stream of $x_1$ discounted (to account for the presence of an option value) at the adjusted rate $(1/2)\sigma^{h2}\underline{\gamma} + r$.

Setting a feasible $C$ requires that:

$$(1/q^{\min})\{x_1/[(1/2)\sigma^{h2}\underline{\gamma} + r]\} \geq x_1/r \tag{A.6.2}$$

which in turn implies that:

$$q^{\min} \leq r/[(1/2)\sigma^{h2}\underline{\gamma} + r] < 1 \tag{A.6.3}$$

In other words, when selecting the cap $C$, the auctioneer may never set 1 as a target in terms of probability of eventual investment. Note also that $\lim_{\sigma^h \to \infty} r/[(1/2)\sigma^{h2}\underline{\gamma} + r] = 0$. This means that $\sigma^h \to \infty$ there does not exist any $C$ with a corresponding positive probability of hitting the investment threshold.

## A.7 Proof of Proposition 6

**Seller (ex-post)** - Taking the derivative of Eq. (A.3.9a) with respect to $\sigma_i$ yields

$$\begin{aligned}
\partial V(x_i;p_i)/\partial\sigma_i &= (\partial(x_i/x_i^*)^{\gamma_i}/\partial\sigma_i)[p_i/(\gamma_i-1)] + (x_i/x_i^*)^{\gamma_i}\partial[p_i/(\gamma_i-1)]/\partial\sigma_i \\
&= V(x_i;p_i)[-(\gamma_i/x_i^*)(\partial x_i^*/\partial\sigma_i) + \ln(x_i/x_i^*)(\partial\gamma_i/\partial\sigma_i) + (1/p_i)(\partial p_i/\partial\sigma_i) + \\
&\quad -(\partial\gamma_i/\partial\sigma_i)/(\gamma_i-1)]
\end{aligned}$$

Note that:

$$\partial x_i^*/\partial\sigma_i = x_i^*[-(\partial\gamma_i/\partial\sigma_i)/\gamma_i(\gamma_i-1) + (1/p(\sigma_i))(\partial p(\sigma_i)/\partial\sigma_i) + (1/\delta_i)(\partial\delta_i/\partial\sigma_i)]$$

Thus,

$$\partial V(x_i;p_i)/\partial\sigma_i = V(x_i;p_i)[\ln(x_i/x_i^*)(\partial\gamma_i/\partial\sigma_i) - (n-1)(f(\sigma_i)/F(\sigma_i)) - (\gamma_i/\delta_i)(\partial\delta_i/\partial\sigma_i)]$$

This implies that the sign of $\partial V(x_i;p_i)/\partial\sigma_i$ depends on the sign taken by the following function:

$$\theta(\sigma_i) = \ln(x_i/x_i^*)(\partial\gamma_i/\partial\sigma_i) - (n-1)(f(\sigma_i)/F(\sigma_i)) - (\gamma_i/\delta_i)(\partial\delta_i/\partial\sigma_i) \tag{A.6.1}$$

**Winner (ex-post)** - Note that

$$R(x_i;p_i) = (x_i/x_i^*)^{\gamma_i} p_i = V(x_i;p_i)(\gamma_i - 1)$$

Hence,

$$\partial R(x_i;p_i)/\partial\sigma_i = R(x_i;p_i)[\theta(\sigma_i) + (\partial\gamma_i/\partial\sigma_i)/(\gamma_i - 1)]$$

This implies that the sign of $\partial R(x_i;p_i)/\partial\sigma_i$ depends on the sign taken by the following function:

$$\vartheta(\sigma_i) = \theta(\sigma_i) + (\partial\gamma_i/\partial\sigma_i)/(\gamma_i - 1) \tag{A.6.2}$$

**Social value (ex-post)** - The ex-post social value attached to the project is

$$S(x_i;p_i) = V(x_i;p_i) + R(x_i;p_i) = V(x_i;p_i)\gamma_i$$

Its derivative with respect to $\sigma_i$ is

$$\partial S(x_i;p_i)/\partial\sigma_i = \partial V(x_i;p_i)/\partial\sigma_i + \partial R(x_i;p_i)/\partial\sigma_i = S(x_i;p_i)[\theta(\sigma_i) + (\partial\gamma_i/\partial\sigma_i)/\gamma_i] \tag{A.6.3}$$

**Losses** - By comparing the social value attached to the project for the case of delegation with the project value without delegation, it is easy to show that:

$$U(x_i;\sigma_i) - S(x_i;p_i) = (x_i/\delta_i) - (x_i/x_i^*)^{\gamma_i}[\gamma_i/(\gamma_i - 1)]p_i = [1 - (x_i/x_i^*)^{\gamma_i - 1}](x_i/\delta_i) > 0$$

Define now the ratio:

$$\Delta^S(x_i;p_i) = S(x_i;p_i)/U(x_i;\sigma_i) = S(x_i;p_i)(\delta_i/x_i)$$

Its derivative with respect to $\sigma_i$ is:

$$\begin{aligned}
\partial\Delta^S(x_i;p_i)/\partial\sigma_i &= (\partial S(x_i;p_i)/\partial\sigma_i)(\delta_i/x_i) + S(x_i;p_i)(\partial\delta_i/\partial\sigma_i)/x_i \\
&= \Delta^S(x_i;p_i)\{[\ln(x_i/x_i^*) + (1/\gamma_i)](\partial\gamma_i/\partial\sigma_i) - (n-1)(f(\sigma_i)/F(\sigma_i)) + \\
&\quad -\lambda\rho(\gamma_i - 1)/\delta_i\}
\end{aligned} \tag{A.6.4}$$

Similarly, define the ratio:

$$\Delta^R(x_i;p_i) = R(x_i;p_i)/U(x_i;\sigma_i) = [1 - (1/\gamma_i)]\Delta^S(x_i;p_i)$$

Its derivative with respect to $\sigma_i$ is:

$$\begin{aligned}
\partial\Delta^R(x_i;p_i)/\partial\sigma_i &= \Delta^S(x_i;p_i)(1/{\gamma_i}^2)(\partial\gamma_i/\partial\sigma_i) + [1-(1/\gamma_i)]\partial\Delta^S(x_i;p_i)/\partial\sigma_i \\
&= [1-(1/\gamma_i)]\Delta^S(x_i;p_i)\{[(\ln(x_i/x_i^*) + 1/(\gamma_i-1)](\partial\gamma_i/\partial\sigma_i) + \\
&\quad -(n-1)(f(\sigma_i)/F(\sigma_i)) - \lambda\rho(\gamma_i-1)/\delta_i] \qquad \text{(A.6.5)}
\end{aligned}$$

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## Notes

[1]See e.g. Hansen (2001), Dasgupta and Hansen (2007) and Boone and Mulherin (2007) on auctions of companies. See Pennings (2008) on the privatisation of real options.

[2]See Quan (1994), Porter (1995), Levin and Athey (2001), Di Corato et al. (2015) for auctions on real estate, oil and gas lease sales, timber and conservation contracts, respectively.

[3]Schwartz (2004) views patents as real options. See Sneed and Johnson (2009) and Gassmann et al. (2010) on auctions for patents.

[4]See Dixit and Pindyck (1994) for an excellent overview of the literature on investment appraisal under a real option approach.

[5]See Skrzypacz (2013) for an excellent survey of this literature.

[6]Canonical security bids are combinations of contingent payments from the cash flow of the project and non-contingent payments. Examples of such bids are royalty contracts (or equity in applications in corporate finance), debt and call option (or royalty rate combined with an advance). In a dynamic context, the non-contingent payments can be viewed as an up-front fee usually representing the non-contingent component of the payment. See DeMarzo et al. (2005) and Cong (2015) for a definition of security bids.

[7]A similar mechanism-design problem is examined by Schummer and Vohra (2003) in the context of electricity markets.

[8]Cong (2015) shows that Board's optimal mechanism can be generalised by using a standard security combining cash and royalty payments.

[9]More precisely, if the underlying risk is unsystematic, the relationship is positive, while, in the presence of systematic risk, the sign is ambiguous. See e.g. Davis (2002) and Wong (2007).

[10]The asset owner is female in this paper, while other agents are male.

[11]It is also worth mentioning Kakade et al. (2013) and Pavan et al. (2014) considering, in a discrete time frame, the problem of designing optimal mechanisms in environments where agents have dynamic private information.

[12]We require $\delta_i > 0$ for securing, as it will become clearer later, a positive project value.

[13]The process (2) is quite standard in the literature (see e.g. McDonald and Siegel, 1984). However, for the convenience of the reader, we provide a detailed derivation in Appendix A.1. We

remind the reader also that a world where the expected growth rate is set equal to $r - \delta_i$ is referred to as a "*risk-neutral*" world (see e.g. Cox and Ross, 1976; Constantinides, 1978; Harrison and Kreps, 1979).

[14]More specifically, if $\sigma_i \neq \sigma_j$ for any $i$ and $j$ (with $i \neq j$) in the considered set of projects, then $\beta_i = \rho \cdot (\sigma_i/\sigma_m) \neq \rho \cdot (\sigma_j/\sigma_m) = \beta_j$, where $\sigma_m$ is the volatility of the market portfolio.

[15]See Merton (1973).

[16]See for instance Davis (2002) and Wong (2007). Note that in our model if the systematic risk component is null, the rate of dividend yield is constant, i.e., $\delta_i = r$. On the analysis of investment decisions where $\delta_i$ does not depend on $\sigma_i$, see McDonald and Siegel (1986), Teisburg (1994), Dixit and Pindyck (1994), Moretto and Cappuccio (2001) and Lund (2005).

[17]We may, however, easily allow for a $x_i(t)$ resulting from taking into account the presence of a periodical constant flow cost.

[18]Note that our framework can easily apply to the case of a single project where the different levels of revenue are the results of new information on the true probability distribution of $x(t)$, where $x_i(t) = E_i(x(t))$ (see Board, 2007). Further, it can be used to represent the different quality of the output produced by each firm once the project is realized (Davis, 2002).

[19]This is done to rule out full surplus extraction à la Cremer and McLean (1988).

[20]Gorbenko and Malenko (2014) discuss the case where the target firm can agree to receive the payment in stock of the combined company, if the bidder is unable to pay cash.

[21]For instance, when considering oil (or gas) leases, even if the market price is observable, reaching an agreement may be rather complicated due to the difficulties in verification of actual extraction costs (see Robinson, 1984) or profits to be shared (see Opaluch et al., 2009).

[22]See Taylor (1993).

[23]PCP involves different suppliers competing through different phases of development. The risks and benefits are shared between the procurers and the suppliers under market conditions. See https://ec.europa.eu/digital-agenda/en/pre-commercial-procurement.

[24]Note that considering commonplace transfer costs would just imply a reduction in the number of potential projects that the asset owner would consider. As we will show later this, is equivalent in our frame to setting a reservation value on the initial cash flows $x_i(0)$.

[25]We drop the time index for notational convenience.

[26]We may easily allow for a payment $p_i$ including a fixed component independent on the agent's type. One, in fact, would simply need incorporating it in the investment threshold $x_i^*$. Note however that including this component would not affect the ranking resulting from the auction.

[27]The expected present value $E_0[\exp(-rT_i)] = (x_i/x_i^*)^{\gamma_i}$ is determined by using dynamic programming (see e.g. Dixit and Pindyck, 1994, pp. 315-316).

[28]See Appendix A.2

[29]This is basically due to the fact that at the moment of contracting the future cash flows are still unknown to both parties. The result is line with previous findings in the literature (see for instance Baron and Besanko, 1984; Besanko, 1985; Eso and Szentes, 2013; Pavan et al., 2014; Arve and Zwart, 2014).

[30]Note that some of these properties characterise the mechanism for the allocation of investment options in Board (2007) and Kruse and Strack (2015).

[31]In a model of investment timing of joint ventures, Yoshida (2012) finds a similar result. Notably, in a simple two-agent model, in which an option model of investment is embedded, he shows that the flexibility chosen by one party creates strategic uncertainty for another party, which causes the other party to choose a higher level of flexibility. The strategic complementarity then leads to delays in the investment.

[32]The rental rate $r$ is adjusted by adding the term $\frac{1}{2}{\sigma_i}^2\gamma_i$ in order to account for the presence of an option value (see Dixit and Pindyck, 1994, p. 145).

[33]It is worth stressing that the frame may be easily adapted for setting $C$ on the basis of other considerations.

[34]See Appendix A.5 for a discussion of this case.

[35]In finance $\partial V(x_i; p_i)/\partial x_i$ is known as $\Delta$ and it measures the rate of change of the option value with respect to changes in the underlying asset's price. For a (perpetual) Call Option it is a number between 0 and 1 and reaches 1 as the option approaches its optimal exercise time (i.e., it is highly in-the-money).

[36]See Cong (2016) for an analysis of the timing of an auction.

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