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Optimizing height measurement for the long-term forest experiments in Sweden

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ABSTRACT

Information on tree height is useful for volume estimation and site productivity assessment and as such, remains one of the most important variables often measured in forest inventories. Measuring a sufficient number of sample trees requires considerable sampling effort and cost. In this study, we developed height functions for optimizing tree height measurement in the Swedish long-term forest experiments (LTFEs). Two large datasets from the LTFE databases: fitting data (from thinning, fertilisation and mixed species experiments) and validation data (tree species and spacing experiments) collected over several decades were used. The fitting and validation data comprise 133,788 and 68,440 observations, respectively, each covering a large range of growth and environmental conditions across Sweden. A multilevel nonlinear mixed-effects modelling approach was used to build the generalised height functions for Scots pine, Norway spruce, birch (Silver and Downy birch united), other conifers and other broadleaves, considering variations in heights and other stand characteristics at sample plot-level and revision-level. The response calibration of the functions was first carried out with all measured heights of the validation data, and second, using heights of one to six sample trees obtained from different tree selection strategies (diameter extremes, largest diameters, and smallest diameters). The mixed-effects height functions explained most of the height variations in the fitting dataset (pseudo R^2: 0.938 – 0.970; RMSE: 0.957 – 1.363 m) without any residual trends. The validation showed that the functions accounted for 95 – 98 % of the height variation in the validation dataset, with RMSE ranging between 0.770 and 1.040 m, confirming the functions' high accuracy. We recommend the measurement of four sample tree heights based on diameter extremes as the ideal threshold for response calibration. These functions and the suggested sampling technique would reduce sampling effort and inventory cost of height measurements for subsequent inventories of the LTFEs.

1. Introduction

Tree height and diameter at breast height (DBH) are important quantities in forest inventories and timber management, as they are necessary for estimating stand volume and biomass. Tree height and DBH are used as input variables in numerous forest models, such as growth and yield models, site productivity models, crown models, biomass models, and carbon budget models (Sharma et al., 2019; West, 2015). These models serve as not only important decision-making tools in forestry but are also used for assessments of forest ecosystem productivity. Tree height is also used together with diameter to determine the stability of trees in a stand, i.e., the ratio of height to diameter at breast height (Zhang et al., 2020). Compared to the measurement of DBH, measurement of tree height is difficult, time-consuming and expensive, especially in dense stands and rough or steep terrain (Ciceu et al., 2020; Magnussen et al., 2020; Özçelik et al., 2018). In this situation, only the heights of a subsample of trees per sample plot are often measured. The heights of the remaining trees of the species of interest are precisely estimated using height functions established from an adequate number of height-diameter observations (Fortin et al., 2019).

The number of height sample trees measured could vary in different countries and types of inventory systems but have all in common to be relatively time-consuming demanding a high proportion of height measurements per tree species (Liziniewicz et al., 2016). In general, the

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methods rely on nonlinear regressions of height to diameter relations for the sample trees (Holmström et al., 2018; Mensah et al., 2022; Nilsson et al., 2010; Persson et al., 2022). The same procedure then has to be repeated on every assessment occasion (referred to here as revisions), creating new height to diameter relations for the stand or plot. This approach seems reasonable but there could be some potential challenges associated with it. Firstly, taking height measurements of e.g. 20 trees per sample plot would require a considerable amount of time and cost; and even more for mixed species plots, where each species requires 20 sample trees (Crecente-Campo et al., 2014). Secondly, since an ordinarily least squares technique is used to obtain estimates for the coefficients of the function, any attempt at reducing the number of subsample trees in the sample plot may lead to unreliable estimates of the function's coefficients (Arcangeli et al., 2014; van Laar and Akça, 2007); thereby, assigning poorly predicted height to the calipered trees with a ripple effect on the other variables derived from tree height.

Rather than fitting a sample plot-wise height-diameter function on every assessment for the individual species, generalised height functions could be developed. A generalised height function in addition to the measured DBH incorporates other stand descriptors as covariates to account for height variations caused by differing stand characteristics (Castedo-Dorado et al., 2006; Krisnawati et al., 2010). Due to the hierarchical and/or temporal structure of height-diameter data, most generalised height functions have been developed based on the mixed-effects modelling approach (e.g., Bronisz and Mehtätalo, 2020a; Ciceu et al., 2020; Patrício et al., 2022; Raptis et al., 2021; Sharma et al., 2019). A mixed-effects height function has both fixed and random effect components, and as such provides flexible and improved predictions of tree height (Sharma et al., 2016). Besides flexibility and improved precision, mixed-effects height functions offer a possible way of optimizing tree height measurements; such that all heights of the calipered trees could be accurately estimated with only a few sample tree heights measured in each experimental plot. There are no bounds to the number of sample tree heights required to calibrate mixed-effects functions (Mehtätalo and Lappi, 2020). The number of sample trees could be as few as one (Trincado et al., 2007) and yet still provide predictions with sufficient precision. In addition to heights, some studies have also used this modelling strategy for different forestry problems, such as tree biomass estimation (Bronisz and Mehtätalo, 2020b; Colmanetti et al., 2020), diameter growth (Bohora and Cao, 2014; Bueno-López and Bevilacqua, 2013), and stem taper (Arias-Rodil et al., 2015; Li and Weiskittel, 2010). There are no height functions established yet for the Swedish long-term forest experiments (LTFEs), but it would be worthwhile to adopt such a modelling approach for estimating tree height in them.

In Sweden, LTFEs have been established across the country since the beginning of the 20th century and new and improved experiments are still being established. Notable among the LTFEs are the thinning & fertilisation, mixed species, spacing and tree species experiments. Since their establishment, the LTFEs have been measured continuously and in a consistent way (Karlsson et al., 2012); and as such has a database with data spanning several decades. LTFE data has been the basis for several forest research studies in Sweden (e.g., Elfving, 2010; Fahlvik et al., 2014; Holmström et al., 2018; Liziniewicz et al., 2016; Mensah et al., 2022; Nilsson et al., 2010; Valinger et al., 1994). Suitable height functions that provide a precise estimate of tree heights also with a reduced number of height measurements would make it possible to assess the LTFEs at a lower cost.

Therefore, the main purpose of this study was to develop multilevel nonlinear mixed-effects generalised height functions that would be used for optimizing tree height measurement in the Swedish LTFEs. In addition, the aim was to quantify the number of necessary height measurements and to define a sample tree selection strategy based on tree diameter. A multilevel mixed-effect approach was adopted so that the functions could adequately explain the variances (sample plot and revision effects) in the LTFE data. The proposed height functions could be used for the characterisation of vertical stand structures, estimations of stand volume and biomass, and simulations of stand dynamics and stand productivity, and thus worthwhile for informed decision-making in forestry at a lower cost.

2. Materials and methods

2.1. Data

The data for this study were obtained from the LTFE database. The experiments are distributed all over Sweden (Fig. 1) and they include thinning & fertilisation, spacing, mixed species and tree species experiments. The thinning (*"Gallring" in Swedish*) & fertilisation (*"Gödsling" in Swedish*) experiments are commonly called the GG experiments on Scots pine (*Pinus sylvestris* L.) and Norway spruce (*Picea abies* [L.] H. Karst.) (Fahlvik et al., 2014). For details on these experiments, see Nilsson et al. (2010) and Valinger et al. (1994). Besides Scots pine and Norway spruce, the LTFE database also contains Silver birch (*Betula pendula* Roth) and Downy birch (*Betula pubescens* Ehrh.) in large numbers. For this study, we did not separate Silver birch and Downy birch, which is hereon referred to as "birch". Silver birch and Downy birch are rarely separated in the Swedish forestry practice or in the national forest inventory because the species have similar phenology and traits (Holmström et al., 2017). Other species in the LTFE database include Pedunculate oak (*Quercus robur* L.), Grey alder (*Alnus glutinosa* [L.] Gaertn), Beech (*Fagus sylvatica* L.), Douglas fir (*Pseudotsuga menziesii* [Mirb.] Franco), European larch (*Larix decidua* Mill.), Siberian larch (*Larix sibirica* Ledeb.), Lodgepole pine (*Pinus contorta v. latifolia* Douglas), and so on (see Appendix Table A1 for the full list). However, too few individuals of each species were available in the LTFE database and therefore, we classified them as "other conifers" and "other broadleaves". Scots pine,

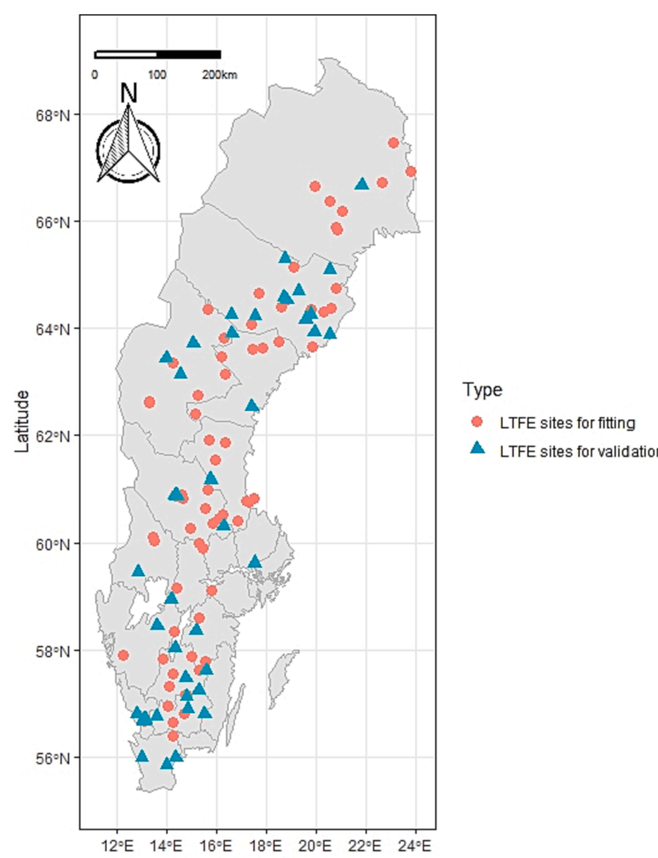

Fig. 1. Location of the LTFE sites used for fitting and validation of the height functions.

Norway spruce and birch are the totally dominating tree species in the Swedish forest and are therefore referred to as the "main species" in this study.

The thinning & fertilisation and mixed species experiments were used as the fitting dataset and included data from 649 sample plots in 85 experimental sites along a latitudinal gradient of 56.39 – 67.48° N. The plot sizes and assessment years had a range of 0.0025 – 0.2517 ha and 1966 – 2020, respectively. The fitting dataset included 133,788 observations, of which 75 % were Scots pine, 12 % Norway spruce and 10 % birch. The remaining 3 % were other conifers and broadleaves.

The validation dataset was based on the spacing and tree species trial experiments, where data was collected from 632 sample plots in 66 experimental sites along a latitudinal gradient of 55.85 – 66.67° N. The plot sizes and measurement years had a range of 0.0072 – 0.6000 ha and 1968 – 2022, respectively. The validation dataset included 68,440 observations, of which 43, 32 and 6 % were Scots pine, Norway spruce and birch, respectively. Altogether, 19 % of the observations were made on other conifers and broadleaves. We combined the fitting and validation data sets to develop specific height functions for other conifers and other broadleaves, respectively. It should be noted that the data from oak were not included in the data used to build the functions for other broadleaves because the species showed height trends that are completely different from others.

The LTFE database, among other things, contains information on the year of revision and stand age at revision. The LTFE data system provides plot-wise calculations of quadratic mean diameter, basal area per ha, trees per ha, etc. based on the diameters and tree heights stored in the database. We also derived other variables, such as the ratio of diameter to quadratic mean diameter and the height of the tree with the largest diameter regardless of species per plot from the database. For inclusion, the interval between successive revisions had to be at least four years. Summary statistics of the fitting and validation datasets used for this study are presented in Table 1. Scatterplots of diameter at breast height and tree height of the fitting and validation (only for the main species) datasets are shown in Fig. 2.

2.2. Model development

Many functions have been used to describe the single-tree height-diameter relationship in forestry for different regions. However, the Näslund function (Näslund, 1936) (equation [1]) is the most frequently used to describe the height-diameter relationship in northern Europe and under Scandinavian conditions (Holmström et al., 2018; Lidman et al., 2021; Liziniewicz et al., 2016; Mehtätalo et al., 2015; Mensah et al., 2022; Persson et al., 2022; Sharma and Breidenbach, 2015). Besides, our preliminary analysis showed that the Näslund function works adequately well for the LTFE data (see Appendix Table A2).

$$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + \beta_1 d_{itj})}\right]^x + \varepsilon_{itj} \quad (1)$$

where h_{itj} is the expected height (m) at a given diameter at breast height d (cm) of tree j at revision t on sample plot i, β_0 and β_1 are model parameters, x is an exponent and ε_{itj} is the error term which approximates a normal distribution with a mean of zero and constant variance.

To develop optimal height functions for the LTFEs, the Näslund function was fitted to each sample plot per revision for Scots pine, Norway spruce, birch, other conifers and other broadleaves at varying exponents (x: 2 – 8) (see Appendix Fig. A1) with ordinary nonlinear least squares. Though not many differences were observed based on the residual standard error for the different exponents, setting $x = 2$ for Scots pine and birch, and $x = 3$ for Norway spruce, other conifers and other broadleaves were the preferable option for the LTFE data. Similar exponent values for this function were recommended for Scots pine, birch and Norway spruce in Sweden by Petterson (1955).

Table 1
Summary of the LTFE data used for fitting and validation of the height functions.

Variables	Fitting data			Validation data		
	Min	Mean	Max	Min	Mean	Max
Number of sample plots	649	–	–	632	–	–
Range of measurement years	1966	–	2020	1968	–	2022
Number of revisions	1		8	1		7
Number of observations	133,788	–	–	68,440	–	–
Diameter at breast height (d, cm)	4.0	15.6	74.2	4.0	14.5	65.7
Tree height (h, m)	1.6	13.8	37.6	2.1	12.7	35.9
Largest diameter regardless of the species in the case of a mixed stand (*ddom*, cm)	4.4	24.6	74.2	4.0	21.8	65.7
Height of the tree with the largest diameter regardless of the species in the case of a mixed stand (*hdom*, m)	1.9	16.7	35.5	2.8	14.9	35.9
Quadratic mean diameter (*QMD*, cm)	1.4	15.3	41.9	0.9	14.5	39.0
Basal area per hectare (G, m^2 ha^{-1})	0.3	19.3	52.8	0.1	20.3	63.0
Stand density (trees ha^{-1})	30.0	1,469	20,100	29.0	1,559	22,978
Ratio of individual tree diameter to QMD (C)	0.11	1.06	8.11	0.11	1.07	22.24
Latitude (°N)	56.39	–	67.48	55.85	–	66.67
Altitude (m)	10.0	211.1	560.0	10.0	182.4	400.0

2.2.1. The generalised mixed-effects height function

The tree height-diameter relationship can be influenced by other tree and stand characteristics, such as site quality, stand density or competition, tree social status, stand development stage, etc. (Fortin et al., 2019; Sharma et al., 2019). Thus, modelling tree height as a function of diameter (d) alone is not suitable for different stand dynamics and silvicultural conditions (Corral Rivas et al., 2019; Krisnawati et al., 2010). For this reason, equation (1) was expanded by inclusion of easily obtained stand characteristics as covariates. Scatterplots were used to examine the relationships between the estimated parameters (obtained for each sample plot and revision) and some stand characteristics (Table 1). The height (*hdom*) of the tree with the largest diameter (*ddom*) (regardless of the species in the case of a mixed stand), total basal area per ha (G, m^2 ha^{-1}) of all living trees, quadratic mean diameter (*QMD*, cm) and stand density per sample plot and revision were considered. A ratio-based index describing the social status of trees in the stand (diameter/QMD, C) was also evaluated. The most correlated variables were used to build the generalised height functions (GHFs).

The parameter β_1 of equation [1] correlated better with the stand variables than β_0; and as such, the GHFs were based on the expansion of parameter β_1. Scatterplots of the association between parameter β_1 and the selected stand variables are presented in Appendix Fig. A2. The best fitting improvement for the LTFE data was obtained when β_1 was modelled as a function of *hdom*, G and C for Scots pine, Norway spruce and birch (equation [2a]), and for other conifers and other broadleaves, relating β_1 with *hdom* and *ddom* performed best (equation [2b]).

$$\beta_1 = f(hdom, G, C) \quad (2a)$$

$$\beta_1 = f(hdom, ddom) \quad (2b)$$

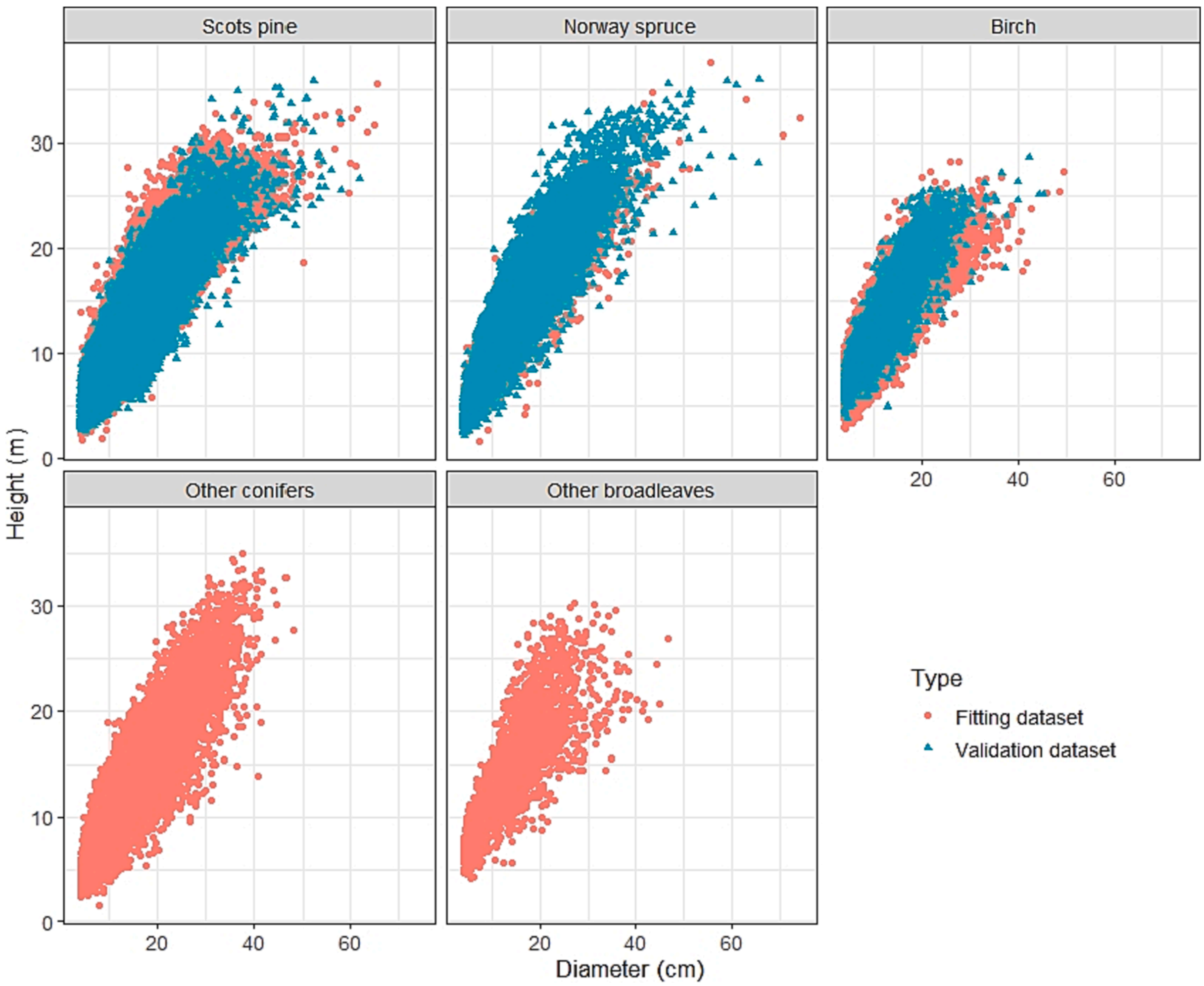

Fig. 2. Scatterplots of tree height vs diameter at breast height of the fitting and validation (only for main species) datasets.

Table 2
Generalised nonlinear mixed-effects height function forms.

Species	Form	Eq.
Scots pine	$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + u_{i1} + v_{it1}) + \left(\beta_1 hdom_{it}^{(\beta_2 + u_{i2} + v_{it2})} + \beta_3 G_{it} + \beta_4 C_{itj}\right) d_{itj}}\right]^2 + \varepsilon_{itj}$	[3]
Norway spruce	$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + u_{i1} + v_{it1}) + \left(\beta_1 \sqrt{hdom_{it}}^{\beta_2} + \beta_3 G_{it} + (\beta_4 + u_{i2} + v_{it2}) C_{itj}\right) d_{itj}}\right]^3 + \varepsilon_{itj}$	[4]
Birch	$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + u_{i1} + v_{it1}) + \left(\beta_1 \sqrt{hdom_{it}}^{(\beta_2 + u_{i2} + v_{it2})} + \beta_3 G_{it} + \beta_4 C_{itj}\right) d_{itj}}\right]^2 + \varepsilon_{itj}$	[5]
Other conifers	$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + u_{i1} + v_{it1}) + \left((\beta_1 + u_{i2} + v_{it2}) \sqrt{hdom_{it}}^{\beta_2} + \beta_3 \sqrt{ddom_{it}}\right) d_{itj}}\right]^3 + \varepsilon_{itj}$	[6]
Other broadleaves	$h_{itj} = 1.3 + \left[\frac{d_{itj}}{(\beta_0 + u_{i1} + v_{it1}) + \left(\beta_1 \sqrt{hdom_{it}}^{\beta_2} + (\beta_3 + u_{i2} + v_{it2}) \sqrt{ddom_{it}}\right) d_{itj}}\right]^3 + \varepsilon_{itj}$	[7]

where $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ are fixed-effect parameters of the functions; u_{i1}, u_{i2} are sample plot-level random effect parameters; v_{it1}, v_{it2} are revision-level random effect parameters; h_{itj} is expected height (m) at a given diameter d (cm) of tree j at revision t on sample plot i; $hdom_{it}$ is the height of the tree with the largest diameter $ddom_{it}$ on sample plot i at revision t, regardless of tree species; G_{it} is the total basal area of living trees (m^2/ha) of sample plot i at revision t; C_{itj} is the ratio of tree diameter j to the quadratic mean diameter of sample plot i at revision t; ε_{itj} is the error term $\left[\varepsilon_{itj}\ N(0, \sigma^2)\right]$; $i = 1, 2, \ldots, M$; $t = 1, 2, \ldots, M_i$; $j = 1, 2, \ldots, n_{it}$; where M is the number of sample plots; M_i is the total number of revisions of the i^{th} sample plot and n_{it} is the number of observations at the revision t of the i^{th} sample plot.

where *hdom* represents the height of the tree with the largest diameter *ddom*, G is basal area per ha, and *C* is the ratio of diameter at breast height to quadratic mean diameter. The fitted generalised functions with ordinary least squares method are presented in Appendix Table A2.

The LTFE database contains multiple sample plots with multiple revisions; following Mehtätalo (2004), multilevel nonlinear mixed-effects GHF was formulated for each tree species using equation [1] (with equations (2a), 2b included) by the inclusion of the sample plot and revision random effects. Fifty-seven mixed-effects model alternatives were formed with different combinations of the fixed parameters in equation [1] (with equations (2a), 2b included) and random effects, were fitted to the fitting dataset (see Appendix Table A3). However, the decision for the optimal combination of fixed parameters and random effects was based on the ease of achieving convergence and the smallest Akaike information criterion (AIC). It was ensured that the random effects structure of the selected height function form was not over-parameterized. The final generalised mixed-effects functional forms are presented in Table 2.

The plot-level random effects (u_{i1}, u_{i2}) i.e., $\boldsymbol{b}_i$ and random effect revision-level (v_{it1}, v_{it2}) i.e., $\boldsymbol{b}_{it}$ in the generalized mixed-effect height function were assumed to follow a multivariate normal distribution with mean equal zero and positive-definite variance–covariance matrices $\boldsymbol{D}_{plot}$ and $\boldsymbol{D}_{revision}$ often expressed as $\boldsymbol{b}_i \sim N(0, \boldsymbol{D}_{plot})$ and $\boldsymbol{b}_{it} \sim N(0, \boldsymbol{D}_{revision})$, respectively. Where:

$$\boldsymbol{D}_{plot} = \begin{pmatrix} \sigma_{u_1}^2 & cov(u_1, u_2) \\ cov(u_1, u_2) & \sigma_{u_2}^2 \end{pmatrix} \tag{8}$$

and

$$\boldsymbol{D}_{revision} = \begin{pmatrix} \sigma_{v_1}^2 & cov(v_1, v_2) \\ cov(v_1, v_2) & \sigma_{v_2}^2 \end{pmatrix}, \tag{9}$$

where $\sigma_{u_1}^2, \sigma_{u_2}^2, \sigma_{v_1}^2, \sigma_{v_2}^2$ are the variances of the random effect parameters $u_{i1}, u_{i2}, v_{it1}, v_{it2}$, respectively for sample plot-level and revision-level; *cov* represents their covariance i.e., correlation times standard deviations. The covariance of the plot-level random effect is given by: $cov(u_1, u_2) = corr(u_1, u_2) \times \sigma_{u_1} \times \sigma_{u_2}$, and of the revision-level as $cov(v_1, v_2) = corr(v_1, v_2) \times \sigma_{v_1} \times \sigma_{v_2}$.

The generalised mixed-effects height functions were fitted with the method of restricted maximum likelihood using the '*nlme*' package (Pinheiro et al., 2021) implemented in R (R Core Team, 2021). Heteroscedasticity is a common problem in modelling height-diameter relationships. In this study, heteroscedasticity in the residual variance, i.e. $var(\varepsilon_{itj}) = \sigma^2$ was stabilized with the power variance-stabilizing function (equation [10]):

$$var(\varepsilon_{itj}) = \sigma^2 d_{itj}^{\;2\delta} \tag{10}$$

where σ and δ are scale and shape parameters to be estimated, respectively; d_{itj} is the diameter of the j^{th} tree on the i^{th} sample plot at the t^{th} revision.

Furthermore, the possibility of using one of the function forms in Table 2 as a general-purpose function was evaluated. To do this, each function was fitted to the respective species. For example, equation [3] for Scots pine was fitted to Norway spruce, birch, other conifers and other broadleaves. The same was applied to the other function forms (equations [3] to [7]). The results were compared with the optimal function for each species in Table 2. Thereafter, the function selected as the general height function was further expanded to include species as additional covariates through a dummy variable modelling approach to check whether there would be a possibility of having a single height function for many species or at least for some tree species of interest.

2.3. Model assessment

We used some versatile fit statistics, such as root mean square error (RMSE, m; equation [11]), mean absolute bias (MAB, m; equation [12]), mean absolute percentage error (MAPE, %; equation [13]), and pseudo coefficient of determination (pseudo R^2; equation [14]) to assess the performance of the generalised mixed-effects height functions. The smaller the RMSE, MAB, MAPE and larger pseudo R^2, the better height functions.

$$RMSE = \sqrt{\frac{1}{(n-1)}\left(\sum_{i=1}^{m}\sum_{t=1}^{n_i}\sum_{j=1}^{n_{it}}\left(h_{itj} - \widehat{h}_{itj}\right)^2\right)} \tag{11}$$

$$MAB = \left(\frac{1}{n}\left(\sum_{i=1}^{m}\sum_{t=1}^{n_i}\sum_{j=1}^{n_{it}}\left|h_{itj} - \widehat{h}_{itj}\right|\right)\right) x\,100 \tag{12}$$

$$MAPE = \left(\frac{1}{n}\left(\sum_{i=1}^{m}\sum_{t=1}^{n_i}\sum_{j=1}^{n_{it}}\frac{\left|h_{itj} - \widehat{h}_{itj}\right|}{h_{itj}}\right)\right) x\,100 \tag{13}$$

$$pseudo\,R^2 = 1 - \left(\left(\sum_{i=1}^{m}\sum_{t=1}^{n_i}\sum_{j=1}^{n_{it}}\left(h_{itj} - \widehat{h}_{itj}\right)^2\right) \Big/ \sum_{i=1}^{m}\sum_{t=1}^{n_i} \times \sum_{j=1}^{n_{it}}\left(h_{itj} - \bar{h}\right)^2\right) \tag{14}$$

where n is the number of observations; m represents the number of model parameters; h_{itj} and $\widehat{h}_{itj}$ represent observed and predicted heights, respectively, of the j^{th} tree on the i^{th} sample plot at the t^{th} revision; and $\bar{h}$ is the mean height.

2.4. Model application/response calibration

The generalised mixed-effects height functions can be used for prediction with or without information on the random effects at sample plot-level random effects (u_{i1}, u_{i2}) and revision-level random effects (v_{it1}, v_{it2}). When applied without the random effect parameters, prediction of the fixed part is obtained (i.e. population average) (Mehtätalo, 2004). However, with response calibration, the random effects could be predicted and included in the generalised mixed-effects height functions, to improve accuracy.

The empirical best linear unbiased prediction (EBLUP) technique (equation [15]) (Vonesh and Chinchilli, 1997) was used, in which the random effect parameter $(\widehat{\boldsymbol{b}})$ was initially set at zero, and then the solution was iteratively searched (Arias-Rodil et al., 2015).

$$\widehat{\boldsymbol{b}} = \widehat{\boldsymbol{D}}\boldsymbol{Z}_i^{'}\left(\widehat{\boldsymbol{R}}_i + \boldsymbol{Z}_i\widehat{\boldsymbol{D}}\widehat{\boldsymbol{Z}}_i^{'}\right)^{-1}\left[\boldsymbol{y}_i - f(\boldsymbol{x}_i, \widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{b}}) + \boldsymbol{Z}_i^{'}\widehat{\boldsymbol{b}}\right] \tag{15}$$

where $\widehat{\boldsymbol{b}}$ represents a vector of the sample plot- and the revision-level random effects; $\boldsymbol{Z}_i$ is the design matrix of random parts, obtained from the partial derivative of the generalised mixed-effect height function, with respect to the fixed-effect parameters to which random effect parameters were added. $\widehat{\boldsymbol{D}}$ represents the estimated variance–covariance matrix of the plot random effect $(\widehat{\boldsymbol{D}}_{plot})$ and revision-level random effect $(\widehat{\boldsymbol{D}}_{revision})$. $\widehat{\boldsymbol{R}}_i$ is an estimate of the error matrix, derived from $\widehat{\sigma}^2 d_{itj}^{\;2\delta}\boldsymbol{I}_{it}$; where $\boldsymbol{I}_{it}$ is the $n_{it} \times n_{it}$ identity matrix; $\boldsymbol{y}_i$ is the vector of the measured tree height(s); $f(\bullet)$ represents the generalised mixed-effect height functions; and $\widehat{\boldsymbol{\beta}}$ is the vector of the fixed-effect parameters. For details about response calibration of the multilevel mixed-effects model, see Mehtätalo and Lappi (2020) and Pinheiro and Bates (2000).

Part of the code by Arias-Rodil et al. (2015) was adapted to calibrate the generalised multilevel nonlinear mixed-effects height functions for the LTFE data. Since no independent data is available for all tree species, only the functions for the main species (Scots pine, Norway spruce and birch) were used for response calibration. All measured tree heights per sample plot and revision occasion were used for the calibration. Numerical statistics and graphical appearance were used to assess the quality of the model predictions.

Since the aim of this study was to reduce the number of sample trees for which height must be measured, we used height measurements of one to six trees for response calibration (localization of the mixed-effects model). As the accuracy of a calibrated mixed-effects model depends to a certain extent on the method of selection of the sample tree(s) (Bronisz and Mehtätalo, 2020a; Zhou et al., 2022), three selection strategies were evaluated: diameter extremes, largest diameters and smallest diameters. For each selection strategy, the random effect parameters were predicted from the measured height of one to six trees of the independent data. In the diameter extremes' strategy, selection of one tree means selecting the maximum; for two trees: one minimum and one maximum; three trees: one minimum and two maximum; four trees: two minimum and two maximum; five trees: two minimum and three maximum; and for six trees: three minimum and three maximum. We also considered the situation where the functions were not calibrated, and as such, only the fixed-effect part was used. Finally, to decide on the optimal number of sample trees required for a response calibration, the percentage reduction in the prediction errors were assessed.

3. Results

3.1. Height functions

The generalised multilevel nonlinear mixed-effects height functions developed for estimating tree height of Scots pine, Norway spruce, birch, other conifers and other broadleaves in the LTFE produced attractive fit statistics (Table 3). Using the height (*hdom*) of the tree with the largest diameter (regardless of the species), basal area and the ratio of diameter to QMD as covariates contributed significantly ($p < 0.05$) to the overall fit of the main species (equations [3], [4] and [5]). Similarly, using the largest diameter (*ddom*) and its height (*hdom*) as covariates contributed significantly to the description of the tree heights of other conifers and other broadleaves (equations [6] and [7]). The RMSE of the height functions ranged from 0.9571 to 1.3630 m and accounted for at least 93.8 % (Pseudo-R^2: 0.938 – 0.970) of the variation in height. In addition, no autocorrelation was observed in the residuals of the functions (see Appendix Fig. A3).

When each function was fitted across the species, only equation [3], i.e. the Scots pine function, produced fits comparable to the species-specific height functions (optimal function), especially for birch (see Appendix Table A4). However, some parameters were not significant ($p > 0.05$) for other conifers and other broadleaves. Thus, equation [3] was used to build a single generalised multilevel nonlinear mixed-effects height function for the main species, in which the effect of tree species was included (equation [16]). All parameters of the function were significant (Table 4). The function had a RMSE of 1.028 m and explained 95.5 % of the variability in tree height.

$$h_{itj} = 1.3 + \left[\frac{d_{itj}}{\left(\beta_0^* + u_{i1} + v_{it1}\right) + \left(\beta_1 hdom_{it}^{\left(\beta_2 + u_{i2} + v_{it2}\right)} + \beta_3 G_{it} + \beta_4 C_{itj}\right) d_{itj}} \right]^2 \tag{16}$$

where $\beta_0^* = \gamma_0 + \gamma_1 S_1 + \gamma_2 S_2$; $\gamma_0, \gamma_1, \gamma_2$ are parameters of the function; S_1 and S_2 are dummy variables. If the species is Norway spruce, $S_1 = 1$ otherwise 0. If the species is birch, $S_2 = 1$ otherwise 0. All other parameters were previously defined in Table 2.

3.2. The reponse calibration

The species-specific functions, i.e. equations [3], [4] and [5] were used to predict the height of Scots pine, Norway spruce and birch,

Table 3
Parameter estimates and fit statistics of the species-specific functions for Scots pine, Norway spruce, birch, other conifers and other broadleaves.

Components	Scots pine (Equation [3])			Norway spruce (Equation [4])			Birch (Equation [5])			Other conifers (Equation [6])			Other broadleaves (Equation [7])		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Fixed part															
β_0	1.0108	0.0105	0.0000	1.5683	0.0162	0.0000	−0.7999	0.0171	0.0000	1.0865	0.0250	0.0000	0.9305	0.0388	0.0000
β_1	0.7921	0.0066	0.0000	0.5236	0.0099	0.0000	−0.6413	0.0148	0.0000	0.7813	0.0126	0.0000	0.8895	0.0650	0.0000
β_2	−0.4615	0.0034	0.0000	−0.3189	0.0141	0.0000	−0.7339	0.0192	0.0000	−0.5127	0.0204	0.0000	−0.7247	0.0751	0.0000
β_3	−0.00016	2.17E-05	0.0000	0.00057	8.18E-05	0.0000	0.0003	6.08E-05	0.0000	−0.00378	0.00107	0.0005	0.00885	0.0026	0.0007
β_4	0.0015	0.0002	0.0000	0.0075	0.0010	0.0000	−0.0012	0.0003	0.0021						
Random part: Plot															
$\sigma^2_{u_1}$	0.1901^2			0.2009^2			0.1918^2			0.2745^2			0.1586^2		
$\sigma^2_{u_2}$	0.0116^2			0.0084^2			0.0390^2			0.0247^2			0.0026^2		
cor(u_1, u_2)	−0.7440			−0.5550			−0.6330			−0.7290			−0.4450		
Random part: Revision															
$\sigma^2_{v_1}$	0.1287^2			0.1385^2			0.1222^2			0.1163^2			0.2122^2		
$\sigma^2_{v_2}$	0.0093^2			0.0047^2			0.0178^2			0.0099^2			0.0021^2		
cor(v_1, v_2)	−0.6760			−0.3830			−0.5950			−0.4900			−0.8230		
σ^2	0.7550^2			0.3316^2			0.5445^2			0.6080^2			0.7674^2		
δ	0.0924			0.4955			0.2398			0.1798			0.2267		
Fit indices															
RMSE	0.9571			1.0692			0.9535			0.9667			1.3630		
MAB	0.7398			0.7745			0.7261			0.7428			1.0293		
MAPE	5.6223			8.3720			6.1999			6.1952			7.6302		
Pseudo-R^2	0.9557			0.9546			0.9538			0.9702			0.9381		

SE: standard error; σ^2: estimated variance; δ: estimated parameter for the power variance-stabilizing function; other symbols and acronyms are the same as defined in the main text.

Table 4
Parameter estimates and fit statistics of equation [16] with grouping effect of Scots pine, Norway spruce and birch.

	Equation [16]		
Components	Estimate	SE	p-value
Fixed part			
γ_0	1.1170	0.0096	0.0000
γ_1	0.1651	0.0033	0.0000
γ_2	−0.1455	0.0035	0.0000
β_1	0.7307	0.0063	0.0000
β_2	−0.4483	0.0035	0.0000
β_3	−0.00025	2.14E-05	0.0000
β_4	0.0041	0.0002	0.0000
Random part: Plot			
$\sigma^2_{u_1}$	0.1938^2		
$\sigma^2_{u_2}$	0.0145^2		
$cor(u_1, u_2)$	−0.6850		
Random part: Revision			
$\sigma^2_{v_1}$	0.1960^2		
$\sigma^2_{v_2}$	0.0155^2		
$cor(v_1, v_2)$	−0.8610		
σ^2	0.9152^2		
δ	0.0507		
Fit indices			
RMSE	1.0288		
MAB	0.7860		
MAPE	6.5768		
Pseudo-R^2	0.9552		

SE: standard error; σ^2: estimated variance; δ: estimated parameter for the power variance-stabilizing function; other symbols and acronyms are the same as defined in the main text.

respectively; and compared with the predictions from equation [16]. The graphical relationship between the predicted and observed tree heights showed that both species-specific (optimal) functions and equation [16] produced a well-organized cluster along the main diagonal with no tendency towards under- and overestimation of tree heights across the latitudinal gradient (Fig. 3). No substantial differences were observed between predictions from the species-specific functions and equation [16]. Furthermore, an assessment of the residuals of the prediction functions showed homoscedastic variance across predicted heights, especially with the species-specific functions (Fig. 4).

The sample plot-specific height-diameter curves were produced by calibrated functions (equation [3], [4], [5] and [16]) and overlaid on the measured height-diameter pairs of the validation data (Fig. 5). As seen in the graph, the height curves covered the entire clouds of the measured data for Scots pine, Norway spruce and birch. The sample plot-specific height-diameter curves of data used for other conifers and other broadleaves are presented in Appendix Fig. A4 (not validated with independent data).

The evaluation of the number of sample trees for height measurements and the method of the selection procedure required for a response calibration showed that the prediction quality of the functions improved with the increased number of sample trees (1, 2, …, 6 tree heights) (Fig. 6). Without response calibration, height prediction was poor, especially in Norway spruce and birch.

The selection of sample trees from the diameter extremes had the lowest residual variations for Scots pine, Norway spruce and birch, compared with the other strategies. Furthermore, an assessment of the percentage reduction in the error for the best selection strategy (i.e., diameter extremes) showed a sharp decline from zero to two trees, and thereafter a steady downward slope from three to six trees for all species (Fig. 7). Equation [16] performed well for Norway spruce and Scots pine but not for birch.

4. Discussion

This study developed multilevel nonlinear mixed-effects height functions for optimizing tree height measurements in the Swedish LTFEs, using data that covered almost all conditions concerning growth, silviculture and the environment. The large variances of the sample plot-level and revision-level random effects confirm the strength of a multilevel nonlinear mixed-effects function for the LTFE data. Complex data structures, like the LTFE, are preferably modelled with such a technique (Arias-Rodil et al., 2015; Mehtätalo, 2004).

The multilevel nonlinear mixed-effects height functions produced attractive statistics and estimated tree height with a high level of precision; showing no tendency for bias across the latitudinal gradient. The predictive performances of the species-specific height functions and the height function with species as a covariate (equation [16]) were quite similar, at least for the main species, indicating that the function with species effects could be used instead of the species-specific functions for predicting tree height of Scots pine, Norway spruce and birch trees. The three main species account for about 92 % of the standing volume of the forest in Sweden (Swedish Forest Agency, 2020). Using common height functions (i.e., equations [6] and [7]) for other conifers and other broadleaves, respectively, also seems appropriate. A further modification of these functions did not improve their performance, and in most cases, some parameter estimates were not significant, especially the function for other broadleaves. One possible explanation for this may be due to the few observations of many of the tree species and their different growth patterns.

Besides the wide range of data used to build the height functions, the inclusion of easily measured variables such as the height (*hdom*) of the tree with the largest diameter (*ddom*) (regardless of the species), basal area (G) and the ratio of diameter at breast height to *QMD* (C) makes the functions easier to apply. These variables have been used consistently together with the diameter at breast height to describe height-diameter relationships in Europe and under Scandinavian conditions. For example, Sharma et al. (2019) reported a significant contribution of these variables to the height-diameter relationships for complex forests in central Europe. In Norway, the tallest tree height and its diameter per sample plot per revision were used to improve the height function for Scots pine, Norway spruce and Downy birch (Sharma and Breidenbach, 2015). Similar variables were used to build a generalised height-diameter function for young Sweet chestnut stands in Portugal (Patrício et al., 2022).

The inclusion of dominant height in the height functions helps account for the effect of stand development and site quality, as tree height increases with increasing stand development and site quality (Castedo Dorado et al., 2006; Sharma et al., 2019). The inclusion of variables describing such stand characteristics makes the resulting height functions more generalized rather than localized, and can be used on forest stands with any site productivity. In addition, dominant height is unaffected by thinning except for thinning from above (Sharma et al., 2016). Thinning from below has been the main thinning type in Sweden for more than a century (Nilsson et al., 2010). Contrary to height-diameter modelling studies (Crecente-Campo et al., 2014; Ogana et al., 2020; Patrício et al., 2022; Raptis et al., 2021; Sharma et al., 2019), we defined the *hdom* as the height of the tree with the largest diameter regardless of the species, to simplify data gathering, thus, only the height of one tree per sample plot and revision is required. This was used as a proxy for dominant height. Estimating dominant height, i.e. the arithmetic average height of the 100 trees with the largest diameter per hectare, will require height measurements of several trees, which means additional inventory costs and as such, nullifies the essence of this study. Moreover, most of the sample plots do not have information on dominant height for some of the revisions. Another proxy for dominant height is the height of the tallest tree (Lam et al., 2017; Sharma et al., 2016; Sharma and Breidenbach, 2015). We did not consider this variable because to identify the tallest tree in a sample plot/stand, the height of several trees must be measured.

Stand density and competition are important considerations in the development of a stand, and thus, influence the height-diameter

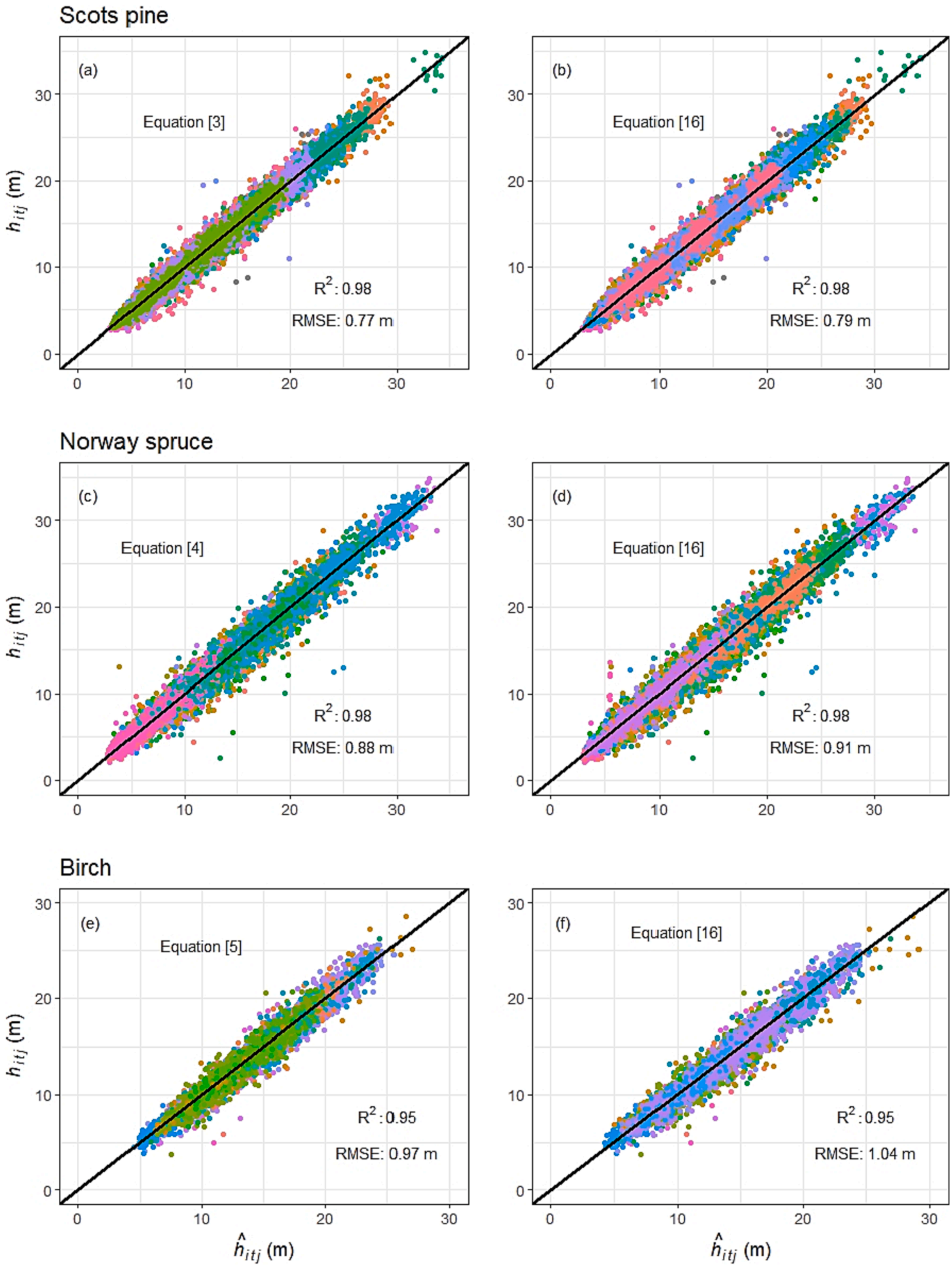

Fig. 3. Plots of observed height (h_{itj}) versus predicted height ($\widehat{h}_{itj}$) by the functions in the validation data for the main species. The colours represent data across the latitudinal gradient of the LTFEs.

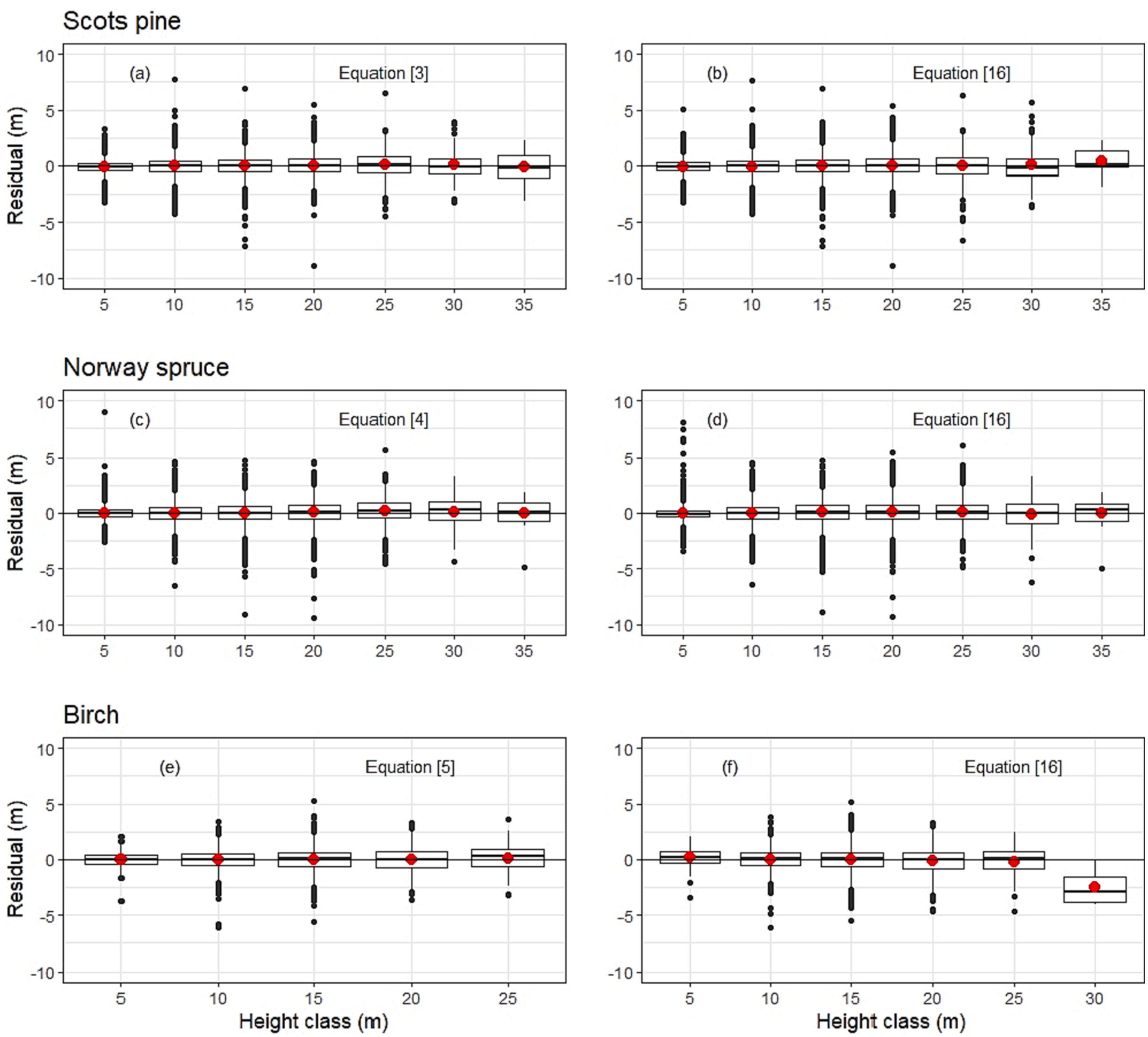

Fig. 4. Plots of residuals versus predicted height for the main species. Red dots are the residual means and the horizontal lines represent the expected mean residual (i.e. $[E(\varepsilon_{itj}) = 0]$).

relationship. Ideally, a more dense stand will result in just as tall trees, but due to competition, the dominated trees are often more slender than those in a less dense stand, compared to the dominating trees, which are not so affected by density (Fortin et al., 2019; Sharma et al., 2019). We used G and C to account for these effects on the height-diameter relationship for the main species.

Our response calibration (localized function) produced the sample plot-specific height-diameter curves for the different revisions that cover almost all the measured height-diameter pairs in the validation dataset (Fig. 5), showing that the calibrated functions could be used for accurate estimation of tree heights in the LTFEs. Adopting the calibrated height functions means that it is not necessary for fitting sample plot and revision-specific height functions. A comparison of the Swedish protocol for estimating heights of the calipered trees in the LTFEs with our functions (response calibrated based on diameter extremes) showed no difference for Scots pine, Norway spruce, birch, larch, alder or Beech from selected sites (see Appendix Fig. A5–A8).

The prediction accuracy of mixed-effects function depends largely on the stand structure of a sample plot, number of trees, and representativeness of height of chosen trees for estimating random effects. For sample plot with the homogenous stand structure, one or two trees may work adequately well, but more trees are needed for heterogeneous stand structure to ensure a higher prediction accuracy (Sharma et al., 2017). With the height functions developed in our study, taking measurements of the height of only three to four trees still yielded desirable results. Though prediction accuracy increases with an increased number of sample trees used for a response calibration, we may consider four trees as a good compromise between sampling cost and prediction accuracy. Numerous studies have also mentioned the use of four sampled trees for a response calibration of the height function. For example, Özçelik et al. (2018) reported a comparable value for the nonlinear mixed-effect height function developed for Brutian pine (*Pinus brutia* Ten.) in Turkey. For the Oriental beech (*Fagus orientalis* Lipsky) in the Hyrcanian forests in Iran, Kalbi et al. (2018) recommended four sampled trees as the best. However, Raptis et al. (2021) reported five as the optimal for black pine (*Pinus thunbergii* Parl.) in Greece. For the mixed uneven-aged forests in northwest Durango, Mexico, Corral-Rivas et al. (2014) reported three performing the best. In Romania, high precision was achieved by sampling six trees around the median and largest diameter of Norway spruce in mixed uneven-aged stands (Ciceu et al., 2020). Besides the tree height functions, various functions developed to estimate biomass have also utilised a comparable value for response calibration (e.g., Bronisz and Mehtätalo, 2020b; Colmanetti et al., 2020; de-Miguel et al., 2014).

In this study, the method of selection of the sample trees should be based on the diameter extremes. The selection of sample trees from

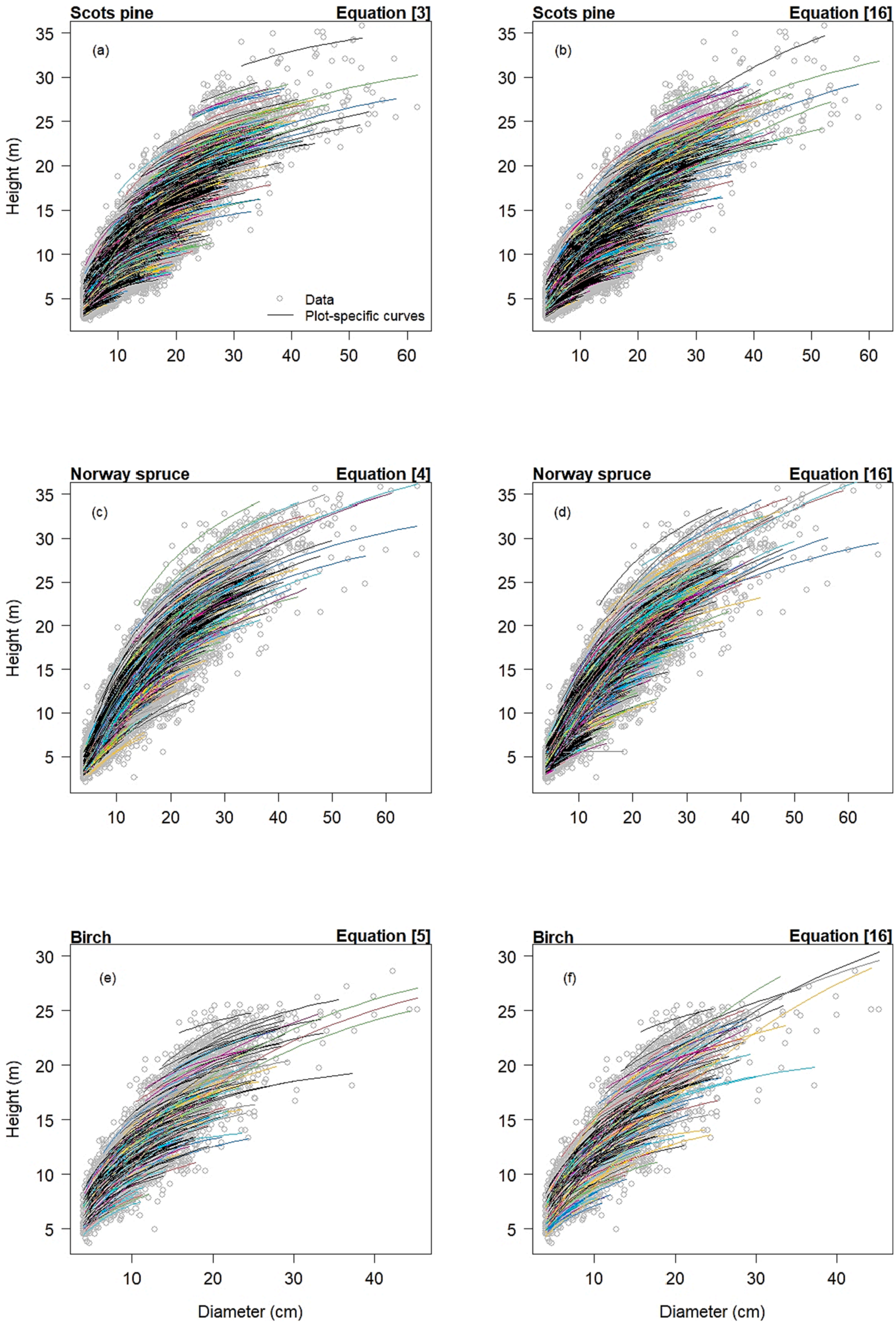

Fig. 5. Sample plot-specific height curves overlaid on the measured height-diameter pairs of validation data. Left panel: species-specific function and right panel: equation [16], i.e. the function with species as a covariate.

diameter extremes outperformed other alternatives evaluated for the LTFE functions in this study. Bronisz and Mehtätalo (2020a) also found diameter extremes as the preferred tree selection strategy for calibrating the height function for young Silver birch stands in central Poland. Other studies (e.g., Castedo Dorado et al., 2006; Crecente-Campo et al., 2010) recommended the selection of sample trees from the smallest diameter for Radiata pine (*Pinus radiata* D. Don) and Tasmanian blue gum (*Eucalyptus globulus* Labill.) in northwest Spain. The selection of sample trees from the smallest diameter was the second best in our study. Using the largest diameter strategy did not perform well. This may be because

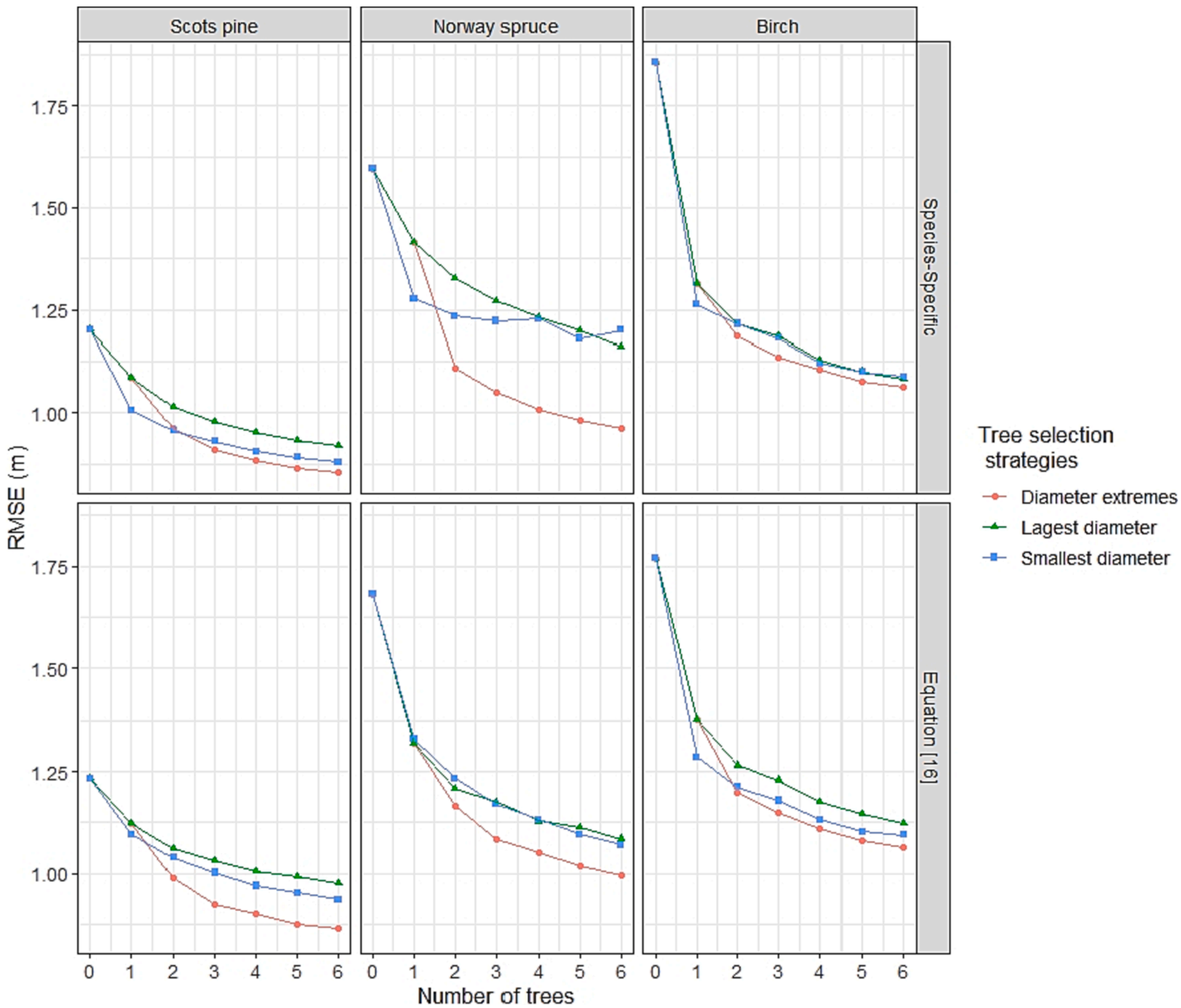

Fig. 6. Performance of sample trees selection strategies for the application of the species-specific functions and equation [16], i.e. the function with species as a covariate, with various number of sample trees selected. Note: zero sample trees means without response calibration.

the functions already contain, as a predictor, the height of the largest tree (Bronisz and Mehtätalo, 2020a). Therefore, taking sample trees from the largest diameter alone may not yield the desired precision. However, Crecente-Campo et al. (2014) observed that though their function did not contain information on the dominant height as a regressor, yet the largest diameter strategy had the worst results in response calibration. Contrary to our result and other studies, Temesgen et al. (2008) found this selection strategy as the best option for Douglas fir (*Pseudotsuga menziesii* [Mirbel] Franco) in Oregon. Application of the height function without response calibration would produce a less accurate prediction of tree heights.

The response calibration of a multilevel mixed-effects function without a programming tool could be a herculean task and would be impossible for application in the real world. Thus, for easy application of our height functions to the Swedish LTFE data, we have provided the complete R syntax and codes for the estimation of sample plot-level random effects and revision-level random effects (see supporting file).

Our height functions will be beneficial for the inventory crew who may measure the heights of only a few trees per sample plot and predict the heights of the remaining trees by applying these height functions. This may reduce the resources and time required for inventorying LTFEs. For forest conditions similar to the basis of this study, which covers a wide range of stand and environmental conditions, our height functions may be applicable to the forests across other Scandinavian countries. However, the functions may require validation before their application in the local context, as numerous factors affecting tree allometry would vary from country to country, even within the same country, from one forested region to another. Our height functions are largely parsimonious, and therefore they would be efficient for practical application. As mentioned earlier, the proposed height functions could be used for characterisation of the vertical stand structures, stand volume and biomass estimation, carbon accounting, simulation of stand dynamics, stand productivity assessment, and thus, largely useful for informed decision-making in forestry.

5. Conclusion

This study developed multilevel mixed-effects height functions for the Swedish long-term forest experiments (LTFEs), using easily measurable stand characteristics. These functions are important for optimizing tree height measurements in the LTFEs. The study showed that with only a few sample trees, it is possible to predict with high precision the height of the main species Scots pine, Norway spruce, Silver and Downy birch, as well as of other conifer and broadleaf species; thereby, reducing the sampling effort and inventory costs. For high precision, measurement of the height of four sample trees per sample plot and revision based on the diameter extremes, i.e. the trees with the two smallest and two largest diameters, was recommended. It is expected that the presented functions will be useful in the strategy for the subsequent inventories of the Swedish LTFEs.

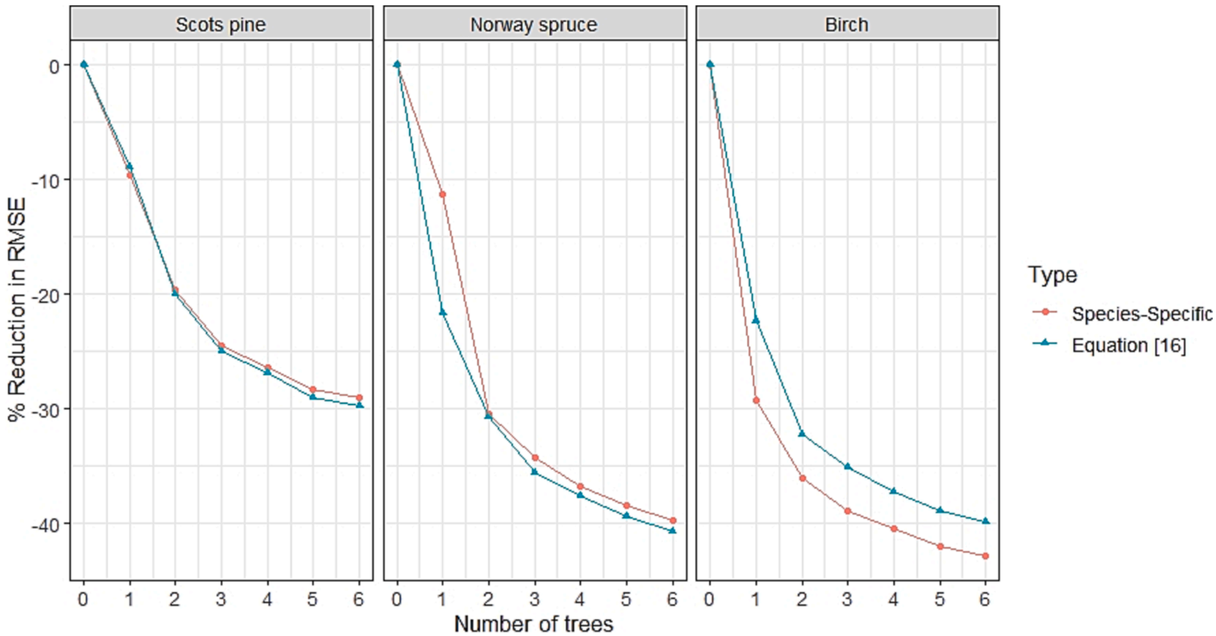

Fig. 7. Percentage reduction in root mean square error (RMSE) for the best sample tree selection strategy, i.e. selecting diameter extremes, with various numbers of selected sample trees. Note: zero sample trees means without response calibration.

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CRediT authorship contribution statement

Friday N. Ogana: Conceptualization, Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Emma Holmström:** Conceptualization, Funding acquisition, Supervision, Writing – review & editing. **Ram P. Sharma:** Methodology, Supervision, Writing – review & editing. **Ola Langvall:** Conceptualization, Data curation, Writing – review & editing. **Urban Nilsson:** Conceptualization, Data curation, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

See supporting file for the R script and sample data on how to perform the response calibration

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.foreco.2023.120843.

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