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20 with the correction factor, and the corresponding variance was estimated. Many factors
21 contribute to the uncertainty of the stand level prediction; in the variance estimator we
22 accounted for the uncertainties due to Kriging prediction and due to estimating a detectability
23 model from the laser scanning data. The results from our new approach were found to
24 correspond fairly well with estimates obtained using field measurements from an independent
25 set of 54 circular sample plots. The predicted number of stems in the stand based on the
26 proposed methodology was 1366 with a 12.9 % relative standard error. The corresponding
27 estimate based on the field plots was 1677, with a 7.5 % relative standard error.

28 Keywords: Covariogram; Detectability function; Forest management; Model-based inference.

29

30 1. Introduction

31 Traditionally, stand level forest inventories have been based on field surveys where the
32 surveyor allocates sample plots either subjectively or by random sampling to assess key stand
33 characteristics such as stem density, age, and growing stock volume (Kangas and Maltamo
34 2006). This information typically is acquired for supporting forest management decisions.
35 However, due to high costs of field data collection, substantial research has been attributed to
36 replacing, or improving the cost-efficiency of, field surveys using remotely sensed data during
37 the last decades. Today, laser scanning has become a widely-used technique to support forest
38 inventories (*e.g.*, Næsset 2002, Boudreau *et al.*, 2008; Andersen *et al.*, 2011). For practical
39 applications, airborne laser scanning (*e.g.*, Naesset 2002, Wulder *et al.*, 2012; Neigh *et al.*,
40 2013) is currently the most common approach. For variables, such as stand height, volume
41 and biomass it can provide stand level estimates that are as precise as, or even more precise,
42 than those obtained from traditional stand level inventories (*e.g.*, Hyypä & Hallikainen,
43 1996; Hyypä & Inkinen, 1999; Næsset, 2002; Ørka *et al.*, 2016). Several studies also point at
44 the potential of laser scanning for providing auxiliary data for improving large-area forest
45 surveys (*e.g.*, Gobakken *et al.*, 2012; Saarela *et al.*, 2015; Ene *et al.*, 2017).

46 Also, terrestrial laser scanning (TLS) has been investigated in many studies (*e.g.*, Watt *et al.*,
47 2005; Ducey *et al.*, 2013; Ducey & Astrup, 2013; Liang *et al.*, 2016; Vaaja *et al.*, 2016;
48 Olofsson & Holmgren, 2016) and it has the potential to become a competitive alternative or
49 adjunct to traditional sample plot inventories. With TLS, currently, a normal procedure is to
50 mount a laser scanning device at several locations in a stand whereby detailed tree level
51 information can be obtained from the measurements. Ducey *et al.* (2013) report that stand
52 conditions and scanner attributes affect how well tree stems can be identified and measured
53 and that none of the scanners evaluated in their study provided sufficiently reliable diameter

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54 measurements to substitute field measurements. On the other hand, Olofsson *et al.* (2014)
55 suggest a method which can measure stem diameters with relative root-mean-square error
56 (RMSE) of 14%. However, with the laser scanner mounted at fixed positions (i.e. single scan
57 mode) there is a problem that some of the trees are hidden by trees located close to the
58 measurement device and thus are not detected. To overcome potentially negative bias due to
59 this, Ducey & Astrup (2013) and Astrup *et al.* (2014) developed procedures to adjust for non-
60 detection and thus make the estimators (almost) unbiased.

61 Terrestrial laser scanning is rapidly developing and modern measurement devices are
62 becoming smaller, cheaper and more accurate. They can also be combined with satellite-based
63 positioning systems, so that all the trees in the vicinity of the measurement device
64 automatically receive a position. Terrestrial laser devices need not be stationary during
65 measurements and, thus, mobile laser scanning (MLS) has emerged as a special branch of
66 TLS. In this case the lasers are mounted on all-terrain vehicles (ATVs), on unmanned aerial
67 vehicles (UAVs) (e.g., Jaakkola *et al.*, 2010; Glennie *et al.*, 2013, Forsman *et al.*, 2016), or
68 they may be held by or be attached to a surveyor walking through the forest (e.g., Liang *et al.*,
69 2014; Rönnholm *et al.*, 2016; Lehtola *et al.*, 2016). For MLS, promising results have been
70 obtained by Rönnholm *et al.* (2016), who evaluated the quality of backpack laser scanning
71 data by comparing them with UAV laser scanning data. Jaakkola *et al.* (2010) performed both
72 automatic and manual tree finding, height determination, and automatic measurement of
73 diameter at breast height (DBH). The tree height bias was 2 cm for manual measurement and -
74 15 cm for automatic measurement. The measurement of DBH was obtained with a root mean
75 square error of 2.1 cm. Forsman *et al.* (2016) reported a root-mean-square error of 14% (3.7
76 cm) in DBH estimation by MLS data using a line-wise intensity-based clustering method.

77 With MLS it is not obvious how estimation procedures should be developed in order to
78 produce precise and unbiased estimators at the level of stands (or larger areas). There are at
79 least three main problems that need to be addressed. These are (i) non-detection errors of
80 similar kind as with ordinary TLS, (ii) difficulties to obtain probability samples and thus
81 unbiased estimators, since accessibility (at least for ATVs) typically is limited in many parts
82 of forest stands, and (iii) measurement errors at the level of single trees due to inexact laser-
83 based determination of key features such as height and diameter.

84 The objective of this study was to develop and evaluate procedures for predicting stand level
85 attributes from MLS data, based on previous experiences with handling non-detection in TLS
86 surveys (Astrup *et al.*, 2014). The developed procedures constitute one way to account for the
87 first two issues outlined above, i.e. adjusting for non-detection errors and handling non-
88 probability samples. The study was performed in a 4.7 ha large study area in southern
89 Norway, based on data collected from a mobile laser scanner mounted on an ATV. The
90 estimates obtained were compared with estimates from an independent set of field sample
91 plots from the same area. We focus on the prediction of stem number of mid- and overstory
92 trees as a characteristic that is relatively directly available from MLS data without a need for
93 further models for assessing characteristics such as stem volume and biomass.

94 **2. Material and method**

95 ***2.1. Material***

96 *2.1.1. Field data*

97 The study area was the 4.7 ha Frydenhaug forest in Ås municipality, approximately 30 km
98 south of Oslo, Norway. The forest is owned by the Norwegian University of Life Sciences
99 and is mainly used for educational and recreational purposes. Frydenhaug is dominated by
100 Norway spruce and Scots pine but is rather heterogeneous with some areas that are dominated

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101 by non-indigenous tree species and broadleaves. In 2014 parts of Frydenhaug were thinned
102 resulting in some areas with very low stand densities. Frydenhaug can be viewed as a very
103 small forest or as a large heterogeneous stand; in this work Frydenhaug is viewed as a single
104 heterogeneous stand.

105 As baseline data for comparison, trees on 54 sample plots on a systematic 20×40 m grid
106 were measured in September-October 2015 according to the field protocol of the Norwegian
107 National Forest Inventory for temporary plots (Landsskogtakseringen 2008). The circular
108 sample plots had a size of 250 m^2 (radius of 8.92 m) and all trees with $\text{DBH} \geq 5$ cm were
109 recorded (but note that for comparing with the MLS survey a 10 cm DBH threshold was
110 applied in the analyses). The measured variables included DBH measured with a calliper, tree
111 height measured using a Vertex hypsometer, and species. The frequency of DBH
112 measurements in the field plots is shown in Figure 1. The basal area weighted mean diameter
113 was found to be 40.2 cm and the stem density $357 \text{ trees ha}^{-1}$.

114 Figure 1 about here

115 2.1.2. *MLS data*

116 MLS data were acquired on 19 November 2014 with a Trimble MX2 system mounted on an
117 ATV. The system consists of two rotating Dynascan S250 scanning devices with a
118 wavelength of 905 nm, a scanning rate of 20 Hz, and a pulse rate of 36 kHz. The sensor's
119 location is determined by two global navigation satellite system (GNSS) antennas with 220
120 channels coupled with an inertial navigation unit (Applanix IMU-42). Images from a Ladybug
121 camera system consisting of five 6-megapixel cameras have been used to colorize the point
122 cloud that was derived using Trimble's generic Trident software.

123 In Figure 2 the location of the study area in Norway, the MLS track, identified tree locations
124 and the field sample plots are presented.

125 Figure 2 about here

126 **2.2.Method**

127 *2.2.1. Overview*

128 An overview of the procedure is provided in Figure 3. The procedure comprises four main
129 components. Firstly, an algorithm developed by Raunonen *et al.* (2015) was applied to
130 recognize individual trees from the MLS data and measure the number of stems within 10 ×
131 50 m sample plots; the width (10 m) was taken along the MLS track and the length (50 m)
132 follows from measuring 25 m on both sides of the MLS track (Figure 4).

133 Secondly, to overcome the problem that probability sampling was not applied (the ATV track
134 was chosen from the point of view of where the ATV could easily pass while measuring a fair
135 portion of the stand) we applied Kriging (*e.g.*, Cressie, 1990; Thompson, 1992) for predicting
136 stem number at stand level, i.e. model-based inference was applied.

137 Thirdly, to overcome the problem of some trees being non-detected, distance sampling theory
138 (Buckland *et al.*, 2005; Ducey & Astrup 2013) was applied to estimate a correction factor.
139 Lastly, Kriging predictions corrected for non-detection were averaged across the stand to
140 obtain a stand level prediction of stem density and the corresponding uncertainty. The details
141 of the four main components of the procedure are provided below.

142 Figure 3 about here

143 *2.2.2. Obtaining tree level and sample plot data from MLS data*

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144 Stem bases were automatically extracted from the scanned point cloud using the method
145 presented in Raumonon *et al.* (2015). In this method, the point cloud is partitioned into small
146 subsets and for a layer near the ground the surface normals of the subsets are estimated.
147 Potential stems are identified by selecting groups of subsets whose surface normals are nearly
148 horizontal, indicating a vertical surface. Expanding these groups with neighbouring subsets
149 completes the stems. Then, cylinders are fitted into sections around 1.3 m from the ground to
150 estimate DBH. In this way, the locations of trees and their corresponding DBHs were
151 estimated. The DBH frequency from the MLS measurements is presented in Figure 1. We
152 applied a DBH threshold of 10 cm in the study to focus on mid- and overstory trees, but also
153 to avoid partially unresolved problems of identifying small understory trees with the tree
154 segmentation algorithm. The number of trees identified with the tree segmentation algorithm
155 within the 10×50 m plots were counted and the corresponding stem density estimate was
156 assigned to the centre point of each plot (Figure 4). In case a plot was located close to the
157 stand boundary, only the 10×25 m subplot located entirely within the stand was used. Thus,
158 stem density values from the tree segmentation algorithm were assigned to 87 points along the
159 MLS track, at 10 m intervals.

160 Figure 4 about here

161 2.2.3. *Kriging prediction of stem densities*

162 One problem with ATV-based MLS inventories of forest stands is that it would be very
163 difficult to select probability samples and thus to apply design-based inference for estimating
164 target parameters, such as stem density and growing stock volume, without bias. The solution
165 we propose in this study is to allow selection of non-probability samples and use of model-
166 based inference (*e.g.*, Gregoire 1998; McRoberts, 2010; Magnussen, 2015; Ståhl *et al.*, 2016)
167 for predicting the target quantities. With model-based inference we rely on certain model-

168 assumptions. In this case our assumption, in general terms, is that values at non-sampled
 169 locations can be adequately predicted through spatial interpolation. Thus, the next step of our
 170 approach was to apply Kriging for predicting MLS-derived stem densities for each centre
 171 point in a dense (5×5 m) square lattice across the stand. Using Kriging, each cell in the lattice
 172 obtains a predicted value through spatial interpolation of the measurements so that the
 173 variance of the predictor is minimised. In our study, we applied ordinary Kriging (*e.g.*,
 174 Thompson 1992) based on a covariogram model estimated from the 87 observations of MLS-
 175 derived stem density along the ATV track. A negative exponential covariogram model,
 176 assuming isotropic conditions (*i.e.* that the spatial autocorrelation is the same in all
 177 directions), was applied, *i.e.*

$$178 \quad Cov_{y_{(t+d)}, y_t}(d) = \beta e^{\left(\frac{-|d|}{\alpha}\right)} \quad (1)$$

179 Where $Cov_{y_{(t+d)}, y_t}(d)$ is the covariance, as a function of distance, between two stem density
 180 values $y_{(t+d)}$ and y_t separated by the distance d ; α and β are model parameters. A simple
 181 covariance estimator was employed in this study

$$182 \quad Cov_{y_{(t+d)}, y_t}(d) = \frac{1}{n_d} \sum_{t=1}^{n_d} (y_{(t+d)} - \bar{y})(y_t - \bar{y})$$

183 where the summation is over the distinct pairs of stem density values from locations along the
 184 scanning track, that are distanced d apart from each other; and n_d is the number of such pairs.
 185 Then, the covariogram model (Eq. 1) was fitted using nonlinear least squares. The statistics of
 186 the fitted model are presented in Table 1.

187

Table 1 about here

188 The fitted model and the observed data are presented graphically in Figure 5. In the figure it
 189 can be seen that the covariance is close to zero at distances greater than 60 m. Using this

190 covariogram we applied Kriging to predict (MLS-derived) stem densities for each centre point
 191 in the square lattice. The Kriging predicted value was obtained as a weighted average across
 192 the $n_{MLS} = 87$ original observations as

$$193 \quad \hat{y}_t = \sum_{i=1}^{n_{MLS}} a_i y_i \quad (2)$$

194 where \hat{y}_t is the predicted stem density at location t , y_i is the predicted stem density value at
 195 the i^{th} location along the scanning track, and a_i is the corresponding weight. The weights
 196 $a_1, a_2, \dots, a_{n_{MLS}}$ are obtained as (cf. Cressie 1990)

$$197 \quad \mathbf{a} = \mathbf{G}^{-1} \left[\mathbf{h} + \mathbf{1} \left(\frac{\mathbf{1} - \mathbf{1}^T \mathbf{G}^{-1} \mathbf{h}}{\mathbf{1}^T \mathbf{G}^{-1} \mathbf{1}} \right) \right] \quad (3)$$

198 where the vectors \mathbf{a} and \mathbf{h} are

$$199 \quad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n_{MLS}} \end{pmatrix}, \quad \mathbf{h} = \begin{pmatrix} c_{1t} \\ c_{2t} \\ \vdots \\ c_{n_{MLS}t} \end{pmatrix} \quad (4)$$

200 with $c_{1t}, c_{2t}, \dots, c_{n_{MLS}t}$ being covariances between the t^{th} location and the different locations
 201 with observations along the scanning track. The covariance values are estimated using the
 202 negative exponential covariogram model (Eq. 1). The matrix \mathbf{G} is

$$203 \quad \mathbf{G} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n_{MLS}} \\ c_{21} & c_{22} & \dots & c_{2n_{MLS}} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n_{MLS}1} & c_{n_{MLS}2} & \dots & c_{n_{MLS}n_{MLS}} \end{pmatrix}_{n_{MLS} \times n_{MLS}} \quad (5)$$

204 each element of which is a covariance value between different locations along the track (the
 205 diagonal contains variances).

206 Figure 5 about here

207 2.2.4. Adjusting for non-detection

208 In a standard case of applying Kriging for predicting stem density at stand level an average
 209 over the lattice level predictions are computed and the corresponding uncertainty assessed
 210 (e.g., Cressie 1990; Thompson 1992). (Note that we speak of predicting rather than estimating

211 stem density at stand level, since in model-based inference this quantity is assumed to be a
 212 random variable rather than a fixed parameter.) However, in the MLS-based approach several
 213 trees remain non-detected within the rectangular sample plots mainly because they are at least
 214 partly hidden by other trees. Thus, there is a need to adjust for non-detection to obtain an
 215 unbiased predictor of stand level stem density (in the model-based sense). We applied
 216 distance sampling theory (Buckland *et al.*, 2005) to compute a correction factor, which was
 217 subsequently applied to all the MLS-derived values that were predicted according to the
 218 description in section 2.2.3. Distance sampling in this case is based on an assumption that the
 219 probability that an object of interest will be detected decreases with the distance from the
 220 survey line (or point) according to a detectability function. The detectability function is
 221 assumed to be well defined within certain conditions, such as those within a forest stand.
 222 Although ideally data should be acquired through probability sampling when distance
 223 sampling methods are applied (Buckland *et al.*, 2005), we used the MLS-derived data from
 224 the 87 rectangular sample plots along the ATV track to estimate a detectability function for
 225 our study stand. A half-normal model was applied, i.e. we used the model

$$226 \quad g(d) = e^{\left[-\frac{1}{2}\left(\frac{d}{\sigma}\right)^2\right]} \quad (6)$$

227 where $g(d)$ is the detection function, σ is a scale parameter, and d is the perpendicular
 228 distance from a detected tree to the ATV track. The maximum likelihood estimator (MLE) for
 229 the parameter σ is

$$230 \quad \hat{\sigma}^2 = \frac{1}{M} \sum_{i=1}^M d_i^2 \quad (7)$$

231 where M is the number of detected trees obtained by pooling the data from the 87 10 m line
 232 transects, and d_i is the perpendicular distance from the i^{th} detected tree to the corresponding
 233 line transect. In Figure 6, the observations and the fitted detection function is shown.

234 Figure 6 about here

235 Using the detection function, a correction factor to be multiplied with each MLS-derived stem
 236 density estimate is

$$237 \quad Q = \frac{w}{\mu} \quad (8)$$

238 where $w = 25$ m is the half-length of our MLS plots, and μ is the effective half-length, which
 239 is estimated as (e.g., Buckland *et al.*, 2005)

$$240 \quad \hat{\mu} = \sqrt{\hat{\sigma}^2 \frac{\pi}{2}} \quad (9)$$

241 *2.2.5. Prediction of stand level stem number*

242 As the final step the stand level stem number was predicted as the average predicted (non-
 243 adjusted) stem density across the $N = 1875$ points in the square lattice, multiplied by the
 244 factor Q and by the stand area, i.e.

$$245 \quad \hat{t}_{MLS} = \hat{Q} \frac{A}{N} \sum_{t=1}^N \sum_{i=1}^{n_{MLS}} a_{ti} y_i = \hat{Q} \frac{A}{N} \sum_{t=1}^N \hat{y}_t = A \hat{Q} \bar{\hat{y}} \quad (10)$$

246 where \hat{Q} is an estimated correction factor based on Eqs. (8) and (9), $A = 46963$ m² is the area
 247 of the stand in square metres, \hat{y}_t is the predicted stem density for the t^{th} point in the square
 248 lattice using Eq. (2), and $\bar{\hat{y}}$ is the average of predicted stem densities across the N points.

249 While the prediction of stem number is straightforward once the values for the lattice centre
 250 points are predicted using Kriging, and corrected for non-detection, the uncertainty of the
 251 prediction is more difficult to assess. The reason is that it involves several sources of
 252 uncertainty: uncertainty due to the Kriging prediction and due to adjusting for non-detection
 253 (estimation of the correction factor Q). In our derivation of the variance of the predictor of
 254 stem number we accounted for these two uncertainty sources, but not for the uncertainty
 255 related to the estimation of the covariogram model, partly since including the latter would
 256 lead to substantial difficulties, partly since it is common practice in Kriging applications to
 257 assume that this source of uncertainty is small (*e.g.*, Thompson 1992). Thus, the variance of
 258 \hat{t}_{MLS} is

$$V(\hat{t}_{MLS}) = V(A\hat{Q}\bar{y}) = A^2V(\hat{Q}\bar{y})$$

259 Using the formula for the variance of the product of two independent random variables (*e.g.*,
 260 Goodman 1960), we can develop the expression further

$$261 \quad V(\hat{t}_{MLS}) = A^2[\bar{y}^2V(\hat{Q}) + \hat{Q}^2V(\bar{y}) + V(\hat{Q})V(\bar{y})] \quad (11)$$

262 By replacing the variances of \hat{Q} and \bar{y} with their corresponding estimators we obtain an
 263 estimator of $V(\hat{t}_{MLS})$.

264 We first derive the variance estimator $\hat{V}(\bar{y})$. Recall that $\bar{y} = \frac{1}{N} \sum_{t=1}^N \sum_{i=1}^{n_{MLS}} a_{ti} y_i$; thus, the
 265 variance of \bar{y} can be written as

$$266 \quad V(\bar{y}) = \frac{1}{N^2} \sum_{k=1}^N \sum_{l=1}^N Cov \left(\sum_{i=1}^{n_{MLS}} a_{ki} y_i, \sum_{j=1}^{n_{MLS}} a_{lj} y_j \right) = \frac{1}{N^2} \sum_{k=1}^N \sum_{l=1}^N \sum_{i=1}^{n_{MLS}} \sum_{j=1}^{n_{MLS}} a_{ki} a_{lj} Cov(y_i, y_j) \quad (12)$$

267 The covariances $Cov(y_i, y_j)$ are estimated using the covariogram function based on distance;
 268 by replacing $Cov(y_i, y_j)$ with its estimator $\widehat{Cov}(y_i, y_j)$ in Eq. (12) we obtain the estimator
 269 $\widehat{V}(\widehat{y})$.

270 The variance of \widehat{Q} can be written as

$$271 \quad V(\widehat{Q}) = V\left(\frac{w}{\widehat{\rho}}\right) = w^2 V\left(\frac{1}{\widehat{\rho}}\right) \quad (13)$$

272 where the variance $V\left(\frac{1}{\widehat{\rho}}\right)$ depends on the number of detected trees M and is given by Quinn &
 273 Gallucci (1980, Eq. 13)

$$274 \quad V\left(\frac{1}{\widehat{\rho}}\right) = \left(\frac{2}{\pi}\right) B^2(M) V\left(\frac{1}{\widehat{\sigma}}\right) \quad (14)$$

275 where $B(M) = \sqrt{\frac{2}{M}} \Gamma\left(\frac{M}{2}\right) / \Gamma\left[\frac{(M-1)}{2}\right]$ is a bias correction factor for $\frac{1}{\widehat{\sigma}}$, and $\Gamma(\cdot)$ is the gamma
 276 function. The bias factor $B(M)$ is close to 1 for $M > 50$ (Quinn & Gallucci, 1980); hence, in
 277 our study we used the approximation $B(M) = 1$. The variance of $\frac{1}{\widehat{\sigma}}$ is given by Quinn &
 278 Gallucci (1980, Eq. 11)

$$279 \quad V\left(\frac{1}{\widehat{\sigma}}\right) = \frac{\left[\frac{M}{(M-2)} - \frac{1}{B^2(M)}\right]}{\sigma^2} \quad (15)$$

280 By replacing σ with the estimator $\widehat{\sigma}$ [Eq. (7)] we obtain an estimator for $V\left(\frac{1}{\widehat{\sigma}}\right)$; employing
 281 Eqs. (13) and (14) we obtain an approximation for $\widehat{V}(\widehat{Q})$:

$$282 \quad \widehat{V}(\widehat{Q}) \cong w^2 \left(\frac{2}{\pi}\right) \frac{\left[\frac{M}{(M-2)} - \frac{1}{\widehat{\sigma}^2}\right]}{\widehat{\sigma}^2} \quad (16)$$

283 Finally, an approximately unbiased variance estimator $\widehat{V}(\widehat{\tau}_{MLS})$ is obtained as:

$$\hat{V}(\hat{t}_{MLS}) \cong A^2 \left[\hat{y}^2 w^2 \left(\frac{2}{\pi} \right) \frac{\left[\frac{M}{(M-2)} - 1 \right]}{\hat{\sigma}^2} + \hat{Q}^2 \frac{1}{N^2} \sum_{k=1}^N \sum_{l=1}^N \sum_{i=1}^{n_{MLS}} \sum_{j=1}^{n_{MLS}} a_{ki} a_{lj} \widehat{Cov}(y_i, y_j) \right. \\ \left. + w^2 \left(\frac{2}{\pi} \right) \frac{\left[\frac{M}{(M-2)} - 1 \right]}{\hat{\sigma}^2} \frac{1}{N^2} \sum_{k=1}^N \sum_{l=1}^N \sum_{i=1}^{n_{MLS}} \sum_{j=1}^{n_{MLS}} a_{ki} a_{lj} \widehat{Cov}(y_i, y_j) \right] \quad (17)$$

284

(17)

285

2.2.6. Comparing with estimates from fixed area plots

286

The circular plots were used to make an independent estimate of stem number for purposes of comparison. We applied the fixed area sampling approach (e.g., Gregoire & Valentine, 2008, p. 215). The population total of the forest attribute parameter was estimated as (Gregoire & Valentine, 2008, Eq. 7.3a, p.216):

290

$$\hat{t}_{FD} = \frac{1}{n_{FD}} \sum_{i=1}^{n_{FD}} \hat{t}_{FDi} \quad (18)$$

291

Where $n_{FD} = 54$ field plots and $\hat{t}_{FDi} = \sum_{k \in S} \frac{y_k}{\pi_k}$, S denotes the sample of n_{FD} field plots, y_k is the attribute for the k^{th} tree, $\pi_k = \frac{a_k}{A}$ is the probability of including the k^{th} tree in the sample, and a_k is the inclusion zone of the k^{th} tree.

294

The variance was estimated as (Gregoire & Valentine, 2008, Eq. 7.6a, p. 216):

295

$$\hat{V}(\hat{t}_{FD}) = \frac{1}{n_{FD}(n_{FD}-1)} \sum_{i=1}^{n_{FD}} (\hat{t}_{FDi} - \hat{t}_{FD})^2 \quad (19)$$

296

2.2.7. Relative standard error and confidence interval

297

Relative standard errors (RSEs) were computed as

$$298 \quad RSE = 100\% \frac{\sqrt{\hat{V}(\hat{\tau})}}{\hat{\tau}} \quad (20)$$

299 The prediction interval (PI) in the case of stem number prediction based on MLS data was
300 estimated as

$$301 \quad \widehat{PI} = \hat{\tau} \pm z_{\alpha/2} \sqrt{\hat{V}(\hat{\tau}_{MLS})} \quad (21)$$

302 and the confidence interval (CI) in the case of stem number estimation based on field plot
303 measurements only was estimated as

$$304 \quad \widehat{CI} = \hat{\tau} \pm z_{\alpha/2} \sqrt{\hat{V}(\hat{\tau}_{FD})} \quad (21)$$

305 where $z_{\alpha/2}$ is the standard normal distribution's critical value, which equals 1.96 for $\alpha =$
306 0.05, i.e. for 95% confidence.

307 **3. Results**

308 The predicted number of stems (with DBH>10 cm) based on MLS data and the estimated
309 number of stems based of field data, their corresponding estimated RSEs, and the PI and the
310 CI, are presented in Table 2. It can be seen that the PI (based on MLS data) covers the stem
311 number estimate using field data and that the CI (based on field data) covers the stem number
312 prediction using MLS data.

313 Table 2 about here

314 In Table 3 the estimated detection model parameter $\hat{\delta}$, the estimated correction factor \hat{Q} , its
315 variance $\hat{V}(\hat{Q})$, the average predicted stem density \bar{y} , and its variance $\hat{V}(\bar{y})$ are presented. It

316 can be noted that the correction for non-detection is substantial, i.e. the stem density based on
317 trees detected with MLS data is increased with 54%.

318 Table 3 about here

319 Figure 7 shows a map of predicted stem densities for each 5×5 m grid-cell across the stand.

320 Figure 7 about here

321 In Figure 7 it can be observed how patches of denser and sparser stem densities were
322 allocated along the MLS track. Further, areas far from the MLS track obtained predicted
323 values equal to the mean stand density, since the spatial autocorrelation was zero at distances
324 greater than about 60 m.

325 4. Discussion

326 Airborne laser scanning has emerged as a very useful technique for forest inventories (*e.g.*,
327 Gregoire *et al.*, 2016) and there is much promise in using TLS for the same purpose (*e.g.*,
328 Raumonon *et al.*, 2013; Astrup *et al.*, 2014). However, several issues related to identifying
329 and measuring trees and setting up adequate sampling protocols remain to be solved before
330 the latter technique will be operational. In this study, we propose an approach for making use
331 of MLS data obtained from subjectively selected MLS tracks in a forest stand. We applied
332 Kriging (*e.g.*, Thompson 1992) to handle the issue that a non-probability sample was selected.
333 With MLS it is possible to select large samples from forest stands and provided that there is
334 significant spatial autocorrelation not all parts of a stand need to be visited. In this study we
335 found that the covariance between plot level stem densities remained non-zero up to about 60
336 m. Thus, in case measurements are obtained through traversing a stand in a manner that leaves
337 few parts further than 60 m away from the survey line, Kriging can compensate for the

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338 subjective sample selection and make the stand level predictions model-unbiased (*e.g.*,
339 Thompson 1992). A second issue addressed in this study was to compensate for non-detected
340 trees. To achieve this we followed an approach suggested by Ducey & Astrup (2013), *i.e.* we
341 used distance sampling theory (*e.g.*, Buckland *et al.*, 2005) to adjust for non-detection. By
342 multiplying the stand level average stem density estimated from MLS data with a correction
343 factor and multiplying it with the study area size we obtained a predicted value of stem
344 number at stand level and its corresponding variance. Assessing the error components
345 contributing to the total variance in the case study, we found that the variance due to Kriging
346 was the major component. We also made comparisons with an independent field sample plot
347 survey within the same stand and found the results to be quite similar. Studying the map of
348 predicted stem densities (Figure 7), patches of denser and sparser densities along the MLS
349 track tend to have smooth shapes due to the spatial interpolation. While sharp boundaries may
350 not be captured, we argue that this is less important when the main objective is to make
351 predictions at stand level. Further, the variance of the grid-cell level stem density predictions
352 was found to increase rather rapidly with distance from the MLS track.

353 The approach we suggest has many shortcomings, and it is easy to make a long list of topics
354 that need further attention. In the bullet list below we suggest some important topics to be
355 addressed in future research studies for improving the methodology.

- 356 • The probability of detecting trees most likely depends on the trees' DBH, and thus it
357 may be advisable to fit detection functions by DBH size groups, or to use DBH as a
358 covariate (*e.g.*, Ducey & Astrup 2013). However, in this study we focused on mid- and
359 overstory trees and used a common detection function for all trees larger than 10 cm.

360

- 361 • How large should the MLS plots be? Perhaps 10×50 m is too large, considering that
362 stands are often relatively small and that large plots will often cross stand boundaries.
363 Within the design-based inventory framework for conventional field plots, procedures
364 for handling plots near the boundary (and trees whose inclusion zones cross the
365 boundary) are well-developed (*e.g.*, Gregoire 1982; Ducey *et al.*, 2004). In general,
366 more work is needed for determining how boundary issues should be treated in MLS
367 surveys. In this study the width (10 m) was chosen to be rather narrow because this
368 made it possible to approximate the ATV track with a straight line.
- 369 • What detection function should ideally be used in MLS-based surveys? In this study,
370 we used the half-normal model, but we did not account for the right-truncation of the
371 detection function in the parameter estimation. Although the literature suggests that
372 this type of truncation has minimal effect on the estimates (*e.g.*, Feller 1971) it would
373 be advisable to study this issue further in case the half-normal model is used. Also,
374 since there cannot be any trees in the ATV track it might be appropriate to let the zero
375 distance in estimating the detection function be located some short distance away from
376 the ATV path. No such correction was employed in the current study, as can be
377 observed in Figure 6.
- 379 • More studies are needed for optimizing the MLS path, considering what spatial
380 autocorrelation can be expected. In our study we found that the distance beyond which
381 the covariance was close to zero was about 60 metres. Thus, the ATV should probably
382 have aimed at traversing the stand in a manner so that only few parts of the stand were
383 located more than 60 metres away from the track.
- 385 • The variance estimation procedures we propose are not entirely straightforward to
386 apply. An alternative could be to apply jackknifing or spatial bootstrapping (*e.g.*,
387

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388 Anselin, 1990; Tomczak, 1998; McRoberts *et al.*, 2011). During the study, we made a
389 simple initial test of jackknifing for the variance estimation and obtained a smaller
390 value compared to the variance estimate reported in Table 2. Also, methods to account
391 for the uncertainty due to the estimation of the covariogram model should be
392 developed.

393
394 • Additional auxiliary data, such as remotely sensed data from digital air photos (*e.g.*,
395 Bohlin *et al.*, 2012; Breidenbach & Astrup 2012; Puliti *et al.*, 2017) or satellites (*e.g.*,
396 Tomppo *et al.*, 2002; McRoberts *et al.*, 2011) could be incorporated into the prediction
397 procedures, in which case cokriging (*e.g.*, Myers 1983) would be applied.

398 Whereas studies such as those by Lin *et al.* (2012) and Forsman *et al.* (2016) focus on
399 obtaining accurate tree-level measurements from the MLS point cloud, the present study is a
400 first step towards utilising MLS data for obtaining stand level estimates once the problems
401 related to tree level measurements are solved. Such applications would potentially be very
402 valuable for obtaining accurate stand level data for forest management purposes. We focused
403 on stem number prediction; however, other tree-based attributes such as basal area, growing
404 stock volume and mean DBH would also be possible to predict from reliable tree level data
405 along the ATV track, perhaps building on the work by Astrup *et al.* (2014) for conventional
406 point-based TLS. Given the fast development of laser technology, we think that procedures
407 like the one we suggest in this study has great potential for providing accurate stand level
408 information at low cost in the future.

409 5. Conclusions

410 MLS coupled with adequate sampling protocols and computational algorithms has a potential
411 to become an efficient tool for stand level forest inventories. This would not require any

412 traditional field measurements at all, or MLS might be operated in tandem with field
413 measurements when those are sparse. However, further studies addressing many remaining
414 uncertainties are needed before MLS-based stand level surveys could become operational.

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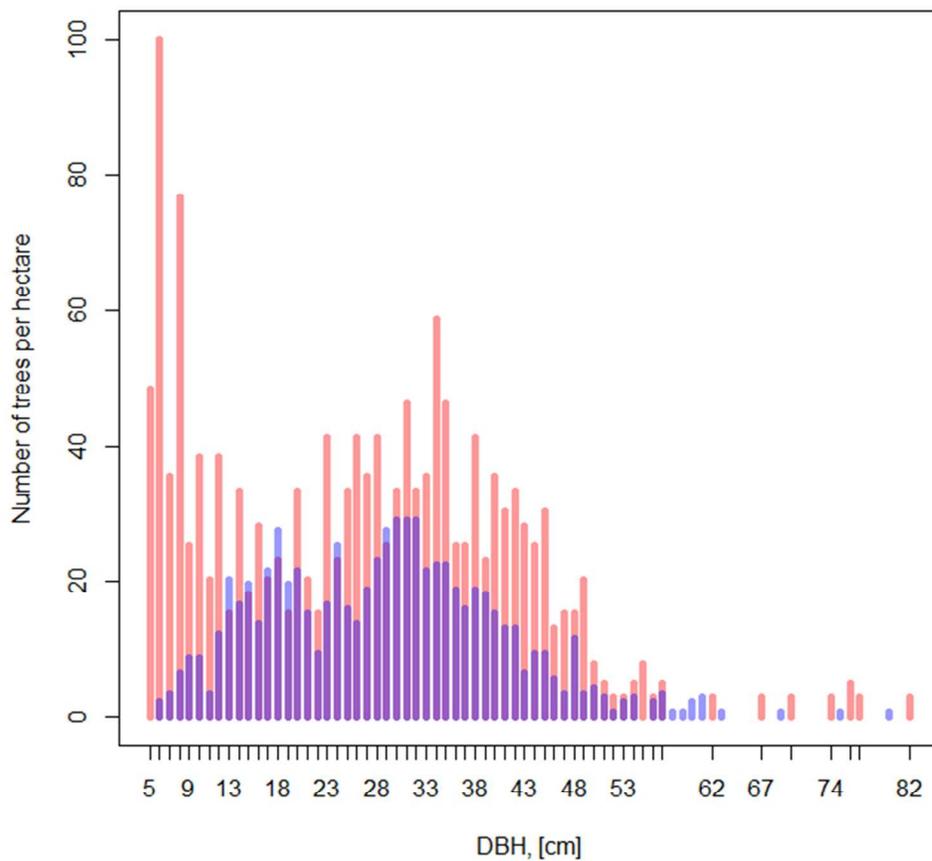


Figure 1: The DBH frequency for trees measured in the field sample plots is indicated with red colour, the DBH frequency for trees identified from the MLS point cloud using tree extraction is indicated with blue colour, the violet colour shows the intersection of these frequencies. The number of trees are shown per hectare.

237x237mm (72 x 72 DPI)

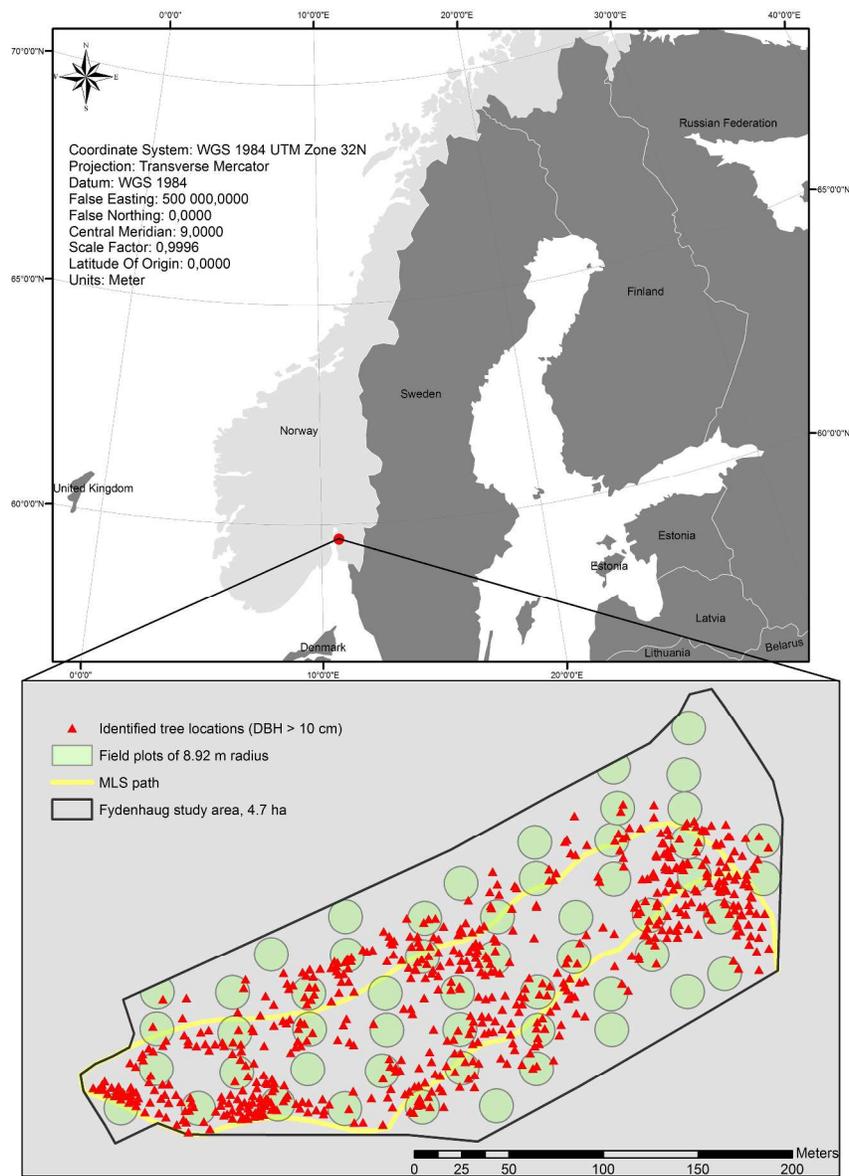


Figure 2: Frydenhaug study area and its location within Norway. Note: the tree locations are identified by the tree detection algorithm using MLS survey data.

296x419mm (300 x 300 DPI)

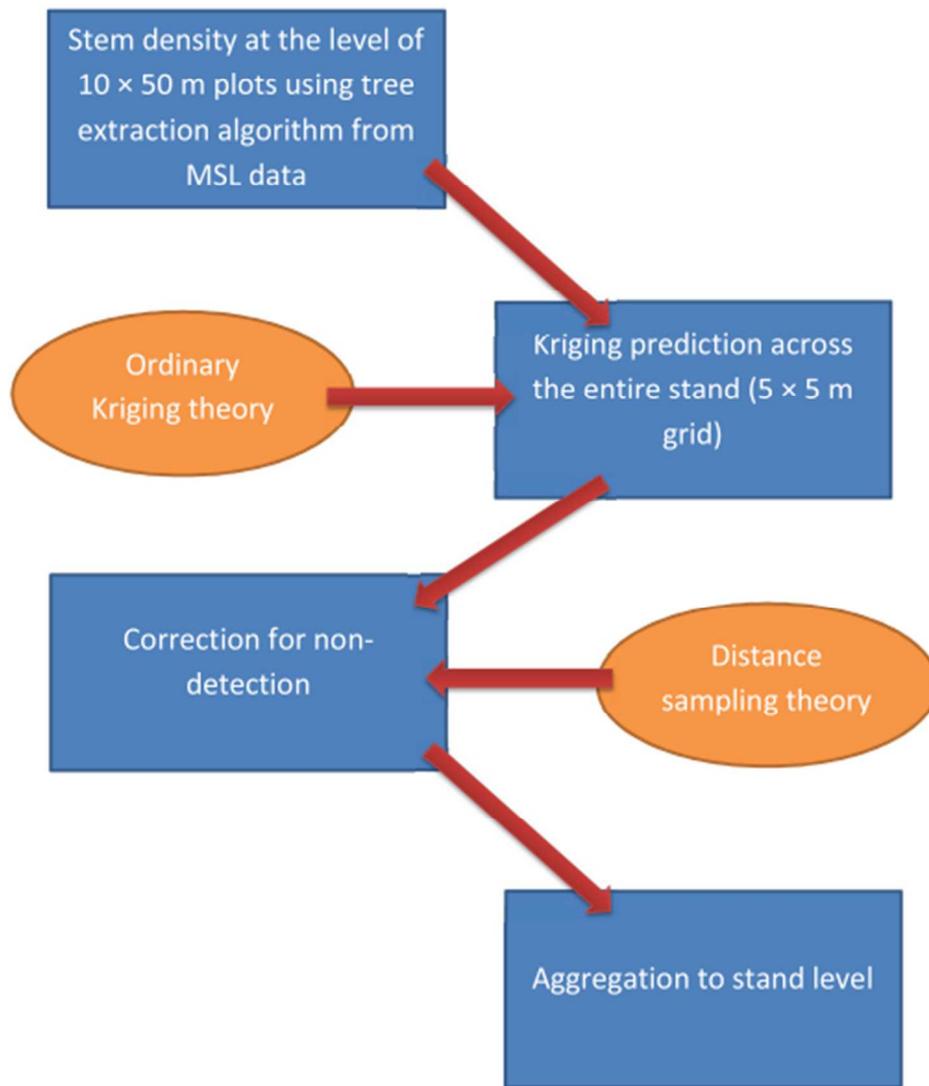


Figure 3: An overview of the procedure to predict stem number at stand level.

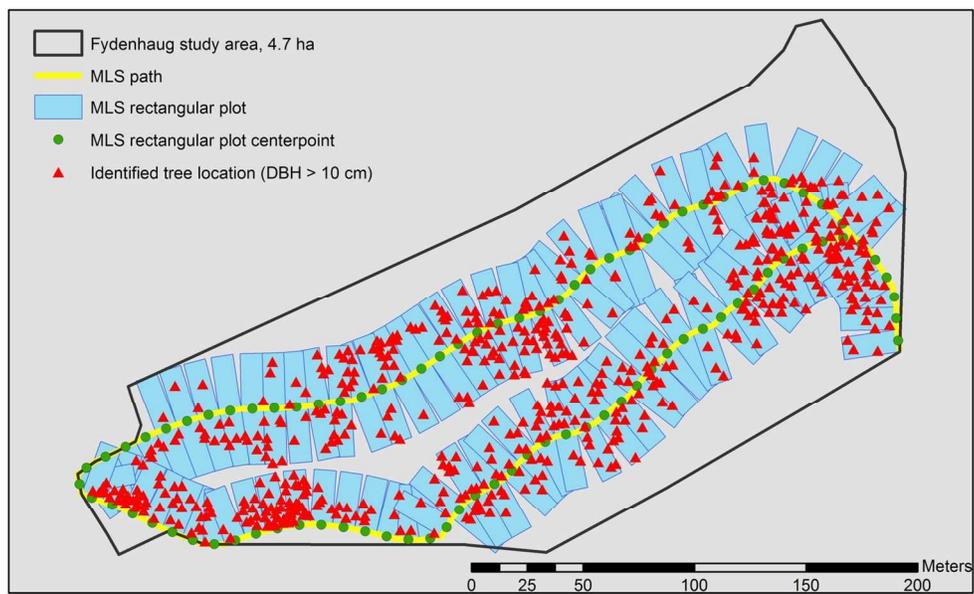


Figure 4: Spatial distribution of the rectangular plots every 10 m along the MLS track. Note: the tree locations are identified by the tree detection algorithm using MLS survey data.

133x84mm (300 x 300 DPI)

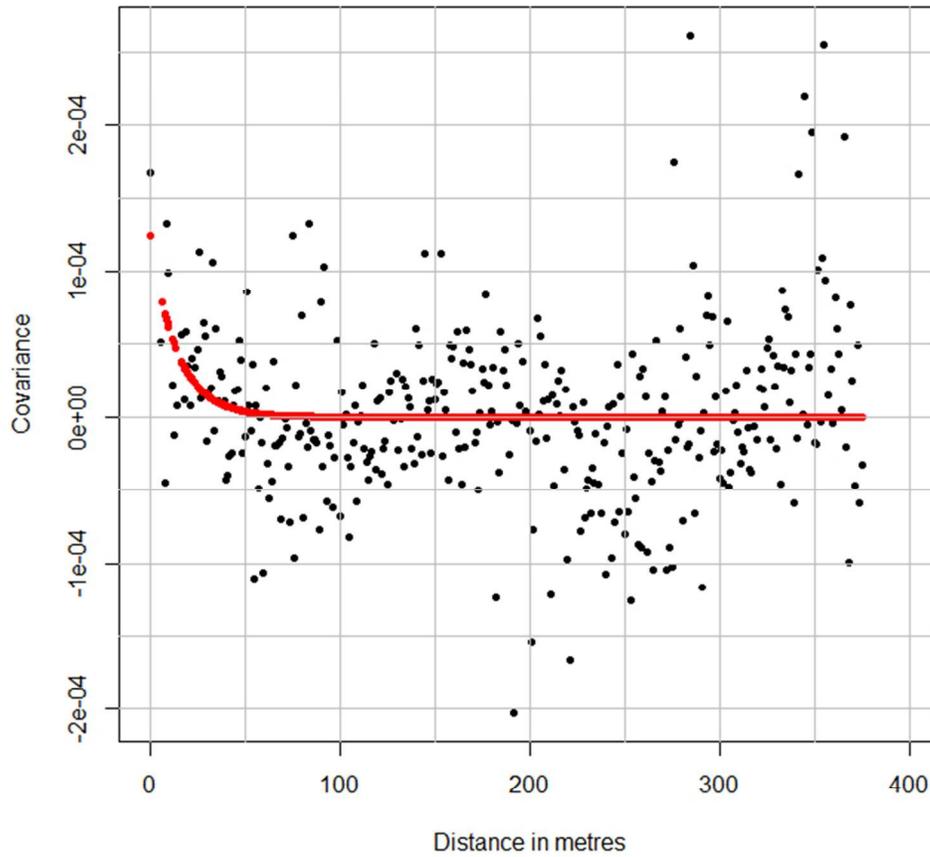


Figure 5: Empirical covariance values (black dots) and the fitted negative exponential covariogram model (red curve) as a function of distance for trees of DBH > 10 cm.

237x237mm (72 x 72 DPI)

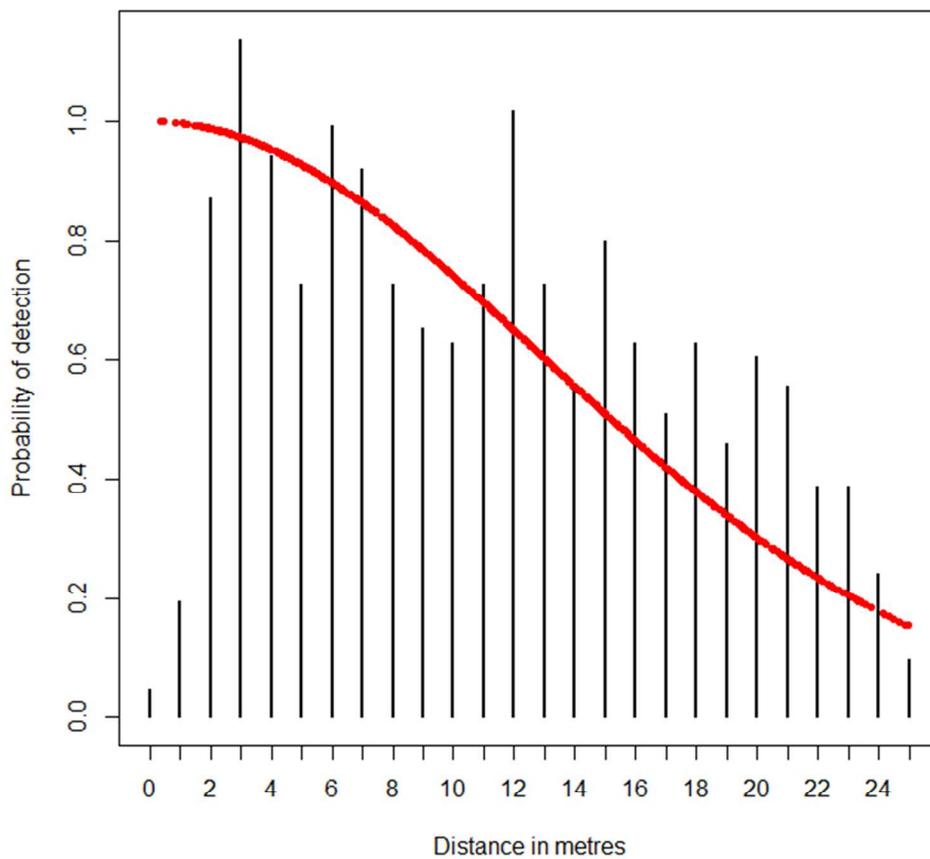


Figure 6: Empirical detection probability values (black bars) and the fitted detection function (red curve).

237x237mm (72 x 72 DPI)

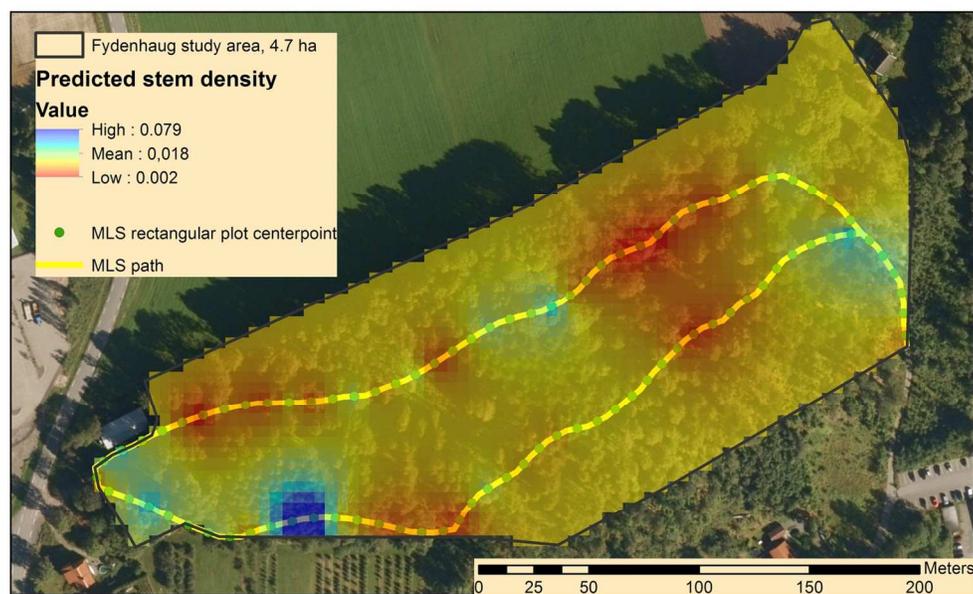


Figure 7: Stem density predictions across Frydenhaug study area.

133x84mm (300 x 300 DPI)

Table 1: The fitted negative exponential covariogram model's characteristics for trees with DBH > 10 cm.

Model parameter	Estimate	Standard Error	<i>t</i> -value	<i>p</i> -value
α	14.22	5.68	2.57	0.012
β	1.24×10^{-4}	4.49×10^{-5}	2.50	0.0060
Residual standard error: 5.72×10^{-5}				

Table 2: The estimated total number of trees, and the corresponding estimated RSE, PI values based on MLS and CI values based on field data.

Based on	\hat{t} [trees]	RSE [%]	PI/CI_{lower} [trees]	PI/CI_{upper} [trees]
MLS data only	1366	12.86	1020	1712
Field data only	1677	7.46	1431	1923

Table 3: The estimated detection function parameter, the correction factor and its variance, and the estimated average of predicted stem density across $N = 1875$ points and its variance.

Kriging prediction of stem densities			Adjusting for non-detection	
\bar{y}	$\hat{V}(\bar{y})$	$\hat{\sigma}$	\hat{Q}	$\hat{V}(\hat{Q})$
1.88×10^{-2}	4.77×10^{-6}	12.90	1.54	7.18×10^{-3}