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## Fungi diversity in Swedish forests: impact of site quality on standing volume and variability in forest growth

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# **Fungi diversity in Swedish forests: impact of site quality on standing volume and variability in forest growth**

## **Abstract**

The impact of biodiversity on forest ecosystem function and provision of ecosystem services has been investigated by many studies, but not the stability in these impacts. This paper calculates the effect of a fungi indicator, forest site quality, on standing volume and variability in forest growth in different forest regions in Sweden. We account for management practices, make use of time series data for a period of approximately 50 years, and estimate forest growth functions totally and for different regions in Sweden. Uncertainty is calculated as the conditional variance in forest growth rate, and the biodiversity indicator together with management practices are used as explanatory variables. Fully modified OLS is used to account for serial correlation and non-stationarity in the variables. The results show that the fungi diversity indicator adds positively to forest growth and growth rate, but increases uncertainty in the growth rate.

Keywords: fungi diversity, site quality, forest growth, impact, variability, econometrics, Sweden

JEL: Q23, Q57, Q59

## 1. Introduction

The role of biodiversity for ecosystem functioning and provisioning of services has been analyzed and empirically estimated for decades within the field of ecology, which have been documented in several surveys (e.g. Cardinale et al., 2012; Balvanera et al., 2014; Harrison et al., 2014). These show that biodiversity measured as diversity in tree species has a positive impact on forest productivity (e.g. Erskine et al 2006). Meta-analyses of field experiments and forest plantation trials have also shown that the increased number of tree species raises the productivity of forest (Piotto, 2008; Zhang et al., 2012; Gamfeldt et al., 2013). It is also argued that biodiversity has effects, not only on functioning of ecosystems and provision of ecosystem services, but also on the stability in these effects (e.g. Ives and Carpenter 2008; Campbell et al., 2011). Biodiversity according to Stokland et al. (2003) can be defined from three perspectives; compositional, structural and functional diversities. The contribution of biodiversity to less volatility in forest outputs, such as timber products, has a value in addition to the value of the good as such for a risk averse society (e.g. Brock and Xepapadeas 2002; Vardas and Xepapadeas, 2011; Gren et al., 2014).

However, much of the diversity is measured by number of tree species and not so much in terms of below ground fungi diversity, which helps in regulating ecosystem processes (Dighton, 2003). Fungi are known to be important in driving or controlling the mineral and energy cycling within ecosystems. They are also able to influence the composition of other organisms within the ecosystem. Generally, the ecosystem services provided by fungi include transformation of carbon dioxide and nutrients into plant biomass by photosynthesis, energy and nutrient transformations among components of food webs, and energy and nutrient cycling. A main impediment in estimating the value of any type of fungi diversity as an input into production of ecosystem services is the lack of production functions quantifying its effects on standing volume and variability on ecosystem services in relation to management practices and forest volume. One reason for this scarcity is the difficulty to obtain constructs of diversity in soil. As suggested by Stokland et al. (2003), site quality can be an indicator of this type of diversity since it usually reflects the quality of the forest soil in terms of potential forest productivity. It has also been found to be highly correlated with fungi diversity in forest soil (Sterkenburg et al., 2015). An advantage with this indicator is its availability for a long period of time through annual national forest inventories in Sweden. The purpose of this study is to calculate effects of fungi diversity, measured as site quality, on the growth of Swedish forests. The growth in forest volume provides the basis for provision of ecosystem

services in terms of e.g timber and fuel wood. We apply econometric analysis to time series data over a 50 year period on forest growth, fungi diversity indicator, and management practices for different forest regions in Sweden.

Our study is most similar to the literature on the estimation of forest production and growth function in economics and ecology. Quantified growth functions have gained dominance in bioeconomic studies on forestry and fishery over the years, where they have been used to optimize the use of the resources over time (e.g. Clark 1990). These functions describe the change in size of an organism or population with age (e.g. Zeide, 1993). Prominence among them includes the logistic growth function in fishery, forestry and other biological sciences. In many branches of science, growth functions have demonstrated complex nonlinear functions where the dependent variable depicts juvenile, adolescent, mature and senescent stages of growth (e.g. Fekedulegn et al., 1999). These characteristics of growth functions are common in trees. As more similar trees are combined, their size growth follows a smoother sigmoid curve. With a sigmoid form which has a starting at the origin of coordinate (0,0), a point of inflection occurring early in the adolescent stage, and either approaching a maximum value, an asymptote, or peaking and falling in the senescent stage. Growth functions which portray these features include theoretical models like the logistic, Gompertz, Chapman-Richards, von Bertalanffy and Schnute functions. Empirically, polynomial models have often been used to estimate growth functions for biological species. However, the above mentioned theoretical models are used in forestry since they have an underlying hypothesis associated with cause or function of the phenomenon described by the response variable unlike the empirical models (Lei and Zhang, 2004). Also, contrary to the empirical models, the theoretical models have meaningful parameters from forestry perspective and may further be more reliable for predictions when dealing with extrapolations far beyond the range that the available data allows (Fekedulegn et al., 1999).

Over the years, estimation of biomass growth has been done by the use of regression analysis and stand tables. This is done by a process called allometry or dimension analysis, which is the sampling of stems of biomass and regressing the weight of each component on the dimension of the standing tree (Baskerville, 1972). These estimations are mostly done at either the eco-physiological, individual tree, stand or succession levels. Considerable empirical studies in time past made use of tree stand growth observations. The growing interest in the use of stand growth is mainly attributed to the supporting role this plays in forestry practice. Thus, it helps in the assessment of wood reserves and

also in the ascertainment of potential sustainable annual cut required information about standing volume or volume growth per unit area (Pretzsch, 2009).

This study makes use of time series data on standing volume, site quality and management practices to estimate nonlinear production functions at the landscape level for four forest regions in Sweden; North Norrland, South Norrland, Svealand and Götaland. The use of the nonlinear growth function makes it possible to examine the effect of fungi diversity on standing volume. Furthermore, the nonlinear models have meaningful parameters from forestry which this study wishes to explore. However, the parameters of the nonlinear models are highly correlated and we therefore use a linearized version of the logistic function to examine the effect of fungi diversity on the forest growth rate. When estimating the impact of fungi diversity on uncertainty in forest growth we measure uncertainty, or volatility, as the conditional variance in growth rate.

In our view, the novelty of this study stems from the fact that aggregate and time series forestry data is used to estimate the growth function and the effect of fungi diversity on standing volume and variability in forest growth. To the best of our knowledge, the effect of diversity on volatility in forest growth has not been estimated by any study. The remainder of this study is organized as follows. Section 2 discusses the relationship between site quality/index and fungi diversity. Whereas Sections 3 and 4 discuss methodological and data issues. Section 5 explains the econometric approach and Section 6 presents the empirical results of the study. Finally, Section 7 concludes the study.

## **2. Site quality as an indicator for fungi diversity**

Fungi are normally a group of organisms that cannot fix energy directly, however, make use of the energy stored in plant and animal biomass to create their own mass. Fungi play an important role in their interactions with other living and dead organisms, together with nonliving components of the environment in regulating ecosystem processes (Dighton, 2003). Fungi are known to be very important in driving or controlling the mineral and energy cycling within ecosystems. They are also able to influence the composition of other organisms within the ecosystem. Generally, the ecosystem services provided by fungi include transformation of carbon dioxide and nutrients into plant biomass by photosynthesis, energy and nutrient transformations among components of food

webs, and energy and nutrient cycling. These ecosystem services provided by fungi are summarized in Table 1.

Table 1: Ecosystem services provided by fungi

<b>Ecosystem Service</b>		<b>Fungal functional group</b>
Soil formation	Rock and dissolution and particle binding	Lichens, Saprotrophs and Mycorrhizae
Soil fertility	Decomposition or organic residues, nutrient mineralization and soil stability	Saprotrophs, Arbuscular mycorrhizae
Primary production	Direct production, nutrient accessibility, plant yield, and defense against pathogens and herbivory	Lichens, Saprotrophs, Mycorrhizae, Pathogens and Endophytes
Plant community structure	Plant-plant interactions	Mycorrhizae and Pathogens
Secondary production	As a food source and population/biomass regulation	Saprotrophs, Mycorrhizae and Pathogens
Modification of pollutants, carbon sequestration and storage		Saprotrophs and Mycorrhizae

Source: Dighton (2003)

As the focus of this section is to establish the relationship between soil quality and fungi diversity, the remainder of the discussion in this section will concentrate on the first two ecosystem services provided by fungi outlined in Table 1. Rock materials underlying the soil, and decomposed dead organisms are the two main sources of nutrients which support plant growth. Scientists have discovered that rocks of the earth's crust contain a variety of the essential mineral nutrients that plants need, however these minerals are bound in very complex chemical forms that make them poorly available for plant uptake. Fortunately, the action of environmental factors in the form of wind, water, and physical disturbance, along with the activities of bacteria, fungi, and plant roots, make it possible for the surface of rocks to weather and degraded to finer particles. Eventually, the mineral nutrients in the rocks are released in a soluble form that can be accessed by plants.

Further, the decomposition process of dead plants and animals are made possible by the actions of microbes (which includes fungi) and animals (Dighton, 2003). During decomposition, mineral nutrients are released in a soluble form, and this process called mineralization, provides fertility to the ecosystem. According to Coleman and Crossley (1996), soils are generally composed of weathered mineral rock and organic material derived from dead plant and animal remains together

with the living biota of fungi and other organisms. Lichens which are a fungal group play a significant role in the formation of soils (Dighton, 2003).

Organisms such as lichens and soil fungi and bacteria produce organic acids in the form of oxalic, citric, lichenic, and tartaric acids. These acids contribute to the chemical weathering of rocks. In addition, the acids increase hydrogen ion concentration in the environment, lowering pH and increasing the solubility of aluminum and silicon. Moreover, Dighton (2003) stipulates that the acids produced by these fungi form chelation products (that is, the complexes between inorganic ions and organic molecules) and release inorganic nutrient elements to enrich the soil. The absence of lichens in the soils reduces the potency of the soil to support primary production plants. Furthermore, both saprotrophic and mycorrhizal which are other fungi groups are associated with mineral rock dissolution. By the action of these fungi groups mineral nutrients are released through the dissolution of rocks.

According to Asta et al. (2001), production of polysaccharides<sup>1</sup> by a fungal called hyphae, is important in the development of organic mineral complexes, which helps in binding mineral particles together. Polysaccharide secretions of both fungi and bacteria penetrate between the soils mineral particles and act as a web to physically retain soil particle. In other words, it acts as a glue to bind mineral particles together. As the mineral particles in the soil are bind together, it reduces the tendency of erosion since soils are mostly prone to erosion due to intensive rainfall which washes top soils and wind which displaces top dry soil. A good blend of inorganic, organic, and biotic components in the development of a soil is of great importance to the physical stability of soils and their ability to support plant life. This physical stability of soils is enhanced by the binding activities of fungi.

Lodge (1993) finds that the biomass of fungi in wet tropical forest soils is significantly and positively correlated with soil moisture and the amount of rainfall in the preceding week. In Sweden, Sterkenburg et al. (2015) finds that fungal community composition is significantly related to soil fertility at the levels of species, genera/orders and functional groups. Accordingly, they observe ascomycetes<sup>2</sup> dominate in less fertile forests whereas basidiomycetes<sup>3</sup> increase in abundance in more

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<sup>1</sup> These are polymeric carbohydrate molecules composed of long chains of monosaccharide units bound together by glycosidic linkages and on hydrolysis give the constituent monosaccharides or oligosaccharides.

<sup>2</sup> These are 'spore shooters'. They are fungi which produce microscopic spores inside special, elongated cells or sacs, known as 'asci', which give the group its name.



fertile forests, both in litter and humus. Also, their results show that the relative abundance of mycorrhizal fungi in the humus layer is even higher in the most fertile soils. The conclusion drawn by Sterkenburg et al. (2015) is based on the analysis of fungal communities in humus and litter from 25 Swedish old-growth forests.

The discussion so far portrays a positive relationship between site/soil quality and fungi diversity. This provides sufficient justification for the use of site quality/index as a proxy for fungi diversity. It is also used as an indicator of biodiversity in the Nordic countries (Stokland et al. 2003). More precise description of the site quality construct collected by Swedish National Forest Inventory and used in current study is presented in the data retrieval section 3.2.

### **3. Theoretical framework**

With the non-linear estimation, the focus is on the Gompertz and Logistic growth functions. As mentioned in the introduction, the non-linear growth functions have some mathematical properties which are meaningful in forestry. The parameters from non-linear estimation provide information about the growth rate, the point of inflection, the initial stands and maximum value (carrying capacity). However, the parameters from the non-linear estimation are mostly correlated since a change in one parameter may cause the other parameters to change in order to maintain its functional form. As such, an introduction of other explanatory variables additively in the function may give a misleading conclusion about the effect of these variables on the growth rate. Therefore, a linearized version of the logistic function is used in this study to examine the effect of fungi diversity (site quality) on forest growth.

The choice of the Gompertz and logistic growth functions is based on the fact that the available forest data is more appropriate for estimating all the parameters when these models are used than with other non-linear models such as Chapman-Richards. Also, Gompertz and Logistic growth functions have demonstrated accuracy in forecasting or predictions (Nguimkeu, 2014). As a result, these growth functions are used in applied research work for modelling and forecasting the behaviour of population growth of various creatures (Nguimkeu and Rekkas, 2011) which include

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<sup>3</sup> Similar to ascomycetes, basidiomycetes are also fungi. However, their spores are produced externally, on the end of specialised cells called basidia.

the forest. The Gompertz and Logistic curves both share the “S-shaped” feature which describes processes that consist of a slow early adoption stage, followed by a phase of rapid adoption which then tails off as the adopting population becomes saturated. Despite these similarities there are fundamental differences between the two curves and one of the most important is that the Gompertz function is symmetric whereas the Logistic function is asymmetric.

The Gompertz growth function used in this study for estimating growth functions in each region  $i$ , where  $i=1,..,k$  regions, is shown in equation (1). From equation (1),  $y_{it}$  represents the volume of forest at each period after harvest,  $H_{it}$  is the harvest at each period,  $Z_{it}$  is site quality/index, and  $X_{it}$  is a vector of control variables, such as thinning and fertilization. .

$$y_{it} = \beta_{i0} + \beta_{i1} * \exp[-\exp(-\beta_{i2}(t - \beta_{i3}))] - H_{it} + Z_{it} + X_{it} \quad (1)$$

The parameters in equation (1) have interesting meanings in forestry economics. The initial standing yield is represented by  $\beta_0$ ,  $\beta_2$  is the growth rate when yield increases from initial stand to final size and  $\beta_3$  shows the time at which the rate of change reaches its maximum, i.e. its inflection point. Given the intercept in equation (1), the final (asymptotic) stand yield is represented as the sum of  $\beta_0$  and  $\beta_1$ . Therefore, as the time period ( $t$ ) approaches infinity, the dependent variable turns to the sum of  $\beta_0$  and  $\beta_1$ <sup>4</sup>.

In a similar vein, the Logistic growth function used in this study is expressed as;

$$y_{it} = \beta_{i0} + \beta_{i1} * \left[ \frac{1}{1 + \exp(-\beta_{i2}(t - \beta_{i3}))} \right] - H_{it} + Z_{it} + X_{it} \quad (2)$$

The meanings of the parameters in equation (2) are analogous to that of equation (1), where  $\beta_0$ ,  $\beta_2$  and  $\beta_3$  respectively represent initial standing yield, growth rate when yield increases from initial stand

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$$y_t + H_t = \beta_0 + \beta_1 * \exp[-\exp(-\beta_2(\infty - \beta_3))]$$

$$y_t + H_t = \beta_0 + \beta_1 * \exp[-\exp(-\infty)] = \beta_0 + \beta_1 * \exp(0)$$

Since  $\exp(-\infty) = 0$

$$\text{Therefore, } y_t + H_t = \beta_0 + \beta_1 * \exp[-\exp(-\beta_2(\infty - \beta_3))] = \beta_0 + \beta_1 * \exp(0) = \beta_0 + \beta_1$$

to final size and the time at which the rate of change reaches its maximum, its inflection point. Also, the sum of  $\beta_0$  and  $\beta_1$  shows the final (asymptotic) stand yield<sup>5</sup>.

In order to facilitate the estimations of impacts of fungi diversity on forest growth and allow for the estimation of impacts on variability in forest growth, we linearize the logistic functions and express it in terms of intrinsic growth rate,  $\beta_{i2}$ , and carrying capacity,  $\beta_{i1}$ . If no intercept is assumed for logistic function in equation (2), the resultant linearized expression of the logistic function is given as;

$$y_{it+1} = y_{it} \left( 1 + \beta_{i2} \left( 1 - \frac{y_{it}}{\beta_{i1}} \right) \right) - S_{it} + Z_{it} + X_{it} \quad (3)$$

where  $S_{it}$  is the annual harvest. Moving  $S_{it}$  to the left hand side and dividing by  $y_{it}$  we obtain;

$$\dot{Y}_i = \beta_{i2} \left( 1 - \frac{y_{it}}{\beta_{i1}} \right) + \frac{Z_{it}}{y_{it}} + \frac{X_{it}}{y_{it}} \quad (4)$$

where  $\dot{Y}_i = \frac{y_{it+1} - y_{it} + S_{it}}{y_{it}}$ . From equation (4),  $\beta_{i2}$ ,  $\beta_{i1}$  and  $y_{it}$  is the intrinsic growth rate, carrying capacity and standing volume per hectare, respectively. We refer to the left hand side of equation (4) as the ‘‘adjusted growth rate’’.

The empirical model for the linearized logistic function is given as;

$$\dot{Y}_i = \alpha_{i1} + \alpha_{i2} y_{it} + \alpha_{i3} \frac{Z_{it}}{y_{it}} + \alpha_{i4} \frac{X_{it}}{y_{it}} + \varepsilon_{it} \quad (5)$$

where  $\alpha_{i1} = \beta_{i1}$  and  $\alpha_{i2} = -\frac{\beta_{i2}}{\beta_{i1}}$ , and  $\varepsilon_{it}$  is the error term which is independently and identically distributed with zero mean and equal variance.

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<sup>5</sup>  $y_t + H_t = \beta_0 + \beta_1 * \left[ \frac{1}{1 + \exp(-\beta_2 (\infty - \beta_3))} \right] = \beta_0 + \beta_1 * \left[ \frac{1}{1 + \exp(-\infty)} \right]$   
 $y_t + H_t = \beta_0 + \beta_1 * \left[ \frac{1}{1+0} \right] = \beta_0 + \beta_1$

Following Gren et al. (2015) we also introduce indirect impacts of site index and other control covariates on the intrinsic growth rate, where  $\beta_{it2} = \gamma_{i1} + \gamma_{i2}Z_{it} + \gamma_{i3}X_{it}$ . This yields equation (6);

$$\dot{Y}_i = (\gamma_{i1} + \gamma_{i2}Z_{it} + \gamma_{i3}X_{it}) \left( 1 - \frac{y_{it}}{\beta_1} \right) \quad (6)$$

and the corresponding regression equation is;

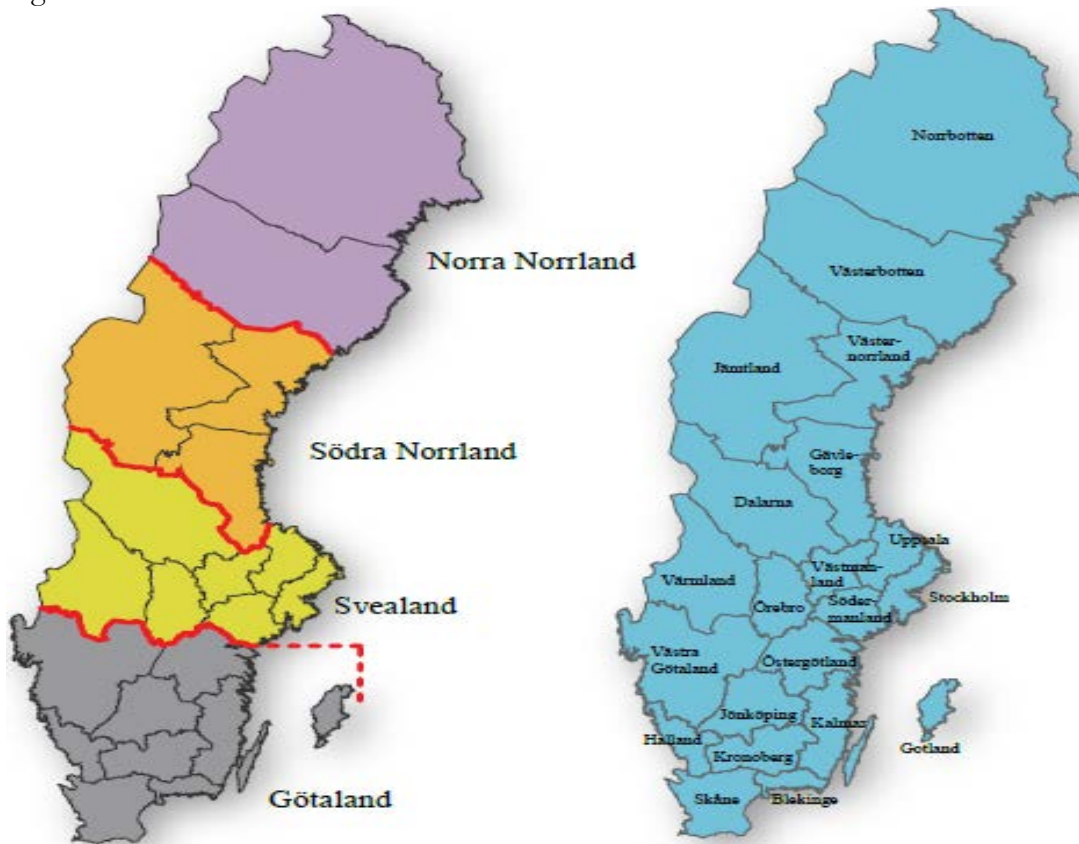
$$\dot{Y}_i = \lambda_{i1} + \lambda_{i2}y_{it} + \lambda_{i3}Z_{it} + \lambda_{i4}X_{it} + \varepsilon_{it} \quad (7)$$

where  $\lambda_{i1} = \gamma_{i1}$ ,  $\lambda_{i2} = -\frac{\beta_{it2}}{\beta_1}$ , and  $\beta_{it2} = \lambda_{i1} + \lambda_{i3}Z_{it} + \lambda_{i4}X_{it}$ .

#### 4. Data retrieval

In total, the forest area in Sweden is about 30.7 million hectares on average between the period 1965 and 2014, which corresponds to approximately 75% of the total land area in Sweden (Table B2). The northern forest zones (that is, Norra and Södra Norrland) have larger forest area than the southern zones (that is, Svealand and Götaland), since the northern part of Sweden has large land space, see Figure 1.

Figure 1: Forest Zones and Counties in Sweden

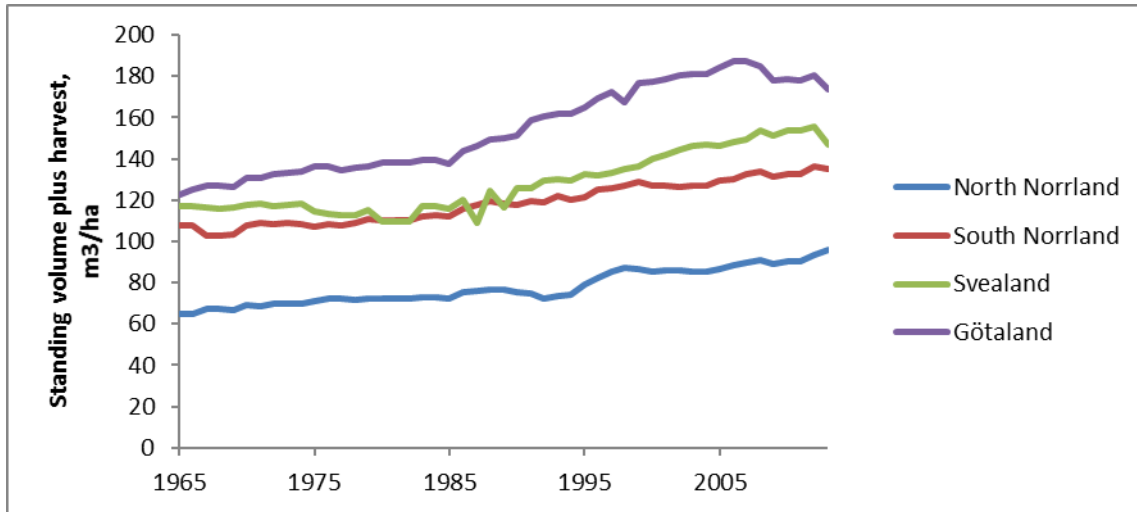


Source: Swedish Forest Data, 2014 report. Department of Forest Resource Management, SLU

Total area of productive forests is largest in North Norrland, but the share of total area of land is smallest and amounts to approximately 0.44. It is highest in Svealand where it corresponds to 0.64 of total land area. Most of the Swedish forest is a part of the Boreal coniferous belt, and Scots pine and Norwegian spruce dominates in all four regions, and range between 76% and 84% of total standing volume on productive forest areas (Swedish Forestry Agency, 2014). The most common broad leaved tree is birch, which amounts to 10% -15% of total standing volume. Beech and oak are found in Götaland where they account for approximately 6% of the total standing volume.

Common to all regions is the increase in standing volume per ha productive forest area over the period 1965-2013, see Figure 2.

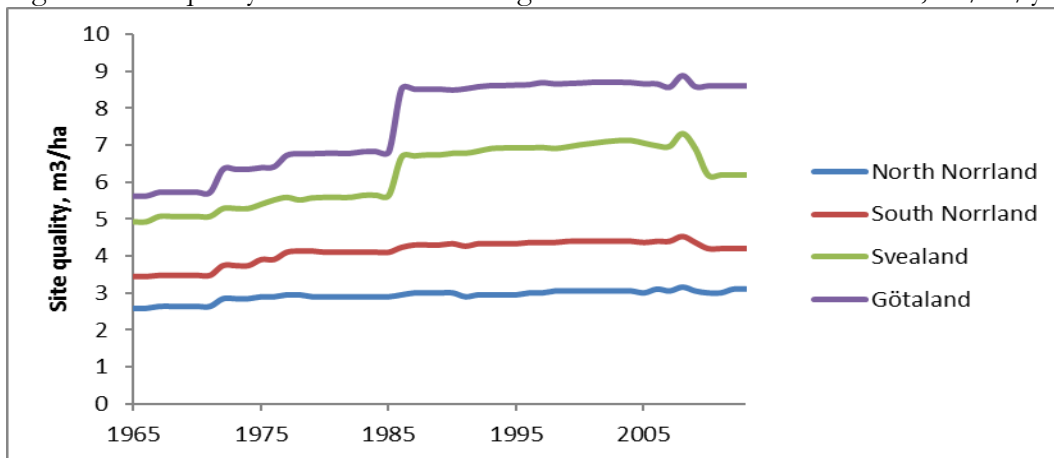
Figure 2: Standing volume plus harvest in Swedish forest regions 1965-2013, m<sup>3</sup>/ha productive forest



Source: Authors' computation from Swedish Annual Forestry Statistics (Swedish Forestry Agency, 2014)

Site quality is measured as the potential productivity of the forest under ideal conditions (Swedish Statistical Yearbook of Forestry, 2014). Potential productivity, in turn, is calculated as the mean annual increment over the life time and is calculated as the maximum standing volume divided by the lifetime of the trees. The lifetime time is 100 years for all forest regions in Sweden, but the maximum potential differs. Maximum potential is based on expert judgement at annual forest inventories, which, in turn, rests on assessments on potential heights, diameter, and density of trees, and soil quality. It is measured in m<sup>3</sup>/ha/year, and varies among the four forest regions, Figure 3.

Figure 3: Site quality in different forest regions in Sweden over 1965-2013, m<sup>3</sup>/ha/year



Sources: Swedish Forestry Agency (2014)

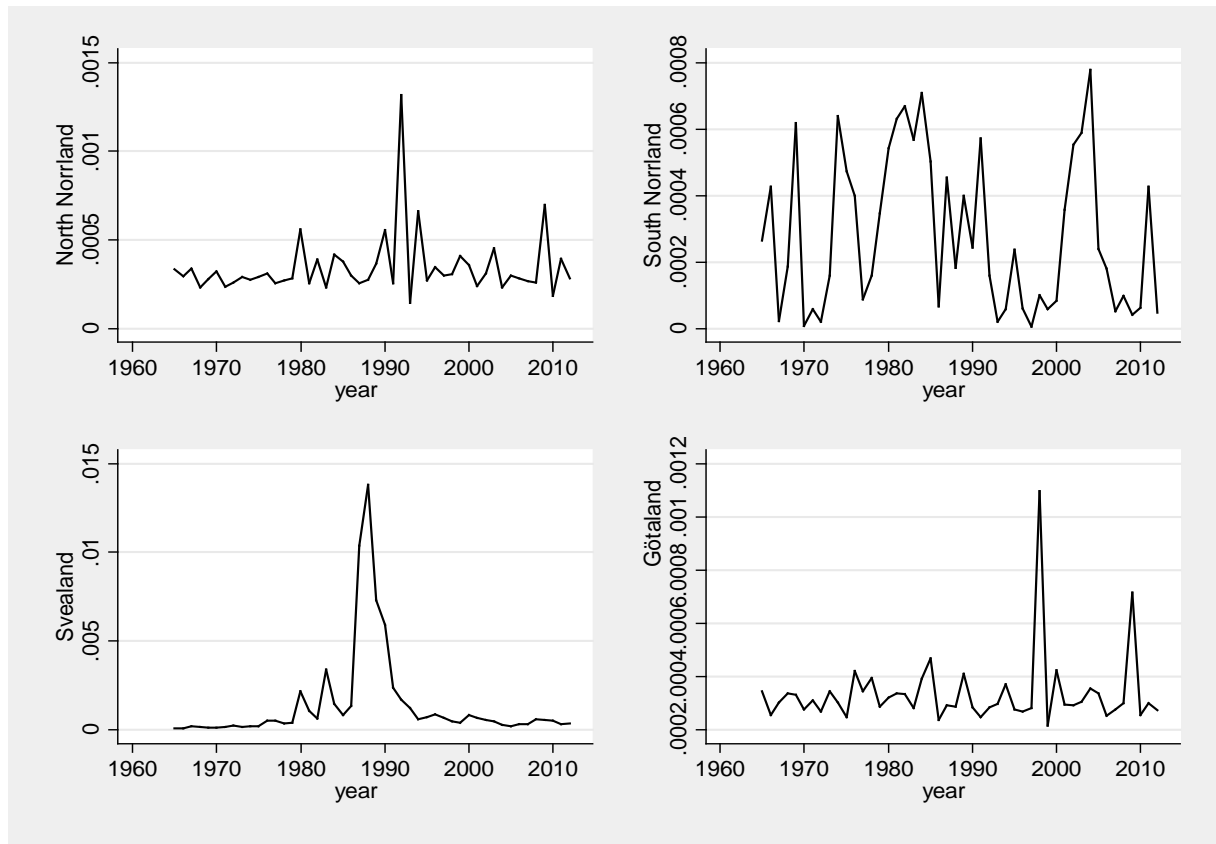
The difference in the levels of site quality among the regions has increased over time, being twice as high in Götaland compared with North Norrland in 1965 and three times as high in 2013. We can also note an upward shift in site quality for Svealand and Götaland in 1985. This can be explained by the change in data collection that took place in 1983-84 which affected the southern regions to a larger degree than the northern (Swedish Forestry Agency, 1985).

As shown in Section 3.1, in addition to standing volume and site quality, data are needed on management variables. The main management practices are thinning, scarification, and fertilization. Data on all these variables over the period 1965-2013 for each of the four forest regions are obtained from Swedish Statistical Yearbook of Forestry 1966-2014 (Swedish Forestry Agency, 2014). The management practices are measured as forest areas subject to thinning, scarification, and fertilization. However, because of eutrophication of coastal waters in Sweden, a fertilization policy was introduced in 1985 which regulated the number and intensity in fertilization. Eutrophication is caused by excess loads of nutrients, and create damage in terms of dead sea bottom areas, increased frequency of toxic blue alga, and changed composition of fish species at the disadvantage of commercial species (e.g. Conley et al., 2009). These damages were particularly severe for Götaland, which therefore faced more strict regulations than the rest of Sweden.

The change in data collection and the introduction of the fertilizer policy occurred in the same year. We therefore introduce a dummy variable which takes value of one in 1985 onwards and zero otherwise, in our empirical estimation to capture the fertilization policy and also the change in data collection for site quality.

Uncertainty in forest growth for this study is measured as the conditional variance for each year. Figures 4 show that the volatility varies with time for each forest region.

Figure 4: Forest growth volatility trend for various forest zones in Sweden (1965-2013)



Variability in the growth of each forest zone displays spikes in different years with Svealand forest zone showing the highest volatility of about 0.014 in the year 1988. With the exception of south Norrland forest zone, the other zones portray relative stability in their forest growth uncertainty though there are spikes in some years.

Descriptive statistics of our data are presented in Table 3.



Table 3: Descriptive statistics for the forest zones in Sweden

VARIABLES	N. Norrland					S. Norrland			
	N	Mean	Sd	min	Max	Mean	Sd	Min	max
Total Area (1000hec)	49	10971.7	2289.1	9096	15431	7563.3	615.54	6178	8982
Productive area (1000hec)	49	7052.8	294.38	6529	7406	5896.4	130.02	5649	6265
Harvest (mil. m <sup>3</sup> )	49	10.36	2.058	4.9	15.1	16.66	3.38	11.2	26.2
Total standing vol.(m <sup>3</sup> )/ha	49	77.77	8.563	64.492	95.766	118.46	9.9533	102.51	136.07
Standing vol (1000 m <sup>3</sup> )/ha	49	0.076	0.0084	0.0632	0.0939	0.116	0.00960	0.100	0.132
Adjusted growth rate	48	0.028	0.0187	-0.0161	0.0843	0.0291	0.0164	-0.0343	0.0820
Volatility in adj. growth rate	48	0.0003	0.0002	0.00015	0.0013	0.0003	0.0002	0.00001	0.0008
Thinning (-1) (1000/ha)	48	0.009	0.0023	0.00566	0.0162	0.0110	0.00302	0.00690	0.0177
Scarification (-1) (1000/ha)	48	0.006	0.0015	0.00261	0.00840	0.00722	0.00225	0.00252	0.0115
Fertilization (-1) (1000/ha)	48	0.003	0.002	0.000212	0.00827	0.00490	0.00289	0.000960	0.0113
Thinning per standing (-1)	48	0.123	0.035	0.0663	0.227	0.0955	0.0249	0.0568	0.161
Scarification per stand.(-1)	48	0.080	0.021	0.0397	0.115	0.0620	0.0181	0.0252	0.108
Fertilization per stand.(-1)	48	0.037	0.029	0.00291	0.119	0.0440	0.0272	0.00774	0.104
Site quality(-1) m <sup>3</sup> /ha/year	48	2.926	0.140	2.600	3.150	4.113	0.330	3.433	4.533
Svealand						Götaland			
Total Area (1000hec)	49	6385.5	323.23	6088	7996	5743.98	235.41	5582	6885
Productive area (1000hec)	49	5368.2	186.63	5155	6152	4899.49	138.48	4517	5476
Harvest (mil. m <sup>3</sup> )	49	18.69	3.53	12.2	26	23.56	6.936	14	44.7
Total standing vol.(m <sup>3</sup> )/ha	49	128.11	14.79	108.73	155.53	154.38	21.192	122.48	187.22
Standing vol (1000 m <sup>3</sup> )/ha	49	0.125	0.014	0.106	0.151	0.150	0.0202	0.118	0.179
Adjusted growth rate	48	0.033	0.034	-0.0784	0.174	0.0393	0.0186	-0.0132	0.0880
Volatility in adj. growth rate	48	0.0014	0.003	0.0001	0.0138	0.0003	0.0001	0.0002	0.0011
Thinning (-1) (1000/ha)	48	0.014	0.0036	0.00601	0.0205	0.0170	0.00448	0.00895	0.0272
Scarification (-1) (1000/ha)	48	0.006	0.0016	0.00175	0.00952	0.00544	0.00182	0.00193	0.0105
Fertilization (-1) (1000/ha)	48	0.005	0.004	0.000690	0.0132	0.00147	0.00184	0	0.00623
Thinning per standing (-1)	48	0.110	0.024	0.0532	0.155	0.114	0.0248	0.0641	0.156
Scarification per stand.(-1)	48	0.050	0.014	0.0155	0.0728	0.0359	0.00953	0.0157	0.0592
Fertilization per stand.(-1)	48	0.045	0.034	0.00484	0.116	0.0112	0.0141	0	0.0485
Site quality (-1) m <sup>3</sup> /ha/year	48	6.203	0.793	4.943	7.314	7.614	1.197	5.622	8.878

## 5. Econometric specification

In order to estimate equations (1)-(2) we move  $H_{it}$  to the left hand side, and the non-linear functions in equations (1)-(2) can be expressed as;

$$z_{it} = f_i(t, \beta_{ij}, Z_i, X_i) + \varepsilon_{it} \quad (8)$$

where  $z_{it} = y_{it} + H_{it}$  is the response variable, and  $\varepsilon_{it}$  is the random error term. The estimators  $\beta_{ij}$ , where  $j=1, \dots, p$  are parameters, to be estimated by non-linear least squares. It can be obtained by minimizing the sum of squares residual (SSR) function shown in equation (9);

$$SSR = \sum_t (z_{it} - f_i(t, \beta_{ij}, Z_i, X_i))^2 \quad (9)$$

The assumptions underlining equation (9) are that the error term in equation (8) is normally and independently distributed with a zero mean and constant variance. The least squares estimates of  $\mathbf{B}$ , are values which when substituted into equation (9) make the sum of squares residual a minimum. These values of  $\mathbf{B}_i$  are found by finding the first derivative of equation (9) and setting the result to zero. This process results in  $k$  times  $p$  simultaneous equations which can be solved for  $\hat{\mathbf{B}}$ . Thus, the system of equations is in the form;

$$\sum_t (z_{it} - f_i(t, \beta_i, Z_i, X_i)) \left( \frac{\partial f_i(t, Z_i, X_i, \mathbf{B}_i)}{\partial \beta_{ij}} \right) = 0 \quad \text{for } i=1, \dots, k \text{ and } j=1, 2, \dots, p \quad (10)$$

Given that the model is non-linear in parameters, it implies that the set of equations in the system are also non-linear. This means that it is impossible to obtain a closed form solution in this setup using least squares by solving for parameters from the system of equations (Fekedulegn et al., 1999). As a result, an iterative method becomes very handy in minimizing the sum of squares residuals to arrive at a closed form solution (Fekedulegn *et al.*, 1999).

The current study makes use of the derivative method<sup>6</sup> to estimate the parameters. This method computes the derivatives of the regression function with respect to each parameter. Two possibilities exist in computing the derivatives: computing analytic expressions or computing finite difference numeric derivatives. This study makes use of the global estimation default setting (in the software) which switches between the two approaches where necessary. While the analytic derivative is used whenever the coefficient derivative is constant, the finite difference numeric derivative is used when derivatives are not constant. Within the numeric derivative approach one has the liberty to choose whether to favor speed of computation or favor accuracy. We opted for the later in our analysis. Since our analysis is an iterative process the convergence rule used is based on changing the parameter values. This approach is much conservative since the change in the objective function (which is the alternative approach) may be quite small as we approach the optimum, while the parameters may still be changing (Kincaid and Cheney, 1996).

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<sup>6</sup> Pertaining to this study, we make use of the derivative method in Eviews software.

Starting values are matter of concern in iterative estimation procedure since they are required for estimating coefficients of the model. We estimate a baseline scenario with zeros as starting values for the parameters. After, we use the estimates from the baseline scenario as starting values for the parameters in the main model which includes fungi diversity and other covariates.

We first estimate a base model without the fungi diversity and control variables. The results from these base line estimates are then used as inputs when we examine the effect of site quality and the control variables displayed in Table 3. The partial derivatives for the baseline models (that is, the logistic and Gompertz) from the derivative method are shown in Table A1 in appendix A.

In relation to the linearized functions in equation (5) and (7), we made use of fully modified ordinary least squares (FMOLS). Fully modified OLS developed by Phillips and Hansen (1990) is a semi-parametric model that is robust to endogeneity and serial correlation problems. Also, it provides a consistent and efficient estimate even in the absence of cointegration relation. Further, it is robust to both stationary and non-stationary series in a single cointegration (see Phillips, 1995). Given that our data is time series, using fully modified OLS helps us to circumvent problems of serial correlation and non-stationarity. In order to estimate the model using fully modified OLS, the variables are first modified and then the system estimates directly to eliminate the existing nuisance parameters. The structure of the fully modified OLS has a correction term for endogeneity and serial correlation. Using this approach is advantageous since the functional form of the linearized logistic function is maintained. Thus, given that our data is time series, variables which are non-stationary need to be transformed by first differencing. This would subsequently change the functional form of the linearized logistic function. As a result, the use of FMOLS makes it possible to maintain the functional form and also circumvent non-stationarity and serial correlation problem.

Based on the adjusted growth rate in equation (10) we derive the volatility in the adjusted growth rate using the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model. The EGARCH is developed by Nelson (1991) and it attempts to address volatility clustering in an innovations process. In this case, volatility clustering occurs when an innovations process does not exhibit significant autocorrelation, but the variance of the process exhibits heteroscedasticity. According to Tsay (2010), EGARCH models are appropriate when positive and negative shocks of equal magnitude might not contribute equally to volatility.

After generating the uncertainty in adjusted forest growth, we proceed by finding how fungi diversity- proxy by site quality- affects the forest growth volatility. The model to be estimated is shown in equation (11);

$$growth\_volatility_t = \psi_1 + \psi_2 Z_t + \gamma \mathbf{X}_t + \nu_t \quad (11)$$

where  $growth\_volatility_t$  is the adjusted growth volatility (uncertainty),  $Z_t$  is site index and  $\mathbf{X}_t$  is a vector of other controls.  $\psi_1, \psi_2$  and  $\gamma$  are parameters to be estimated. The term  $\varepsilon_t$  is the usual error term which is independently and identically distributed with zero mean and equal variance. Similar to equations (5) and (7), we make use of the fully modified ordinary least squares (OLS) to estimate equation (11). We first estimate equation (11) separately for each forest zone. After, we form a panel data from the four forest zones and carry out panel estimation for the entire Sweden.

## 6. Results

The baseline estimation is shown in Tables B1-B2 (see appendix) for the Gompertz and Logistic growth functions, which are used as starting values for the parameters in estimating the effects of site quality and management practices. We tested the effect of lagged and non-lagged variables for management practices, since there could be some delay in the effects of management. The results showed that the lagged management variables gave a better fit than the non-lagged, and we therefore use the lagged variables for all regression estimates. The results of the Gompertz production function specification is presented in Tables 4.

Table 4: Regression results from the Gompertz specification (standard deviations in parentheses)

Variables	N.Norrland	S.Norrland	Svealand	Götaland	Sweden
$\beta_0$	154.15 (720.28)	79.41 (500.93)	120.56*** (10.4)	141.89*** (12.42)	103.69*** (6.47)
$\beta_1$	-91.02 (737.55)	1350.52 (95550.7)	44.97*** (7.47)	56.01*** (7.95)	41.47*** (3.07)
$\beta_2$	-0.041 (0.098)	0.0039 (0.11)	0.142*** (0.033)	0.154*** (0.039)	0.105*** (0.015)
$\beta_3$	67.69 (271.92)	321.96 (13588.7)	26.98*** (1.613)	23.017*** (1.375)	26.486*** (1.381)
Site Index (-1)	0.734 (5.293)	-4.484 (3.741)	-1.469 (1.96)	-0.594 (1.837)	-0.411 (1.55)
Policy	-0.412 (1.611)	3.925** (1.753)	-2.377 (3.67)	-3.707 (3.45)	0.376 (1.49)
Fertiliz.perfect(-1)	11.21 (178.75)	86.08 (139.96)	253.71 (283.295)	-16.899 (449.18)	410.57 (295.37)
Thinnperfect.(-1)	-125.37 (178.55)	-337.14** (134.47)	425.37* (249.02)	-360.66 (238.13)	-336.28** (130.40)
Scarifi. perfect.(-1)	-239.06 (363.73)	25.23 (191.74)	-882.11** (396.33)	163.74 (755.24)	348.01 (315.87)
$\rho_1$	0.733*** (0.130)			0.676*** (0.145)	
Observations	47	47	47	47	47
Adj. R-squared	0.971	0.973	0.96	0.983	0.99
Log likelihood	-77.497	-86.22	-115.19	-107.99	-74.97
F-statistic	170.23***	216.8***	143.3***	291.52***	617.44***
AIC	3.723	3.968	5.174	5.021	3.499
DW stat.	1.392	1.687	2.23	2.06	1.905
Inverted AR roots	0.74			0.68	

Probabilities: \*\*\*  $p < .01$ , \*\*  $p < .05$ , \*  $p < .10$ .  $\rho_1$  is serial correlation coefficient

The results in Table 4 show good statistical performance as revealed by the adjusted  $R^2$  which exceeds 0.95 for all regions, by the absence of serial correlation shown by the DW statistic, and by the significant estimates of several explanatory variables, especially in the southern regions and the entire Sweden. The significant estimates of  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  for the southern regions (that is Svealand and Götaland) indicate non-linear production functions. The growth rates, i.e. the estimates of  $\beta_2$ , are higher for the southern than the northern regions, which is expected. The levels are in the same order of magnitude as in the base case (Table B3 in appendix). The constants show the initial standing volume per ha, which are in the range of the report for all region. The maximum increase in growth, i.e.  $\beta_3$ , is achieved after approximately 27 and 23 years for Svealand and Götaland respectively, and the asymptotic standing volume ( $\beta_0 + \beta_1$ ) is highest for Götaland where it amounts to 198 m<sup>3</sup>/ha. On average, the growth rate for Swedish forest is about 0.105 and the maximum

increase in growth is achieved after approximately 26 years. In addition, the asymptotic standing volume is about 144 m<sup>3</sup>/ha.

However, site index has no significant effect in any of regions and the entire Sweden. Most of the estimates of the management practices are also insignificant. Whereas thinning has negative effect on standing volume in South Norrland and the entire Sweden, a positive effect is observed in the case of Svealand. The results from the logistic production function are very similar, but with less satisfactory statistical performance, which are presented in Table B5. One reason for the insignificant estimates of management practice can be associated with endogeneity in the function, where several parameters, such as the growth rate and asymptotic standing volume, can depend on management practices.

Based on the linearized logistic function we examine the effect of site quality and management practices on forest growth rate on the two different models, direct and indirect effects, shown by equations (5) and (7) in Section 3. The dependent variable shows the change in the growth rate of productivity. Simple plotting of growth rate and site quality for each region shows positive relations, see Figures C2. However, augmented Dickey-Fuller unit root tests revealed non-stationarity, and Durbin Watson statistics revealed serial correlation. Therefore, we used fully modified ordinary least square (FMOLS) for all regressions as described in Section 5.

Results separate for each forest region showed expected positive signs of the constant, i.e. intrinsic growth rate, and negative sign on standing volume, see Table B6 and B7. However, the levels of the significant estimates of the constants were unrealistically large and we therefore focus on the results for entire Sweden, see Table 5.

The statistical performance improves from the inclusion of site index and management practices for both models. The constant, or intrinsic growth rate, is significant in all equations, but increases when including the variables for management practices. Standing volume has the expected negative sign, but is significant only for the models with the management variables.

Table 5: Regression results from FMOLS of adjusted growth rate for Sweden

Variables	Basic model	Direct model	Indirect model
Constant	0.0324*** (0.0077)	0.1321*** (0.0145)	0.1037*** (0.0120)
Standing per hect.	-0.0810 (0.0940)	-2.1031*** (0.2459)	-1.8648*** (0.2287)
Fertilizer (-1)		0.0875 (0.0986)	0.3306 (1.2093)
Policy		-0.0001 (0.0002)	-0.0000 (0.0002)
Thinning (-1)		-0.0679 (0.0448)	-0.5115 (0.3953)
Scarificat. (-1)		-0.4243*** (0.0987)	-3.8138*** (0.9565)
Site index (-1)		0.0228*** (0.0028)	0.0219*** (0.0028)
Trend		0.0012*** (0.0002)	0.0012*** (0.0002)
South Norrland	0.0045 (0.0052)	-0.0076 (0.0081)	-0.0050 (0.0078)
Svealand	0.0090 (0.0058)	-0.0937*** (0.0175)	-0.0914*** (0.0172)
Götaland	0.0160** (0.0078)	-0.1227*** (0.0242)	-0.1226*** (0.0242)
Observations	194	190	190
R-squared	0.0287	0.2151	0.2131
Long Run SE	0.0180	0.0149	0.0148
Bandwidth(neweywest)	8.974	14.32	14.26

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ . In carrying out the estimation for entire Sweden we use the information for the various forest zones to form a panel data. This provides us with sufficient observations without compromising on the accuracy and efficiency of our estimates. We accounted for zonal fixed effect since each forest zone may have unique characteristics which will affect the response variable

Site quality is significant and positive for both models. Scarification is the only management practices which has significant effect in both models. This effect is observed to be negative. The marginal impact of site quality is about 0.023, i.e growth rate increases by 0.023 from one unit increase in the site quality (see direct model). It can also be noticed that the estimates of site quality are significant for only the northern regions in the case of the direct model, however, the indirect model shows a significant effect of site quality in all forest regions except Götaland (Tables B6-B7).

However, the signs are negative for North and South Norrland and positive for the Svealand for both the direct and indirect model.

Scarification shows a negative effect for both models in all regions and the entire Sweden. However, fertilization and thinning have significant effect only in some regions (Tables B6-B7). When significant, management practices show negative signs for both models except fertilization which show positive sign (but not in Svealand for indirect model). One reason can be that they contribute to growth in standing volume per ha, but to a lower degree at higher levels. The growth rate then decreases as the magnitude of the practices increase. Another reason can be that our one period lag of these variables does not capture the long term effect of the practices. Interestingly, the growth rate for both direct and indirect models during the time of regulation is significantly higher than periods without regulation for all regions with the exception of Svealand where growth rate is lower. On the contrary, no significant difference is observed in the growth rate for the entire Swedish forest between the two periods.

With respect to uncertainty, simple plots of volatility in adjusted growth rate and site quality do not show a clear relation for any forest region (Figures C3). Nevertheless, when estimating volatility as a function of site quality and management practices, the biodiversity indicator has a significant impact in all regions but one, Table 6.



Table 6: FMOLS regression results of conditional variance in adjusted growth rate

Variables	N. Norrland	S. Norrland	Svealand	Götaland	Sweden
Site index (-1)	-0.00057*** (0.00019)	0.00054*** (0.00010)	0.00109 (0.00151)	0.00001 (0.00002)	0.00037*** (0.00012)
Fertili. per hect. (-1)	-0.01724* (0.00921)	0.02815*** (0.00848)	0.15477 (0.22914)	-0.01406* (0.00832)	0.09154*** (0.02967)
Policy	0.00009* (0.00005)	-0.00019** (0.00008)	0.00468** (0.00212)	0.00003 (0.00005)	0.00117*** (0.00023)
Thinning per hect. (-1)	-0.01297* (0.00752)	0.00018 (0.00790)	0.23013 (0.17234)	-0.01389*** (0.00321)	0.00101 (0.02021)
Scarifi. per hect (-1)	0.06350*** (0.01286)	-0.04474*** (0.01308)	0.56931 (0.38537)	0.04727*** (0.01042)	0.00557 (0.04492)
South Norrland					0.00121*** (0.00045)
Svealand					0.00345*** (0.00089)
Götaland					0.00412*** (0.00128)
Trend			-0.00025*** (0.00009)	-0.00000* (0.00000)	-0.00004*** (0.00001)
Constant	0.00174*** (0.00054)	-0.00167*** (0.00040)	-0.00955 (0.00896)	0.00036** (0.00014)	-0.00069** (0.00033)
Observations	46	46	46	46	187
R-squared	0.06581	0.14763	0.39853	0.06234	0.10409
Long Run SE	7.13e-05	0.000112	0.00256	5.49e-05	0.000738
Bandwidth(neweywest)	12.34	13.18	2.640	6.667	97.34

Standard errors in parentheses, \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ . In estimation for entire Sweden, we use the information for the various forest zones to form a panel data. This provides us with sufficient observations without compromising on the accuracy and efficiency of our estimates. We accounted for zonal fixed effect since each forest zone may have unique characteristics which will affect the response variable. The estimation for the northern regions exhibit no trend

Considering the long run standard errors, the regression results in Table 6 pass the fitness criteria as the long run standard errors are very small. The long run standard error is mostly used to ascertain the fitness of the model since FMOLS technique transforms the data before estimating the parameters. The lower the long run standard error, gives an indication of a good fit model. The mean volatility in adjusted growth rate varies between 0.0003 and 0.0014 between the four forest regions and amounts to 0.0006 for entire Sweden. The impact of a marginal change in site quality

thus raises the average volatility in Sweden by approximately 62%<sup>7</sup>, and at the most by 180% which occurs for South Norrland. On the other hand, the results indicate that the volatility in North Norrland could be reduced by, in average, 190%. The results in Table 6 also show diverse effects of management practices. There is no management variable with the same direction of marginal impact in all regions, except thinning which has a negative significant in North Norrland and Götaland. Generally, fertilization shows a positive effect on growth volatility for Swedish forest and the growth volatility in periods of regulation is significantly higher than periods without regulations.

## 7. Summary and conclusions

The main purpose of this study has been to estimate the effect of fungi diversity on total standing volume per hectare and variability in forest standing volume and its growth rate. To this end, we used econometric tools for estimation of forest growth functions. Different functional forms were tested and a common feature was the assumption of a sigmoid shape in forest standing volume over time. A panel data set was used which covers the period 1965-2013 for four different forest zones. Fungi diversity was measured by means of a site quality index which reflects the potential of the average forest site in each forest zone. Admittedly, the index does not directly measure fungi diversity, but it is highly correlated with the diversity and is therefore regarded as a good approximation. The main advantage of the index is the availability for quite many years and for different forest zones.

In addition to a proxy of fungi diversity, explanatory variables on management practices (thinning, scarification, and fertilization) were included. However, neither site quality nor management practices showed any significant effects on the development in standing volume. On the other hand, significant impacts were obtained for their effects on growth rate in standing volume. The estimated intrinsic growth rate for entire Sweden was 0.13, which is similar to the estimates obtained for development in standing volume. The estimated marginal effect of site quality was an increase in growth rate by 0.023 which is considerable when considering that the average growth rate is 0.031. This result supports that of a positive effect of another indicator of diversity, tree species, on forest productivity (Piotto, 2008; Zhang et al., 2012). On the other hand, our regression results indicated

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<sup>7</sup> Calculated as the estimated coefficient in Table 6 divided by the mean volatility in Table 3

that site quality also increases volatility in growth rate, where volatility was measured as the conditional variance in each time period. A marginal increase in site quality could raise the average volatility by 0.0004, which can be compared with the average volatility of 0.0006. This is in contrast to the expectations of a stabilizing impact on growth rate of biodiversity (e.g. Ives and Carpenter 2008; Campbell et al., 2011). On the other hand, an input, such as site quality, that contributes positively to production and its uncertainty is common for several production functions in economics (e.g. Pope et al. 1979).

Our results thus point out a tradeoff between increase in productivity and stability in forest growth rate of fungi diversity measured as site quality. Productivity increases contribute by higher values from timber and other ecosystem services. The cost of increased volatility depends on risk aversion and on the availability of options to reduce or insure against risk. Admittedly, our empirical results rest heavily on a positive correlation between site quality and fungi diversity. Nevertheless, our approach and estimations could contribute to the calculations of benefits and costs of increases in fungi diversity as measured by other indicators.

## Appendix A: Derivation of the linearized logistic growth function

Linearizing the logistic function

$$\text{Assume } Y_t = \beta_1 * \left[ 1 / \left( 1 + \exp(-\beta_2(t - \beta_3)) \right) \right] \quad (\text{A1})$$

This can be rewritten as;  $Y_t = \beta_1 * \left( 1 + \exp(-\beta_2(t - \beta_3)) \right)^{-1}$

Finding the time derivative of the expression above yields;

$$\frac{\partial Y_t}{\partial t} = -\beta_1 \left[ 1 + \exp(-\beta_2(t - \beta_3)) \right]^{-2} \left( -\beta_2 \exp(-\beta_2(t - \beta_3)) \right)$$

$$\frac{\partial Y_t}{\partial t} = \frac{\beta_1}{1 + \exp(-\beta_2(t - \beta_3))} \cdot \frac{\beta_2 \exp(-\beta_2(t - \beta_3))}{1 + \exp(-\beta_2(t - \beta_3))} \quad (\text{A2})$$

Substituting equation (A1) into equation (A2) yields;

$$\frac{\partial Y_t}{\partial t} = \beta_2 Y_t \left[ \frac{\exp(-\beta_2(t - \beta_3))}{1 + \exp(-\beta_2(t - \beta_3))} \right] \quad (\text{A3})$$

From equation (A1), we can rewrite the equation as  $\frac{Y_t}{\beta_1} = \frac{1}{1 + \exp(-\beta_2(t - \beta_3))}$ . Plugging this

equation into equation (A3) gives as;

$$\frac{\partial Y_t}{\partial t} = \beta_2 Y_t \left[ \frac{Y_t}{\beta_1} \left( \exp(-\beta_2(t - \beta_3)) \right) \right] \quad (\text{A4})$$

We can rearrange equation (A1) as;

$$1 + \exp(-\beta_2(t - \beta_3)) = \frac{\beta_1}{Y_t} \Rightarrow \exp(-\beta_2(t - \beta_3)) = \frac{\beta_1}{Y_t} - 1 \quad (\text{A1a})$$

$$\therefore \exp(-\beta_2(t - \beta_3)) = \frac{\beta_1 - Y_t}{Y_t}$$

Now, substituting equation (A1a) into equation (A4) gives as;

$$\begin{aligned}\frac{\partial Y_t}{\partial t} &= \beta_2 Y_t \left[ \frac{Y_t (\beta_1 - Y_t)}{\beta_1 Y_t} \right] = \beta_2 Y_t \left[ \frac{\beta_1 - Y_t}{\beta_1} \right] = \beta_2 Y_t \left[ 1 - \frac{Y_t}{\beta_1} \right] \\ \therefore \frac{\partial Y_t}{\partial t} &= \beta_2 \left[ 1 - \frac{Y_t}{\beta_1} \right] Y_t \Leftrightarrow \dot{Y} = \beta_2 \left[ 1 - \frac{Y_t}{\beta_1} \right] Y_t\end{aligned}\tag{A5}$$

## Appendix B: Tables

Table B1: Partial derivatives of growth functions with respect to parameters

	Gompertz growth function	Logistic growth function
Parameter	Specification of derivatives	Specification of derivatives
$\beta_0$	-1	-1
$\beta_1$	$-\exp(-\exp(-\beta_2(t-\beta_3)))$	$-1/(1+\exp(-\beta_2(t-\beta_3)))$
$\beta_2$	$\beta_1 * (\beta_3 - t) * \exp(-\beta_2(t-\beta_3)) * \exp(-\exp(-\beta_2(t-\beta_3)))$	$\beta_1 * (\beta_3 - t) * \exp(-\beta_2(t-\beta_3)) / (1 + \exp(-\beta_2(t-\beta_3)))^2$
$\beta_3$	$\beta_1 * \beta_2 * \exp(-\beta_2(t-\beta_3)) * \exp(-\exp(-\beta_2(t-\beta_3)))$	$\beta_1 * \beta_2 * \exp(-\beta_2(t-\beta_3)) / (1 + \exp(-\beta_2(t-\beta_3)))^2$
$\beta_4$	Site Index	Site Index
$\beta_5$	Fertilization	Fertilization
$\beta_6$	Policy	Policy
$\beta_7$	Thinning	Thinning
$\beta_8$	Scarification	Scarification

Table B2: Descriptive Statistics for Sweden

Variables	N	mean	sd	min	max
*Total Area (1000hec)	49	30719.47	3292.79	27249	39294
*Productive area (1000hec)	49	23216.98	499.9	22379	24449
*Harvest (mil. m <sup>3</sup> )	49	69.276	12.47	52.3	97.5
Total Area (1000hec)	196	7679.87	2370.02	5582	15431
Productive area (1000hec)	196	5804.25	828.31	4517	7406
Harvest (mil. m <sup>3</sup> )	196	17.32	6.42	4.9	44.74
Total standing vol.(m <sup>3</sup> )/ha	196	119.68	31.13	64.493	187.22
Standing vol (1000 m <sup>3</sup> )/ha	196	0.117	0.0298	0.0632	0.179
Adjusted growth rate	195	0.0313	0.0296	-0.143	0.174
Volatility in adj. growth rate	192	0.0006	0.0014	0	0.0138
Thinning (-1) (1000/ha)	192	0.0005	0.0018	0	0.0197
Scarification (-1) (1000/ha)	192	0.0127	0.00449	0.00566	0.0272
Fertilization (-1) (1000/ha)	192	0.00624	0.00192	0.00175	0.0115
Thinning per standing (-1)	192	0.00357	0.00311	0	0.0132
Scarification per stand.(-1)	192	0.111	0.0291	0.0532	0.227
Fertilization per stand.(-1)	192	0.0572	0.0228	0.0155	0.115
Site quality (-1) m <sup>3</sup> /ha/year	192	0.0343	0.0302	0	0.119
Total Area (1000hec)	189	0.915	1.543	0	7.780

\* denotes aggregates

Table B3: Regression results from base line nonlinear Gompertz production function

Variables	N.Norrland	S.Norrland	Svealand	Götaland	Sweden
$\beta_0$	69.326*** (5.1375)	103.41*** (3.346)	115.07*** (0.727)	135.05*** (2.787)	102.49*** (0.698)
$\beta_1$	78.823 (159.30)	43.307*** (12.66)	42.187*** (3.471)	46.193*** (4.991)	38.309*** (2.576)
$\beta_2$	0.0333 (0.047)	0.052** (0.0194)	0.127*** (0.0212)	0.189*** (0.053)	0.106*** (0.014)
$\beta_3$	51.38 (53.89)	25.961*** (3.490)	29.484*** (33.526)	24.415*** (1.125)	26.589*** (0.791)
$\rho_1$	0.77*** (0.0949)	0.452*** (0.133)		0.717*** (0.112)	0.421*** (0.136)
Observations	48	48	48	48	48
Adj. R-squared	0.974	0.977	0.956	0.984	0.98
Log likelihood	-80.056	-85.17	-123.07	-112.45	-79.68
F-statistic	443.91***	499.98***	346.56***	713.51***	1117.04***
AIC	3.544	3.757	5.186	4.894	3.528
DW stat.	1.465	1.838	1.776	2.10	2.293
Inverted AR roots	0.77	0.45		0.72	0.42

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1,  $\rho_1$  is serial correlation coefficient

Table B4: Regression results from the baseline nonlinear logistic production function

Variables	N. Norrland	S.Norrland	Svealand	Götaland	Sweden
$\beta_0$	66.444*** (9.211)	99.638*** (5.514)	114.72*** (0.873)	132.48*** (2.652)	101.16*** (0.681)
$\beta_1$	62.094 (121.27)	41.979*** (12.082)	38.821*** (2.416)	48.72*** (4.305)	36.735*** (1.549)
$\beta_2$	0.0646 (0.078)	0.0796** (0.0303)	0.220*** (0.033)	0.241*** (0.056)	0.169*** (0.015)
$\beta_3$	50.351 (50.176)	27.608*** (2.9694)	31.702*** (0.784)	26.177*** (0.995)	28.829*** (0.536)
$\rho_1$	0.768*** (0.096)	0.458*** (0.131)		0.634*** (0.128)	0.254* (0.145)
Observations	48	48	48	48	48
Adj. R-squared	0.974	0.977	0.95	0.984	0.99
Log likelihood	-80.108	-85.181	-125.77	-111.53	-76.63
F-statistic	442.93***	499.73***	308.82***	741.85***	1269.5***
AIC	3.546	3.758	5.297	4.855	3.401
DW stat.	1.458	1.848	1.592	2.015	2.144
Inverted AR roots	0.77	0.46		0.63	0.25

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1,  $\rho_1$  is serial correlation coefficient

Table B5: Regression results from the logistic production function

Variables	N.Norrland	S.Norrland	Svealand	Götaland	Sweden
$\beta_0$	156.21 (403.55)	57.24 (820.56)	119.4*** (10.405)	135.8*** (11.43)	104.09*** (6.27)
$\beta_1$	-91.75 (415.42)	1485.62 (260798.0)	42.52*** (6.835)	57.49*** (9.11)	40.06*** (3.385)
$\beta_2$	-0.0499 (0.102)	0.0093 (0.183)	0.226*** (0.054)	0.195*** (0.047)	0.160*** (0.024)
$\beta_3$	61.456 (146.57)	334.08 (25007.9)	29.53*** (1.335)	25.36*** (1.44)	28.81*** (1.235)
<i>Site Index (-1)</i>	0.797 (5.297)	-4.502 (3.737)	-1.208 (1.889)	-0.121 (1.828)	-0.623 (1.482)
Policy	-0.415 (1.613)	3.948 (1.712)	-1.67 (3.126)	-3.144 (3.319)	1.054 (1.374)
Fertiliz.perhect(-1)	14.12 (179.13)	85.168 (140.36)	262.99 (291.01)	-86.396 (441.88)	380.99 (298.80)
Thinnperhect.(-1)	-123.9 (178.8)	-338.73 (134.79)	483.96* (273.69)	-317.34 (230.39)	-259.64* (145.97)
Scarifi. perhect.(-1)	-233.83 (364.8)	24.40 (191.09)	-1190.86*** (426.16)	-7.496 (724.36)	15.48 (314.19)
$\rho_1$	0.731*** (0.13)			0.583*** (0.158)	
Observations	47	47	47	47	47
Adj. R-squared	0.971	0.973	0.96	0.983	0.991
Log likelihood	-77.49	-86.22	-116.5	-107.22	-73.66
F-statistic	170.27***	216.81***	135.48***	301.3***	652.16***
AIC	3.72	3.967	5.23	4.99	3.44
DW stat.	1.393	1.69	2.21	1.99	1.997
Inverted AR roots	0.73			0.58	

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1,  $\rho_1$  is serial correlation coefficient



Table B6: FMOLS regression results for the direct model of adjusted growth rate for separate regions

Variables	North Norrland	South Norrland	Svealand	Götaland
Intrinsic growth rate	0.6237*** (0.0540)	0.7117*** (0.1070)	0.5900*** (0.1332)	0.2214*** (0.0717)
Standing per hect.	-3.5584*** (0.4124)	-5.3402*** (0.8465)	-5.4849*** (0.8781)	-1.4139*** (0.5421)
Fertili.perstand (-1)	0.2865*** (0.0464)	-0.0317 (0.0730)	-0.2693 (0.1996)	0.4209* (0.2266)
Policy	0.0144*** (0.0036)	0.0237*** (0.0060)	-0.0516*** (0.0172)	0.0300*** (0.0107)
Thinning perstand (-1)	-0.1853*** (0.0376)	-0.1029 (0.0693)	0.1289 (0.1636)	-0.0989 (0.1035)
Scarificat.perstand (-1)	-0.2948*** (0.0646)	-0.0259 (0.0960)	-1.8983*** (0.4416)	-0.8791*** (0.3283)
Site index (-1)	-0.1248*** (0.0164)	-0.0371*** (0.0103)	0.0167 (0.0117)	0.0019 (0.0047)
Trend	0.0028*** (0.0003)	0.0036*** (0.0006)	0.0061*** (0.0013)	0.0014* (0.0008)
Observations	46	46	46	46
R-squared	0.3116	0.2870	0.4093	0.3285
Adjusted R-squared	0.3438	0.4647	0.3556	0.3724
Long Run SE	0.223	0.366	0.237	0.257
Bandwidth(neweywest)	0.00461	0.00754	0.0195	0.0107

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

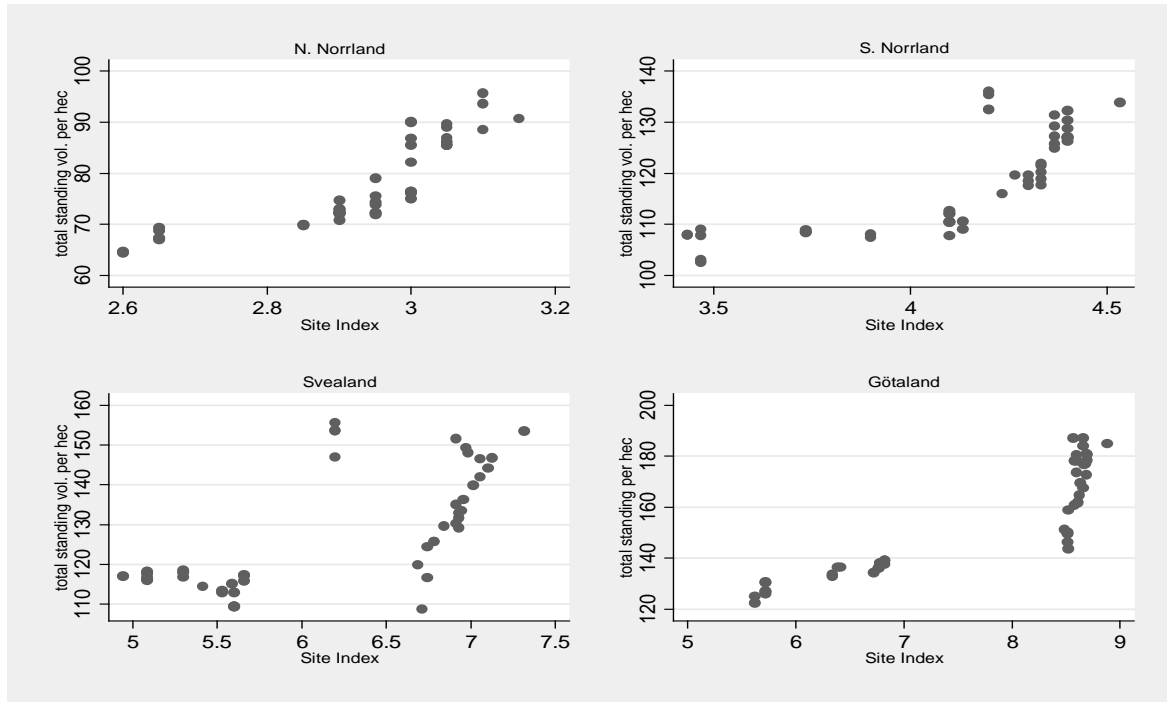
Table B7: FMOLS regression results for the indirect model of adjusted growth rate for the four Swedish regions

Variables	N. Norra	S. Norra	Svealand	Götaland
Constant	0.5590*** (0.0564)	0.6499*** (0.0840)	0.3974*** (0.1249)	0.1924*** (0.0602)
Standing per hect.	-3.1932*** (0.4258)	-4.8613*** (0.6625)	-4.0558*** (0.7850)	-1.1362** (0.4580)
Fertilization per hect. (-1)	3.8946*** (0.6924)	-0.1700 (0.5190)	-2.8228* (1.7001)	2.8751* (1.5692)
Policy	0.0136*** (0.0038)	0.0222*** (0.0048)	-0.0470*** (0.0162)	0.0308*** (0.0097)
Thinning per hect. (-1)	-2.3303*** (0.5509)	-0.5788 (0.5005)	1.7712 (1.2806)	-0.4327 (0.6273)
Scarification per hect. (-1)	-4.6495*** (0.9883)	-0.4513 (0.7006)	-12.8119*** (3.9149)	-5.5367*** (1.9602)
Site index (-1)	-0.1108*** (0.0174)	-0.0346*** (0.0082)	0.0208* (0.0116)	-0.0008 (0.0042)
Trend	0.0028*** (0.0003)	0.0034*** (0.0005)	0.0045*** (0.0014)	0.0015** (0.0008)
Observations	46	46	46	46
R-squared	0.3288	0.4626	0.3207	0.3423
Adjusted R-squared	0.205	0.364	0.196	0.221
Long Run SE	0.00503	0.00596	0.0190	0.00978
Bandwidth(neweywest)	14.49	5.246	6.390	4.940

Standard errors in parentheses, \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

## Appendix C: Figures

Figure C1 : Relationship between total standing volume per hectare and site quality



Figures C2: Adjusted growth rate and site quality for the four Swedish forest regions

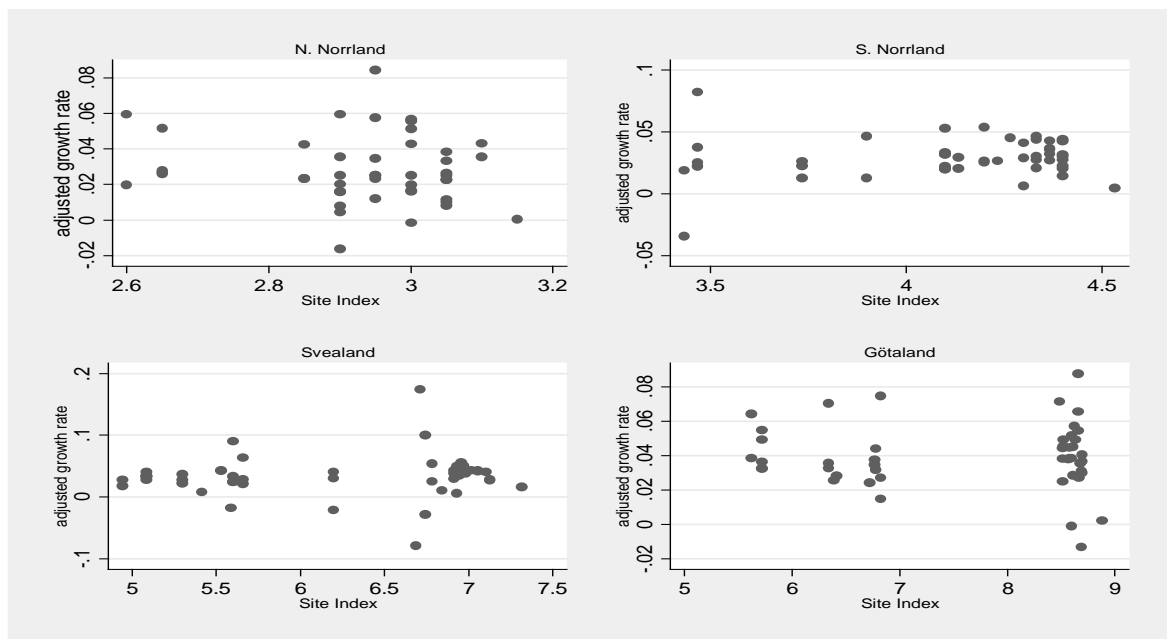
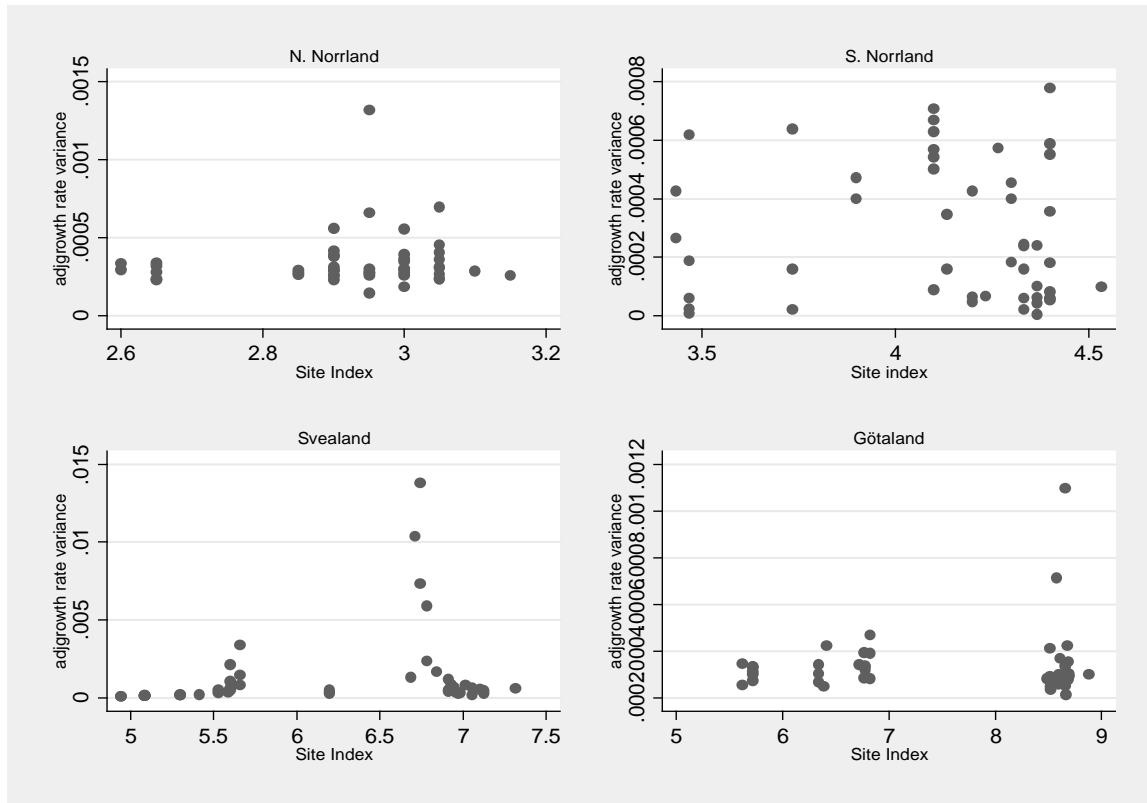


Figure C3 : Relationship between adjusted growth rate conditional variance and site quality



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