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Collision Avoidance for Wheeled Mobile Robots in Smart Agricultural Systems Using Control Barrier Function Quadratic Programming

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Abstract: The primary challenge is to design feedback controls that enable robots to autonomously reach predetermined destinations while avoiding collisions with obstacles and other robots. Various control algorithms, such as the control barrier function-based quadratic programming (CBF-QP) controller, address collision avoidance problems. Control barrier functions (CBFs) ensure forward invariance, which is critical for guaranteeing safety in robotic collision avoidance within agricultural fields. The goal of this study is to enhance the safety and mitigation of potential collisions in smart agriculture systems. The entire system was simulated in the MATLAB/Simulink environment, and the results demonstrated a **93% improvement in steady-state error over** rapidly exploring random tree (RRT). These findings indicate that the proposed controller is highly effective for collision avoidance in smart agricultural systems.

Keywords: smart agriculture; control barrier function; wheeled mobile robot; autonomous system; collision avoidance; quadratic programming; obstacle

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1. Introduction

By monitoring speed, distance, and potential hazards, collision avoidance systems (CASs) enable wheeled mobile robots (WMRs) to prevent collisions and reduce crash risks in agricultural environments [1]. Multi-agent systems have recently gained significant attention due to their broad applications in distributed robotics, mobile sensor networks, air traffic management, and agricultural automation. These systems consist of multiple interacting agents that collectively perform tasks more efficiently than individual complex agents. However, their implementation often requires novel control strategies, particularly in scenarios like formation control and collision avoidance [2].

Existing robot navigation methods frequently rely on navigation functions mathematical constructs derived from environmental geometry to design gradient-based control laws. Other approaches draw inspiration from natural swarm behaviors observed in bird flocks, fish schools, and bee colonies [3]. Control barrier functions (CBFs) offer additional advantages by ensuring safety across diverse operational conditions through adaptable parameter configurations [4]. Enhanced safety in agricultural fields not only minimizes accidents [5] but also improves operational efficiency and cost savings by ensuring uninterrupted WMR functionality [6].

Despite these advancements, existing research lacks a unified framework for evaluating WMR performance in real-time optimization-based control systems. Furthermore, safety conditions often defined via the forward invariance of safe sets remain insufficiently validated for agricultural WMR applications [7].

The aim of this study is to address the multi-agent collision avoidance problem within a centralized framework, where each agent has continuous access to the positions of all other group members. This approach is particularly relevant in agricultural settings, where multiple wheeled mobile robots (WMRs) often operate simultaneously, necessitating coordinated collision avoidance strategies. By leveraging control barrier functions (CBFs), this study ensures that each WMR can navigate safely while avoiding both static obstacles and other moving agents. This represents a significant improvement over traditional single-agent navigation methods, as it enhances safety and efficiency in dynamic and complex agricultural environments.

The novelty of this work lies in the application of control barrier function-based quadratic programming (CBF-QP) for real-time, safety-critical collision avoidance in dynamic agricultural environments. While the use of control Lyapunov functions (CLFs) for asymptotic stability in trajectory tracking is well established, the key innovation here is the focus on safety-critical control through the integration of CBFs with QP. This approach ensures forward invariance of safe sets, which is crucial for navigating unpredictable agricultural environments where collision avoidance is essential [8].

The proposed CBF-QP framework provides a robust solution for enforcing safety constraints in real time, enabling wheeled mobile robots (WMRs) to avoid collisions while maintaining performance. Unlike traditional methods that primarily focus on stability and trajectory tracking, CBFs offer a systematic way to ensure the system operates within safe bounds, even in the presence of dynamic obstacles and disturbances. This is particularly relevant in agricultural settings, where obstacles such as crops, other robots, or static barriers are common.

By combining CBFs with QP, the proposed method represents a significant advancement over traditional approaches, making it highly suitable for smart agricultural systems. The integration of these techniques ensures both safety and efficiency, addressing the unique challenges of real-time collision avoidance in dynamic environments.

The remainder of this work is structured as follows: Section 2 reviews the relevant literature, while Section 3 details the WMR's mathematical model and control design. Sections 4 and 5 present the results/discussion and conclusions, respectively.

2. Literature Review

This paper addresses the multi-agent collision avoidance problem for a system of wheeled mobile robots (WMRs). Assuming unicycle dynamics for the WMRs, the objective is to design feedback control mechanisms that enable autonomous navigation to predefined targets while avoiding collisions with obstacles and other agents [9]. To achieve this, this study leverages two key control-theoretic frameworks: control barrier functions (CBFs) and control Lyapunov functions (CLFs). CBFs ensure safety by enforcing the forward invariance of a predefined safe set of states [10], while CLFs provide asymptotic stability guarantees for trajectory tracking and goal convergence [11]. These frameworks form the foundation for developing robust and adaptive control strategies in dynamic environments.

Control theory broadly aims to manipulate dynamic systems to achieve desired outputs through reference signals. Classical approaches often focus on linear time-invariant (LTI) single-input single-output (SISO) systems, which are limited in their ability to handle dynamic load variations, real-time fluctuations, and the complexity of multi-input multi-output (MIMO) systems. While modern methods prioritize robustness, adaptability, and MIMO system compatibility, they may lack the intuitive design principles of traditional approaches, complicating system analysis and implementation [12]. This trade-off between classical and modern methods highlights the need for innovative solutions that balance simplicity, adaptability, and computational efficiency.

For motion planning in cluttered environments, rapidly exploring random trees (RRTs), a subclass of rapidly exploring dense trees (RDTs), offer probabilistic completeness and efficient exploration of high-dimensional configuration spaces through stochastic sampling [13]. Despite their advantages, such as computational simplicity and the ability to expand into unexplored regions, standard RRTs suffer from limitations, including slow convergence, inconsistent path quality, and inaccuracies in practical implementations [14]. These shortcomings underscore the need for more reliable and efficient motion planning techniques, particularly in dynamic and unpredictable environments.

In multi-agent decision-making scenarios involving non-cooperative agents, game theory provides valuable tools for analyzing strategic interactions and optimizing collective outcomes [15]. While game theory enhances risk management and strategic reasoning, it is not without limitations. Computational complexity, the oversight of psychological factors, and idealized knowledge assumptions can hinder its practical applicability [16]. These challenges highlight the importance of integrating game-theoretic approaches with other control strategies to achieve more robust and realistic solutions.

Artificial intelligence (AI) technologies further augment WMR autonomy by enabling advanced perception, language processing, data analysis, and decision-making capabilities [17]. Despite their benefits, such as reduced human error and 24/7 availability, AI-driven systems face challenges, including algorithmic bias, high deployment costs, security vulnerabilities, and limited adaptability in unstructured environments [18]. Addressing these challenges is critical for realizing the full potential of AI in autonomous systems, particularly in safety-critical applications like agriculture.

Among these approaches, control barrier functions (CBFs) are particularly critical for ensuring WMR safety. CBFs provide a systematic framework to define and maintain safe sets during operation, enabling collision avoidance, resilience to disturbances, and adaptability in dynamic environments [19]. By integrating CBFs with Lyapunov-based controllers, WMRs can achieve both safety and performance goals, making them well suited for applications in smart agriculture.

When comparing control barrier function-based quadratic programming (CBF-QP) and adaptive model predictive control (AMPC) for robotic systems, it is essential to evaluate their strengths, weaknesses, and suitability for specific tasks. CBF-QP explicitly enforces safety constraints, ensuring the system remains within a safe set, such as avoiding collisions. It is particularly effective in dynamic environments with moving obstacles and offers computational efficiency, making it suitable for real-time applications. Additionally, CBF-QP can be combined with other control strategies, such as CLFs, to achieve both safety and stability. However, CBF-QP may not always produce globally optimal trajectories, especially in complex environments, and its effectiveness depends on the design of the barrier functions, which can be challenging for highly nonlinear systems. Furthermore, the QP solver may converge to local solutions, which might not be ideal for long-term planning [12].

On the other hand, AMPC optimizes control inputs over a finite horizon, providing globally optimal or near-optimal solutions for trajectory planning. It can adapt to changes in system dynamics or environmental conditions, making it suitable for uncertain or time-varying scenarios. AMPC also considers future states and constraints, enabling better long-term planning compared to reactive methods like CBF-QP. However, AMPC's computational complexity can be a significant drawback, especially for systems with long prediction horizons or high-dimensional states. Additionally, AMPC relies on accurate system models, and inaccuracies can lead to suboptimal or unsafe control actions. The computational burden of AMPC may also limit its applicability in real-time systems with strict timing requirements [14].

In summary, the choice between CBF-QP and AMPC depends on the specific requirements of the application, such as the need for safety, optimality, and computational efficiency. While CBF-QP excels in real-time safety-critical applications, AMPC is better suited for scenarios requiring long-term planning and adaptability. A hybrid approach that combines the strengths of both methods could offer a promising solution for complex and dynamic environments, such as those encountered in smart agriculture [5].

3. Mathematical Models and Control Design

To visualize the simulation scenario in the MATLAB/Simulink environment, the mathematical model of the wheeled mobile robot (WMR) is first defined in the code compiler. Next, the system checks for the presence of an obstacle. If an obstacle is identified, the distance to the obstacle is calculated. If no obstacle is detected, the system proceeds to monitor the WMR's status (Figure 1).

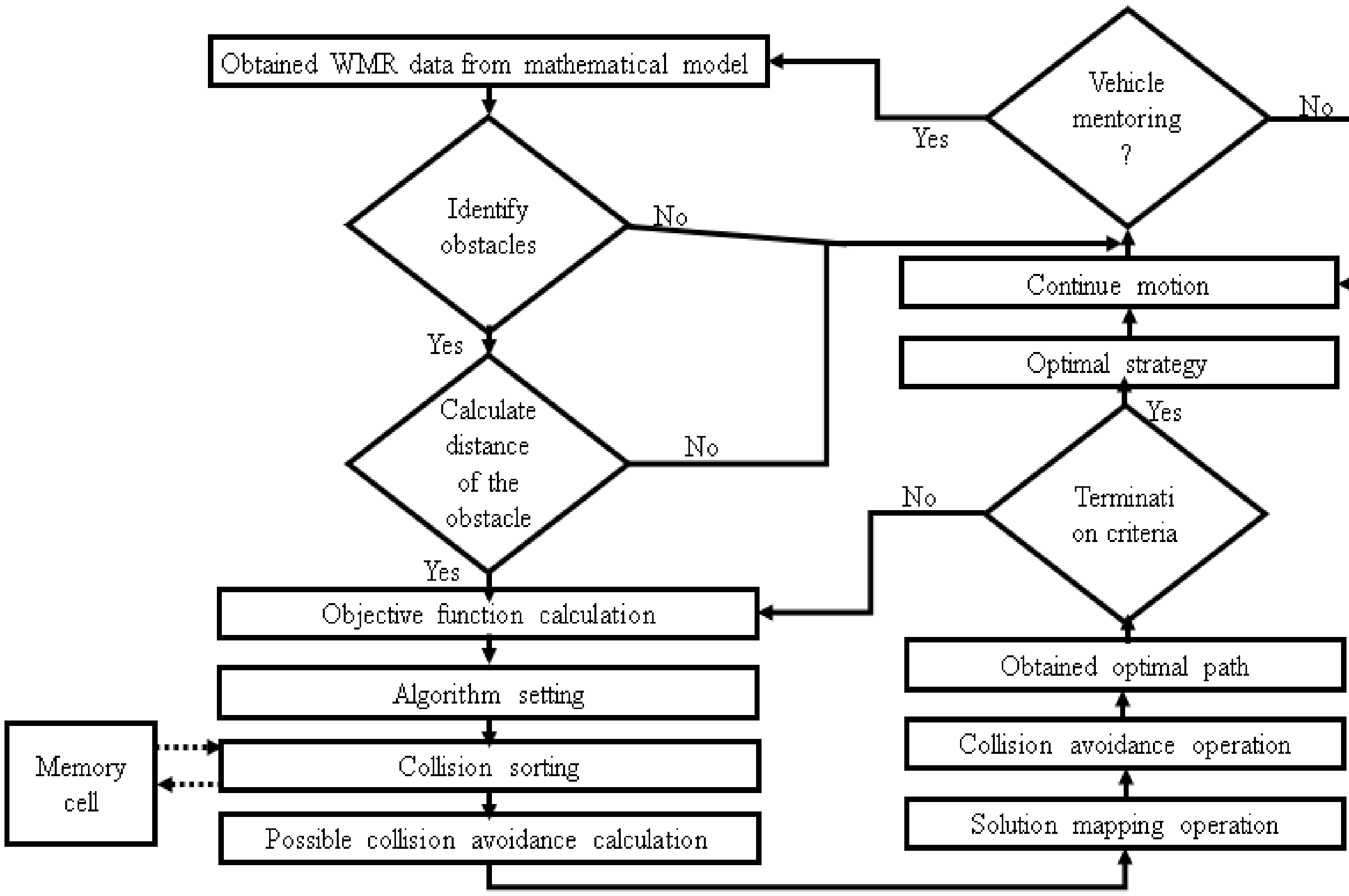

Figure 1. Simulation scenario of the current work.

If the distance to the obstacle is calculated, the system proceeds to compute the objective function. Following this, the algorithm settings are configured, collision detection is performed, and the data are stored in the memory cell. Subsequently, the system calculates possible collision avoidance strategies and maps the solution for the collision avoidance operation. This process yields an optimal path for the WMR.

If the terminal criteria are not met, the system recalculates the objective function. If the terminal criteria are satisfied, the optimal strategy is evaluated, and the WMR continues its motion. After this, the system checks the WMR's monitoring status. If the WMR is not being monitored, the system verifies the continuation of the motion. If the WMR is being monitored, the process restarts from the beginning and repeats.

The current work is motivated by the growing interest in autonomous wheeled mobile robots, a topic that holds significant promise for both scientific research and practical applications in agriculture. Research into the control of autonomous wheeled mobile robotic platforms in unstructured, obstacle-filled environments is particularly relevant and feasible [20]. For safe navigation in such challenging conditions, wheeled mobile robots must be equipped with robust collision avoidance systems. These systems rely on sensors, cameras, and advanced algorithms to detect obstacles and generate safe paths. Ensuring reliable and consistent performance of these collision avoidance mechanisms is critical for the safe operation of wheeled mobile robots.

In Figure 2, the wheeled mobile robot's planned path to its goal is represented by a solid red line. A potential collision with an obstacle is indicated by a dotted red line, while the robot's mobility and maneuvering area are depicted by a dotted rectangle. This visualization highlights the importance of effective path planning and collision avoidance in dynamic and complex environments.

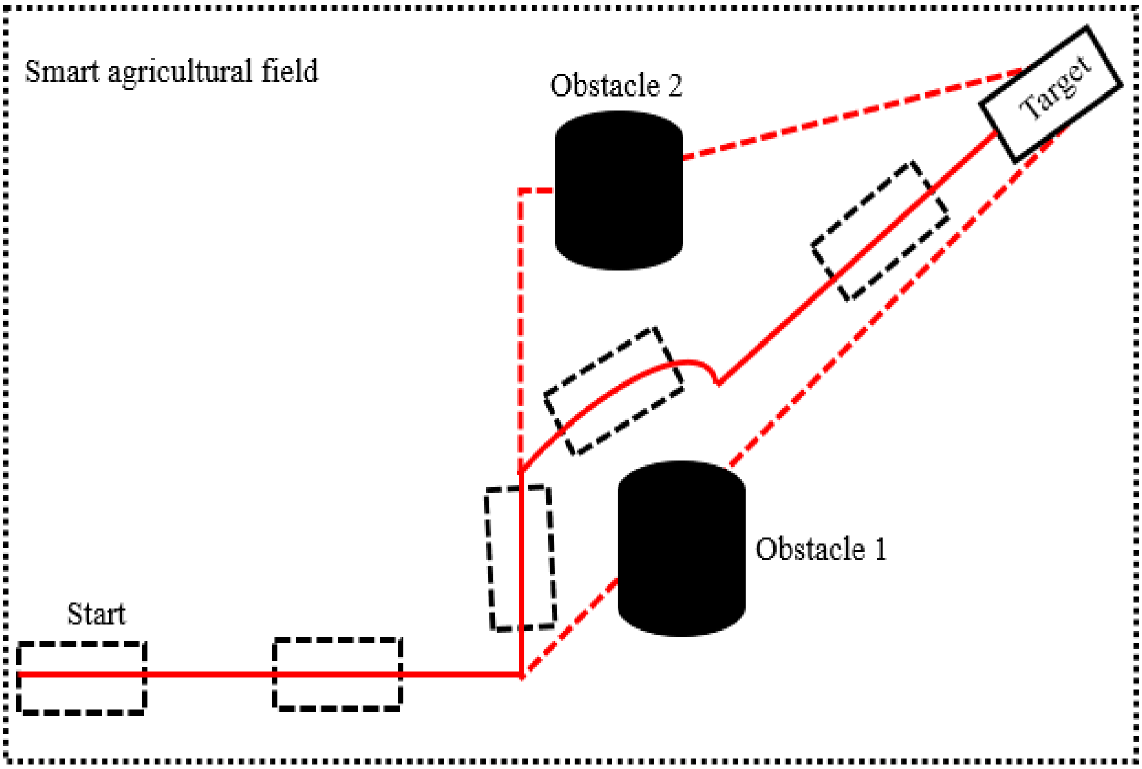

Figure 2. A wheeled mobile robot platform's trajectory for avoiding obstacles [3]. The solid red line indicates the path traveled by the wheeled mobile robot. The red dashed line indicates the possibility of a colli-sion. The black dashed rectangle represents the robot's past positions, and the black dotted rectangle outlines the entire agricultur-al field.

- Wheeled Robots: Typically used in relatively flat and even terrains such as orchards, vineyards, or open fields. They are ideal for environments where the ground is stable and free from large obstacles.
- Other Environments: In uneven, rocky, or muddy terrains (e.g., hilly areas or dense forests), wheeled robots may struggle. In such cases, tracked robots or legged robots might be more suitable due to their better stability and traction.
- Wheeled Robots: Generally, they offer high speed and efficiency on flat surfaces. They are easier to control and require less energy compared to other types of robots.

- Other Environments: In environments with dense vegetation or narrow pathways, robots with higher maneuverability (e.g., drones or legged robots) might be preferred. Drones can fly over obstacles, while legged robots can navigate through rough terrain.
- Wheeled Robots: Often used for tasks like planting, spraying, harvesting, and soil analysis in open fields. They can carry heavy payloads and are suitable for repetitive tasks over large areas.
- Other Environments: In environments requiring precise navigation or access to hard-to-reach areas (e.g., treetops or steep slopes), drones or specialized robots might be more effective. For example, drones can monitor crop health from above, while climbing robots can inspect vertical structures.

In agricultural fields, the collision avoidance algorithm is depicted in Figure 3. The algorithm works as follows: after starting up, it moves to the desired position and orientations. The wheeled mobile robot then enters drive mode, which activates the motors. Next, it begins by sweeping, then detecting objects; then, a wheeled mobile robot controls its speed and makes a decision; and last, it identifies and detects edges. Fault tolerance in the system is implemented in the event of a malfunction, coordinating decision-making and final determination. If not, the algorithm goes back to the beginning of the sweep process. All the cycles were repeated.

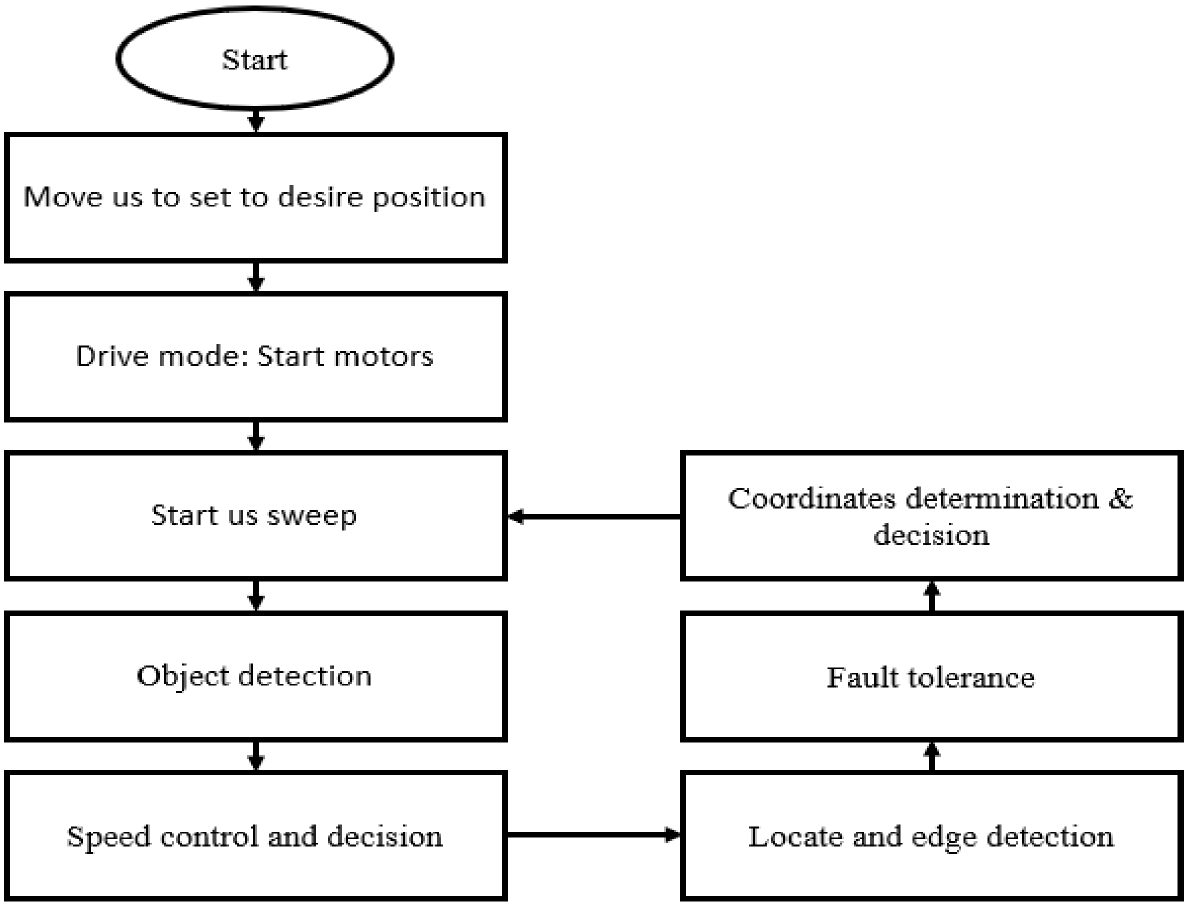

Figure 3. Overview of the collision avoidance and object detection algorithm.

3.1. Mathematical Model

Problem formulation: The problem of multi-agent collision avoidance is presented, developed, and evaluated in a centralized environment. All remaining members of the group have constant access to each agent's positions. A group of N WMRs traveling on the ground (the Euclidean plane) is taken into consideration; they may be distinguished by the existence of (static) impediments. Specifically, a model resembling a unicycle with a passive wheel describes each WMR [1,21].

$$\begin{cases} \dot{X}_i = \cos(\theta_i) v_i \\ \dot{Y}_i = sin(\theta_i) v_i \\ \dot{\theta}_i = \omega_i \end{cases} \tag{1}$$

where $(X_i,\ Y_i) \in R^2$ is the location of the middle point between the two activated wheels on the Euclidean plane, along the axle that connects them, $i = 1, \ldots, N$; θ_i is the orientation of the ith robot; v_i is longitudinal velocity, and ω_i is angular velocity. $\left(X_i^*,\ Y_i^*\right) \in R^2$. For every agent, find the control inputs for its center of mass, v_i, and ω_i in such a way that the i^{th} agent is guided to the intended location while avoiding collisions with both static barriers and other team members (Figure 4).

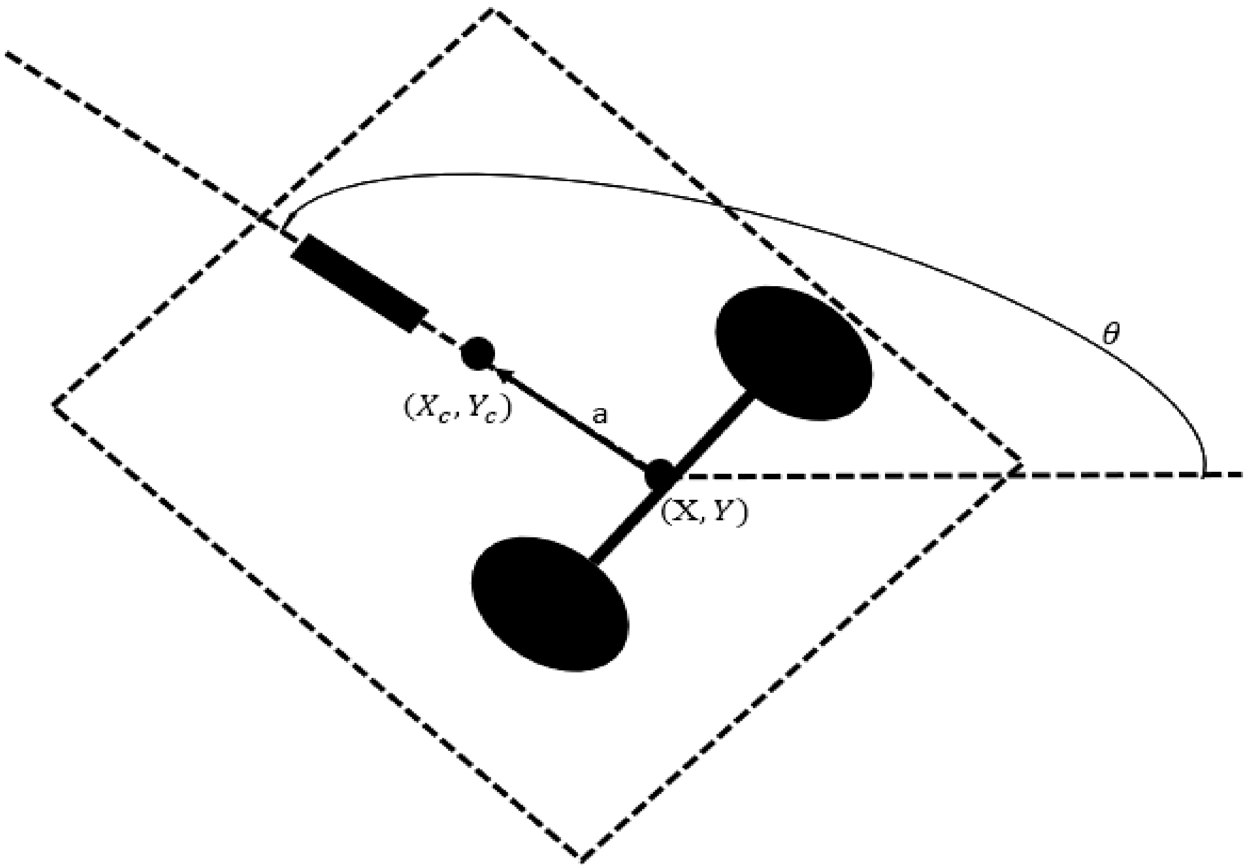

Figure 4. Diagram of the WMR.

Take into consideration the scenario where single-integrator dynamics describes each virtual [1] agent's motion.

$$\dot{x}_i = u_i \tag{2}$$

The boundary of the region in the multi-agent collision avoidance problem for WMRs is a fundamental concept that defines the safe operational space for the robots. It ensures that the WMRs maintain a minimum distance from obstacles and other agents, thereby avoiding collisions. The boundary is enforced using control barrier functions (CBFs), which guarantee forward invariance of the safe set. This approach is particularly important in dynamic and unpredictable environments, such as agricultural fields, where collision avoidance is critical for the safe and efficient operation of WMRs. Therefore, the boundary of the region would be

$$\partial P_j = \left\{ x \in R^2 : \left\| x - p_j \right\|^2 E_j - p_j^2 = 0 \right\} \tag{3}$$

where $p_j > 0$, and $E_j = E_j^T > 0$. Initial deployment without obstacle collisions: the agents' starting locations satisfy

$$\left\| x_i(0) - x_f(0) \right\| > r_i + r_j \tag{4}$$

Initial deployment of agents without collisions: the agents' starting locations satisfy

$$\left\| x_i^* - p_j^c \right\|^2 > \left(r_i + \overline{p}_j(\varnothing^*) \right) \tag{5}$$

Desired deployment without obstacles colliding: the agents' intended positions fulfill

$$\left\| x_i^* - x_j^* \right\| > r_i + r_j \tag{6}$$

Desired deployment of agents without collisions: each agent's target placements meet

$$l_i \cap \left(\cup_{j=1,\ldots,m} \partial p_j \right) = \varnothing \tag{7}$$

A multi-agent system with $N > 1$ WMRs presents significant challenges, including scalability, coordination, and adaptability to dynamic environments. By leveraging control barrier functions (CBFs) and control Lyapunov functions (CLFs), the system can achieve collision-free navigation while ensuring stability and goal convergence.

Consequently, take into consideration a multi-agent system with dynamics that consists of $N > 1$.

$$p_i\left(\tilde{x}\right) = \left[p_{kj}^i \right]^T + \gamma_i I \tag{8}$$

where $p_{kj}^i \in R^{2N \times 2N}$, $k = 1, \ldots, N$, and $\gamma_i > 0$ is a constant parameter:

$$p^i{}_{ii}\,(\check{x}) = \left[\sqrt{\propto_i + \beta_i^s g_i^s(\check{x}) + \beta_i^d g_i^d(\check{x}) I} \right] \tag{9}$$

The statical equation becomes

$$g_i^s(\check{x}) = \sum_{j=1}^{m} \frac{1}{\left(\left\| (\check{x}_i + x_i^* - p_j) \right\|^2 E_j - p_i^2 \right)^c} \tag{10}$$

The dynamics equation becomes

$$g_i^d(\check{x}) = \sum_{j=1, j \neq i}^{N} \frac{1}{\left(\left\| (\check{x}_i + x_i^*) - (\check{x}_j + x_j^*) \right\|^2 - r_i^2 \right)^c} \tag{11}$$

For $c \geq 1$, and $p_{kj}^i = 0$ for $k \neq i$, and $j \neq i$. Let $R_i = R_i^T \in R^{2N \times 2N}$. $\Omega \subseteq R^{2N} \times R^{2N}$ the dynamics strategies for the WMRs involve the use of control barrier functions (CBFs) and control Lyapunov functions (CLFs) to ensure collision avoidance and stability. The control inputs v_i and ω_i are computed using a quadratic programming (QP) framework, which minimizes a cost function while satisfying safety constraints. The Hamilton–Jacobi–Isaacs (HJI) variational inequality is used to ensure safety in the presence of disturbances, and the optimal control policy is determined to maximize the value function. These strategies enable the WMRs to navigate safely and efficiently in dynamic and complex environments.

The dynamics strategies would be

$$u_i = -\check{x}_i\left(\sqrt{\alpha_i+\beta_i^s g_i^s(\xi)+\beta_i^d g_i^d(\xi)}+\gamma_i\right)-\sum_{j=1}^{N} N_{ij}^i(\check{x}_j-\xi_j) \tag{12}$$

The derivative of the dynamic's strategies along i direction would be

$$\dot{\xi}_i = -k\sum_{i=1}^{N}\left(\frac{\check{x}_i^T\check{x}_i}{2\sqrt{\alpha_i+\beta_i^s g_i^s(\xi)+\beta_i^d g_i^d(\xi)}}\left(\beta_i^s\frac{\partial g_i^s(\xi)^T}{\partial\xi}+\beta_i^d\frac{\partial g_i^d(\xi)^T}{\partial\xi}\right)-R_i(\check{x}-\xi)\right) \tag{13}$$

The derivative of the dynamic's strategies along j direction would be

$$\dot{\xi}_j = -k\sum_{j=1}^{N}\left(\frac{\check{x}_j^T\check{x}_j}{2\left(\sqrt{\alpha_j+\beta_j^s g_j^s(\xi)+\beta_j^d g_j^d(\xi)}\right)}\left(\beta_j^s\frac{\partial g_j^s(\xi)^T}{\partial\xi}+\beta_j^d\frac{\partial g_j^d(\xi)}{\partial\xi}\right)+R_i(\check{x}-\xi)\right) \tag{14}$$

The state equation for WMRs in collision avoidance describes the evolution of the system's state over time, incorporating the dynamics of the WMRs and the constraints imposed by collision avoidance. The state equation is derived using unicycle dynamics and incorporates control barrier functions (CBFs) to enforce safety constraints. The control inputs v_i and ω_i are computed using a quadratic programming (QP) framework, which minimizes a cost function while satisfying the CBF constraints. This approach ensures that the WMRs navigate safely and efficiently in dynamic and complex environments, avoiding collisions with obstacles and other WMRs.

Then, the state equation becomes

$$\begin{bmatrix}\dot{x}_i^1\\ \dot{x}_i^2\end{bmatrix} = -\check{x}_i\left(\sqrt{\alpha_i+\beta_i^s g_i^s(\xi)+\beta_i^d g_i^d(\xi)}+\gamma_i\right)-\sum_{j=1}^{N} N_{ij}^i(\check{x}_j-\xi_j) \tag{15}$$

Velocity and angular velocity states equation becomes

$$\begin{bmatrix}V_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}\cos(\theta_i) & sin(\theta_i)\\ -\frac{1}{a}sin(\theta_i) & \frac{1}{a}cos(\theta_i)\end{bmatrix}\begin{bmatrix}k(x_i^1-X_i-a_i cos(\theta_i)+\dot{x}_i^1)\\ k(x_i^2-Y_i-a_i sin(\theta_i)+\dot{x}_i^2)\end{bmatrix} \tag{16}$$

The coordinates of the center of mass of each robot may be written as

$$P_i = (X_i + a_i\cos(\theta_i) + Y_i + a_i sin(\theta_i)) \tag{17}$$

The equations describe the dynamics of each agent's center of mass when the aforementioned relation is differentiated with regard to time.

$$\dot{p}_i = \begin{bmatrix}\cos(\theta_i) & -a_i sin(\theta_i)\\ sin(\theta_i) & a_i\cos(\theta_i)\end{bmatrix} \triangleq T_i(\theta_i)\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} \tag{18}$$

3.2. Control Design

Problem formulation: Constructing a control barrier function (CBF) that recovers the entire forward invariance part of the safe set is difficult. However, constructing a CBF that only recovers a subset of the safe set is shown in Figure 5.

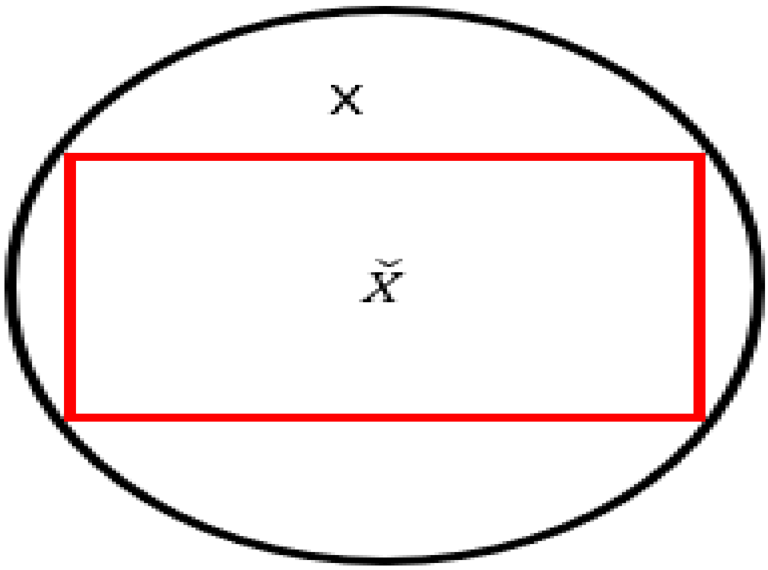

Figure 5. Venn diagram of the problem formulation [22]. Black circle line indicates initial state of the wheeled mobile robot, and red rectangle is new updated state of the wheeled mobile robot.

Define the zero-superlevel set of the handcrafted control barrier function (HCBF) as $\breve{x} = \left\{ x \middle| \breve{h}(x) \geq 0 \right\} \subseteq x$; without loss of generality, it can assume the relationship $h(x) = \breve{h}(x) + \Delta h(x)$. Based on preference ($u_{perf}$) algorithms (CBF-QP), as shown in Figure 6, the algorithms are transferred to the wheeled mobile robot system (plant) if both model prediction control and learned CBF-QP are safe. The system then relays the feedback back to the reference and performs a rollout check using CBF-QP. The control algorithms evaluated whether the feedback signal was safe to roll out to the preferred setting based on the feedback signals [23]. The wheeled mobile robot's signals would be sent to a model-based rollout with a data buffer and input preference. Wheeled mobile robot signals are sent, checked for unsafe rollout using CBF-QP, and then sent to the data buffer. Data buffer would be used for model-based rollout with input preference [24].

Let us look at a state trajectory of the time-invariant, continuous-time controlled system with a disturbance.

$$\begin{cases} \dot{X}(s) = f(X(s), u(s), d(s), s \in [t, t'] \\ x(t) = x \end{cases} \tag{19}$$

where t and x are the initial time and state. $u \in U \subset R^m$ is control inputs, $d \in D \subset R^\omega$ is disturbance, where U and D are compact and convex sets. $f : R^n \times U \times D \to R^n$ is Lipschitz continuous in the state and bounded. The zero-superlevel set of a bounded Lipschitz continuous function $l : R^n \to R$.

$$L = \{x : l(x) \geq 0\} \tag{20}$$

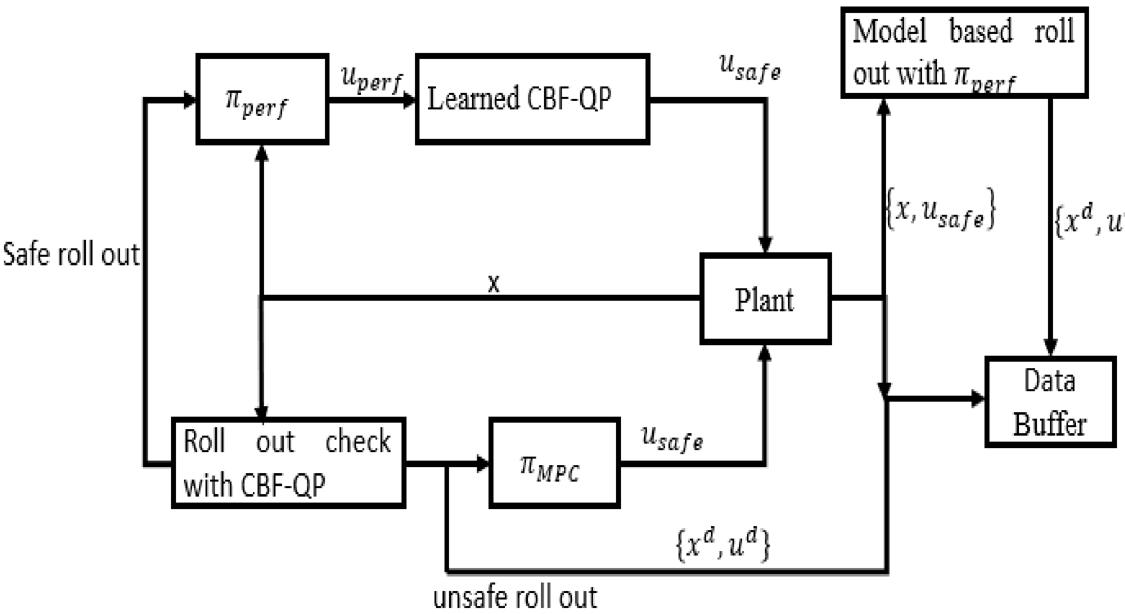

Figure 6. Schematic diagrams of CBF-QP [25].

The Equation (20) where safety target function. The safety control's goal is to ensure that the trajectory stays in L for $s \in [t, 0]$, in the event of the worst disturbance, resolved by the use of Hamilton–Jacobi (HJ) reachability analysis. The cost function could be

$$J(x, t, u(.), d(.)) := \min_{s \in [t,0]} l(X(s)) \tag{21}$$

At some point throughout the time span, the trajectory violated the safety requirement (obtaining a negative value of l), making it unsafe. In the worst scenario, the disturbance would act to reduce J as much as possible, while the goal of the safety control is to make J as large as feasible. This allows for us to define the value function $V : R^n \ (-\infty, 0] \to R$ as

$$V(x, t) := \min_{\xi_d \in E_{[t,0]}} \max_{u \in U_{[t,0]}} J(x, t, u(.), \xi_d[u](.)) \tag{22}$$

The viscosity solution to the subsequent Hamilton–Jacobi–Isaacs variational inequality (HJI-VI) is represented by the value function V (x, t).

$$0 = \min \left\{ \begin{array}{c} l(x) - V(x, t) \\ D_t V(x, t) + maxmin\ D_x V(x, t).f(x, u, d) \end{array} \right\} \tag{23}$$

This indicates that by applying the HJI-VI at each location in the state space, V (x, t) may be directly calculated using dynamic programming backwards in time. The optimal policy $\pi^* V\ (x, t) : R^n \times (-\infty,\ 0] \to U$ is determined in a different way:

$$\left\{ \begin{array}{c} V(x, t) < l(x) \\ \pi^* V(x, t) = \underset{u \in U}{\text{argmax}} \min_{d \in D} D_x V(x, t).f(x, u, d) \end{array} \right. \tag{24}$$

$$k_v(x, t) = \left\{ \mathrm{u} \in \mathrm{U} : \mathrm{D_t V(x, t)} + \min_{d \in D} D_x V(x, t).f(x, u, d) \geq 0 \right. \tag{25}$$

For any optimal:

$$\dot{l}(x(t)) = \dot{V}(x(t), t) \geq 0 \tag{26}$$

For a reference control signal, such a perfect control policy is frequently too tight to serve as a safety filter. To address this, the reachability community frequently uses the so-called least-restrictive control law, which states that the safe optimal control should be used only when $V\ (x(s),\ s)$ is near zero. Suppose that C is a zero-superlevel set of a function that differentiates continuously $B : R^n \to R$. Take into consideration a Lipschitz continuous, disturbance-free controlled system.

$f = f(X(s), u(s))$. If an expanded class exists, then B is a control barrier function for the system k_∞.

$$\max_{u \in U} D_x B(x).f(x,u) \geq - \propto (B(x)) \tag{27}$$

C is any Lipschitz continuous controller, and the zero-superlevel $\pi : C \to U$, such that $\pi(x) \in k_B(x)$

$$k_B(x) := \{u \in U : D_x B(x).f(x,u) \geq - \propto (B(x))\} \tag{28}$$

will make C a forward invariant set. Stated differently, C is control invariant. In order to reduce the norm of the difference between u and the reference control u_{ref}, a controller based on online optimization can use a condition. A control barrier function-based quadratic program (CBF-QP), which may be utilized as an online safety filter for any reference control signal, can be created for control-affine systems.

3.3. Validation of the Control Algorithms (Prove)

Assume that the closed-loop system's equilibrium points are the outcome of applying the control law.

$$\varepsilon = \{0\} \bigcup \varepsilon_{int} \bigcup \varepsilon_{\partial c} \tag{29}$$

where $0 \epsilon R^n$ is the origin of the state space and

$$\varepsilon_{int} = \left\{ x \in \Omega \frac{clf}{cbf} \backslash \{0\} \middle| f(x) = p\gamma(v(x))G(x)\nabla v(x) \right\} \tag{30}$$

$$\varepsilon_{\partial c} = \left\{ x \in \Omega_{cbf}^{clf} \bigcap \partial c \middle| N\left(\begin{bmatrix} f(x)^T \\ \nabla v(x)^T G(x) \\ \nabla h(x)^T G(x) \end{bmatrix}^T \right) \backslash \{0\} \neq \varnothing \right\} \tag{31}$$

where $G(x) = g(x)g(x)^T$, ε_{int} is the set of interior equilibria, and $\varepsilon_{\partial c}$ is the set of boundary equilibria. CLF denotes the active constraints, and CBF the inactive constraints. $\Omega \frac{clf}{cbf}$ are the active states for both CLF and CBF. The interplay between the CLF and CBF constraints is crucial in collision avoidance. The system must ensure stability (via CLF) while avoiding collisions (via CBF), especially in dynamic environments with moving obstacles and other WMRs. The distinction between interior equilibria and boundary equilibria helps in understanding the system's behavior under different conditions. Interior equilibria represent safe and stable states, while boundary equilibria represent critical states where the system is at risk of violating safety constraints.

$$\Omega \frac{clf}{cbf} = \left\{ x \in R^n : \begin{array}{c} x \in R^n : L_f V + \gamma(V) \geq 0, \\ L_g V L_g h^T \left(L_f V + \gamma(V) \right) < \left(L_f h + \alpha(h) \right) \left(p^{-1} + ||L_g V|| \right) \end{array} \right\} \tag{32}$$

The concepts of interior equilibria, boundary equilibria, and states where both CLF and CBF are active are central to the design of control strategies for collision avoidance in WMRs. These concepts help in balancing the competing objectives of stability (via CLF) and safety (via CBF), ensuring that the system operates safely and efficiently in dynamic environments. The mathematical formulations provide a framework for designing control inputs that achieve these objectives, making them essential for real-world applications like smart agriculture and autonomous navigation.

$$\Omega_{cbf}^{clf} = \left\{ \begin{array}{l} x \in R^n : L_g V L_g h^T \left(\frac{L_f h + \alpha(h)}{L_f V + \gamma(V)} \right) \leq ||L_g h||^2, \\ L_g V L_g h^T \geq \left(\frac{L_f h + \alpha(h)}{L_f V + \gamma(V)} \right) \left(||L_g V||^2 + p^{-1} \right) \end{array} \right\} \tag{33}$$

The Lagrangian associated with the control law provides a mathematical framework for deriving optimal control inputs that ensure both stability (via CLF) and safety (via CBF) in collision avoidance for WMRs. By minimizing the control effort while satisfying the CLF and CBF constraints, the Lagrangian enables the system to achieve efficient, stable, and safe navigation in dynamic environments. The Karush–Kuhn–Tucker (KKT) conditions ensure that the optimal solution is both feasible and efficient, making the Lagrangian a powerful tool for control design in complex systems.

$$L = \frac{1}{2}||u||^2 + \frac{1}{2}p\omega^2 + \lambda_1 \Big(L_f V + L_g V u + \gamma(V) - \omega \Big) - \lambda_2 \Big(L_f V + L_g h u + \alpha(h) \Big) \tag{34}$$

The KKT conditions becomes

$$\frac{\partial L}{\partial u} = u + \lambda_1 L_g V - \lambda_2 L_g h = 0 \tag{35}$$

$$\frac{\partial L}{\partial \omega} = p\omega - \lambda_1 = 0 \tag{36}$$

$$\lambda_1 \Big(L_f V + L_g V u + \gamma(V) - \omega \Big) = 0 \tag{37}$$

$$\lambda_2 \Big(L_f V + L_g h u + \alpha(h) \Big) = 0 \tag{38}$$

With $\lambda_1,\ \lambda_2 \geq 0$, it must now differentiate between four distinct scenarios based on when each constraint is activated.

Case 1. *Both restrictions are passive in this instance, meaning* $L_f V + L_g V u + \gamma(V) - \omega < 0$, $L_f V + L_g h u + \alpha(h) > 0$, *and* $\lambda_1 = \lambda_2 = 0$. *From Karush–Kuhn–Tucker (KKT) conditions,* $k(x) = 0$, $\omega(x) = 0$. *However, since* $\gamma(V) < 0$, *this solution is never applicable in this situation.*

Case 2. *Only the CLF constraint is in effect in this instance, which means* $L_f V + L_g V u + \gamma(V) = \delta$, $L_f V + L_g h^T u + \alpha(h) > 0$, *and* $\lambda_1 \geq 0,\ \lambda_2 = 0$; *then, the solution:*

$$k(x) = -\frac{L_f V + \gamma(V)}{p^{-1} + ||L_g V||^2} L_g V^T \tag{39}$$

CBF is inactive; then, from the equilibrium points:

$$f_{cl}(x) = f(x) - \frac{L_f V + \gamma(V)}{p^{-1} + ||L_g V||^2} G \nabla V = 0 \tag{40}$$

There are two possible solutions from Equation (40):

(i). $f(x) = 0$, *and* $G(x)\nabla V(x) = 0$ *for some* $x \in \Omega_{cbf}^{clf}$;

(ii). $f(x) = G(x)\nabla V(x)$; *then, the Equation (40) becomes:*

$$k(x) = p\gamma(V), x \in \Omega\frac{clf}{cbf} \tag{41}$$

Such that the equilibrium points could be

$$f(x) = p\gamma(V)G\nabla V \tag{42}$$

Equation (42) provides set of interior equilibria.

Case 3. *CBF constraint is active,* $L_f V + L_g V u + \gamma(V) - \delta < 0,\ L_f h + L_g h u + \alpha(h) = 0,$ $\lambda_1 = 0,\ \lambda_2 \geq 0$. *Then, the KKT condition becomes:*

$$k(x) = -\left|\left|L_g h\right|\right|^{-2}\left(L_f h + \alpha(h)\right)L_g h^T \tag{43}$$

CLF is inactive; then:

$$\Omega_{cbf}^{\overline{clf}} = \left\{ \begin{array}{c} x\epsilon R^n : L_f h + \alpha(h) \leq 0, \\ L_g V L_g h^T\left(L_f h + \alpha(h)\right) > \left(L_f V + \gamma(V)\right)\left|\left|L_g h\right|\right|^2 \end{array} \right\} \tag{44}$$

At the equilibrium points $f_{cl}(x) = 0$; *then*:

$$f_{cl}(x) = f(x) - \left|\left|L_g h\right|\right|^{-2}\left(L_f h + \alpha(h)\right)G\nabla h = 0 \tag{45}$$

The solutions of Equation (45):

(i) $f(x) = 0$, *and* $G(x)\nabla V(x) = 0$ *for some* $x \in \Omega_{cbf}^{\overline{clf}}$.

(ii). $f(x) = kG(x)\nabla V(x)$; *then, Equation (45) becomes* $\alpha(h) = 0$, *for* $x \in \partial \mathbb{C}$.

Case 4. *When both constraints are active, then* $L_f V + L_g V^T u + \gamma(V) - \delta = 0,\ L_f h + L_g h^T u + \alpha(h) = 0,\ \lambda_1,\ \lambda_2 \geq 0$; *then, the KKT equation becomes*

$$\begin{bmatrix} p^{-1} + \left|\left|L_g V\right|\right|^2 & -L_g V L_g h^T \\ L_g V L_g h^T & -\left|\left|L_g h\right|\right|^2 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_f V + \gamma(V) \\ L_f h + \alpha(h) \end{bmatrix} \tag{46}$$

The determinant of the matrix on the left-side of (46):

$$\Delta = \left(L_g V L_g h^T\right)^2 - \left(p^{-1} + \left|\left|L_g V\right|\right|^2\right)\left(\left|\left|L_g h\right|\right|^2\right) \tag{47}$$

Consider two cases:

(i). $\Delta = 0$ *; then, the matrix on the left side of the Equation (47) loses rank when* $L_g h = 0$. *Then, the solution* $L_g h + \alpha(h) = 0$. *The KKT equation becomes equal as Equation (39).*

(ii). $\Delta < 0$; *then, KKT equation becomes*

$$k(x) = -\lambda_1 L_g V^T + \lambda_2 L_g h^T \tag{48}$$

λ *values are obtained as*

$$\left\{ \begin{array}{c} \lambda_1 = \frac{1}{\Delta}\left(\left(L_f h + \alpha(h)\right)L_g V L_g h^T - \left(L_f V + \gamma(V)\right)\left|\left|L_g h\right|\right|^2\right) \\ \lambda_2 = \frac{1}{\Delta}\left(\left(L_f h + \alpha(h)\right)\left(\left|\left|L_g V\right|\right|^2 + p^{-1}\right) - \left(L_f V + \gamma(V)\right)L_g V L_g h^T\right) \end{array} \right. \tag{49}$$

When $\Delta = 0$ *or* $\Delta > 0$, *then the solution becomes*

$$\Omega_{cbf}^{clf} = \left\{ \begin{array}{c} x\epsilon R^n : L_g V L_g h^T\left(\frac{L_f h + \alpha(h)}{L_f V + \lambda(V)}\right) \leq \left|\left|L_g h\right|\right|^2, \\ L_g V L_g h^T \geq \left(\frac{L_f h + \alpha(h)}{L_f V + \lambda(V)}\right)\left(\left|\left|L_g h\right|\right|^2 + p^{-1}\right) \end{array} \right\} \tag{50}$$

The equilibrium conditions become

$$f_{cl}(x) = f(x) - \lambda_1 G\nabla V + \lambda_2 G\nabla h = 0 \tag{51}$$

The following fundamental criteria for the occurrence of a valid solution can be inferred from Equation (51).

(i). $f(x) = 0$, *for all* $x \in R^n$, *and* $\nabla V(x)||\nabla h(x)$;
(ii). $\nabla h = 0$ *or* $\nabla h \in N(G)$, *and* $f(x)||G(x)\nabla V(x)$;
(iii). $\nabla V = 0$, *or* $\nabla V \in N(G)$, *and* $f(x)||G(x)\nabla h(x)$;
(iv). $\nabla V(x)||\nabla h(x)$, $f(x)||G(\mathrm{x})\nabla h(\mathrm{x})$.

These solutions satisfy the equilibrium points.

4. Results and Discussion

In agricultural environments, collision avoidance for wheeled mobile robots (WMRs) relies on situational awareness and visual cues to navigate through complex and dynamic conditions. As illustrated in Figure 7, the WMRs successfully avoid collisions by dynamically maneuvering around obstacles, demonstrating the effectiveness of the proposed control algorithms.

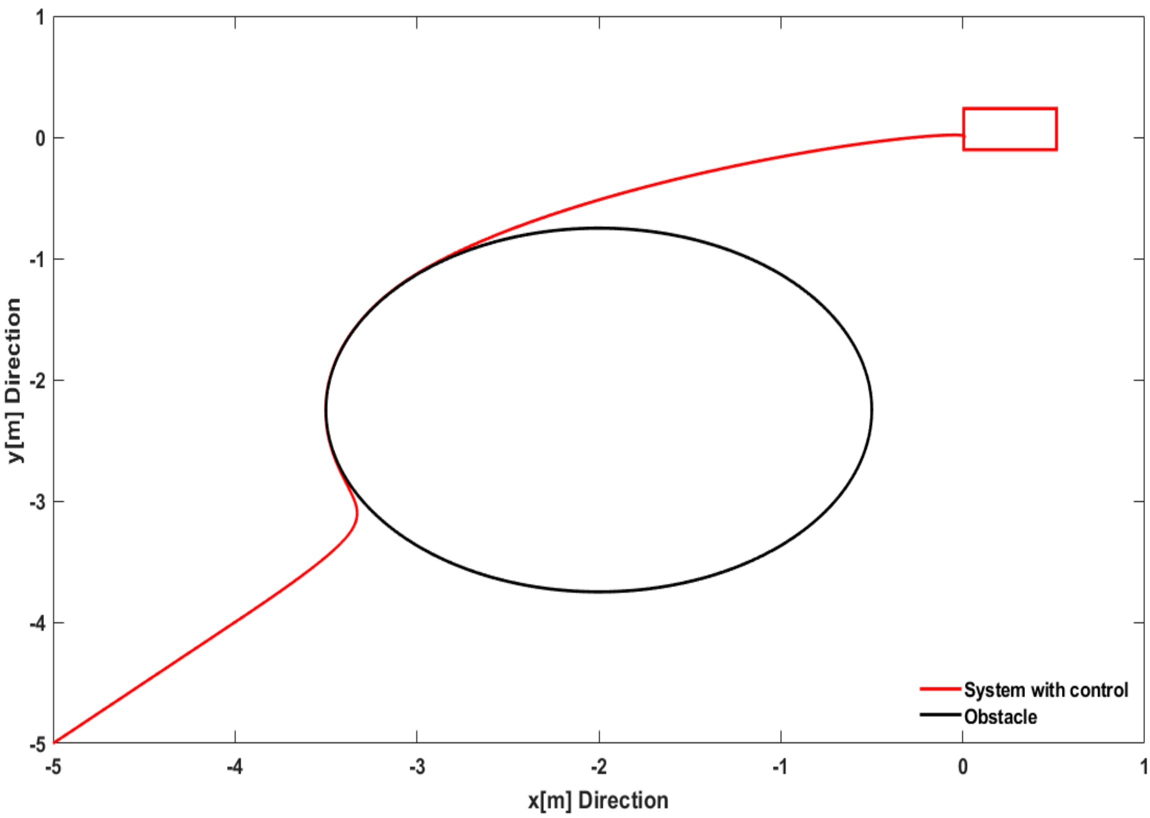

Figure 7. Wheeled mobile robotic collision avoidance at single obstacle.

The graphical representation of the agricultural area provides a clear visualization of the system's operation. Black circles denote potential barriers or obstacles, while red lines represent the trajectories generated by the control algorithms guiding the WMRs. To enhance clarity, both the vertical and horizontal axes are labeled in meters, providing a precise spatial context for the WMRs' movements.

Initially, the WMR follows a straight path toward its destination. However, upon detecting an obstacle, it dynamically adjusts its trajectory, opting for an efficient shortcut to avoid collisions while maintaining its course. The obstacles are strategically positioned across the first and second quadrants, while the WMRs operate primarily in the third quadrant, as indicated by the negative values on the axes. This spatial arrangement highlights the system's ability to navigate complex environments, even when obstacles are distributed across multiple regions.

The ability of the WMRs to dynamically adapt their paths in real time underscores the robustness of the proposed control algorithms. By leveraging situational awareness and visual cues, the system ensures efficient and collision-free navigation, making it highly suitable for agricultural applications where dynamic obstacles and confined spaces are common.

Wheeled mobile robots (WMRs) are designed to navigate agricultural environments without halting upon encountering obstacles or impacts. Instead, they dynamically adjust their trajectories to continue moving toward their intended positions. This capability requires WMRs to maintain appropriate stability and possess the ability to traverse obstacles effectively, as depicted in Figure 8. The position and velocity of the WMRs exhibit nonlinear characteristics during navigation, particularly when avoiding obstacles or recovering from disturbances, until they ultimately reach their destination.

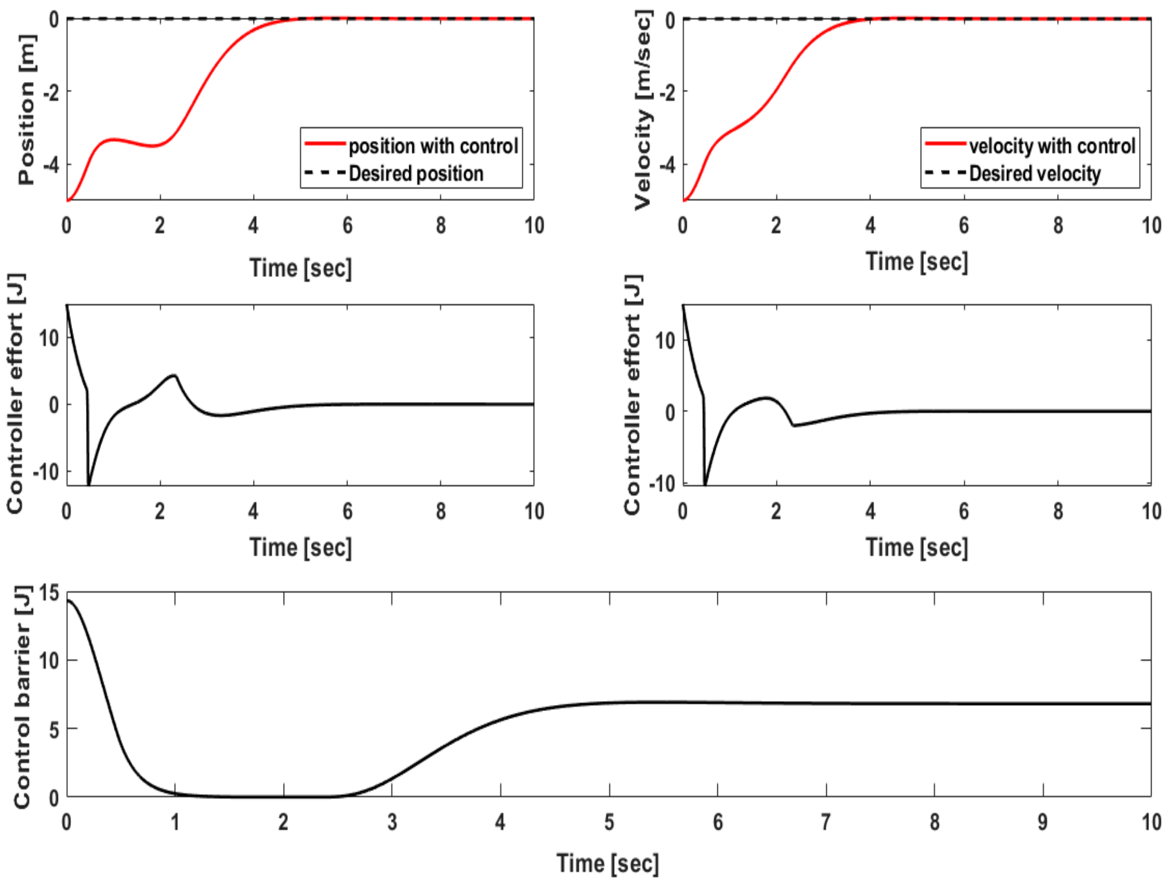

Figure 8. Wheeled mobile collision avoidance at single obstacle states response.

When a WMR experiences an impact, it may undergo temporary instability. However, the system's states are quickly stabilized through the instantaneous activation of control barrier mechanisms. The control algorithms expend a specific amount of energy to restore the system to its initial conditions. A positive control effort is applied to drive the states back to their desired values, while a negative control effort dissipates excess energy, ensuring the system remains stable and efficient.

Initially, the control barriers are reduced, occasionally reverting to their initial conditions, before gradually increasing until equilibrium is achieved. Over time, both the control barriers and the control effort decline, but at different rates: the control barriers decrease moderately, while the control effort diminishes rapidly. This differential reduction ensures that the system maintains stability and safety while minimizing energy consumption.

Control barrier functions (CBFs) are essential mathematical tools in control theory, designed to enforce safety requirements during system operation. Analogous to how Lyapunov functions ensure stability, CBFs provide a systematic framework to guarantee that a system operates within predefined safe bounds. By integrating CBFs into control algorithms, robotic systems achieve enhanced safety and autonomy, particularly in dynamic and unpredictable environments such as agricultural fields.

When a double-wheeled mobile robot (DWMR) encounters an impact, it actively employs collision avoidance strategies, as illustrated in Figure 9. In this scenario, the environment is characterized by asymmetry, with two obstacles strategically positioned within the agricultural area. The black circles represent potential obstructions, while the red line depicts the trajectory of the DWMR, guided by advanced control algorithms. Both the horizontal and vertical axes are clearly labeled, providing a precise spatial representation of the scenario and highlighting the robot's ability to navigate complex environments.

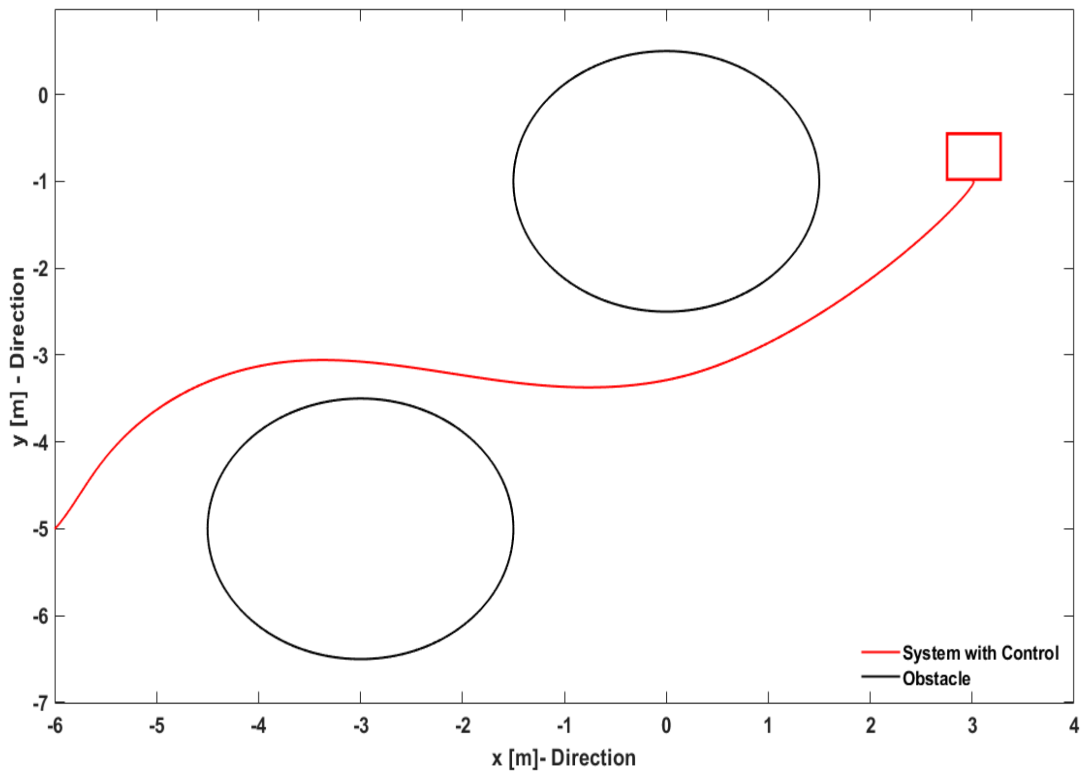

Figure 9. Wheeled mobile robotic collision avoidance at double obstacle.

The development and integration of control barrier functions (CBFs) have revolutionized the implementation of safety constraints in robotic systems. Unlike traditional methods, which often rely on overly conservative constraints, CBFs enable a less restrictive and more flexible approach to ensuring safety. This adaptability is particularly crucial in dynamic environments, where the system must respond to unpredictable changes while maintaining robust safety guarantees. By enforcing specific state constraints, CBFs ensure that the system operates within predefined safe bounds, significantly reducing the risk of collisions even in highly nonlinear and uncertain scenarios.

The effectiveness of CBFs lies in their ability to provide real-time safety guarantees without compromising the system's performance. For instance, in the case of the DWMR, the control algorithms dynamically adjust the robot's trajectory to avoid obstacles while minimizing deviations from the desired path. This is achieved by continuously evaluating the system's state and applying corrective actions to satisfy the CBF constraints. The result is a highly adaptive and resilient system capable of operating safely in complex and asymmetric environments, such as agricultural fields with uneven terrain and dynamic obstacles.

Moreover, the use of CBFs enhances the scalability of multi-robot systems, where multiple DWMRs must navigate shared spaces without interfering with one another. By ensuring that each robot adheres to its safety constraints, CBFs enable efficient coordination and collision-free operation, even in densely populated environments. This capability is particularly valuable in agricultural applications, where multiple robots may be deployed simultaneously to perform tasks such as planting, harvesting, or monitoring.

In summary, the integration of CBFs into the control framework of DWMRs represents a significant advancement in robotic safety and autonomy. By providing a flexible and robust mechanism for enforcing safety constraints, CBFs enable robots to navigate dynamic and asymmetric environments with confidence, significantly reducing the risk of collisions and enhancing overall system performance. This approach not only improves the reliability of robotic systems in agriculture but also paves the way for their broader adoption in other safety-critical applications.

Figure 10 presents the simulation results of a wheeled mobile robot (WMR) navigating a scenario with double obstacles. The results reveal distinct recovery patterns for the WMR's velocity and position states: the velocity state exhibits a nonlinear recovery, while the position state is restored linearly. As depicted in Figure 9, the control effort demonstrates a rapid decline for velocity regulation but decreases more gradually for position stabilization. This difference highlights the varying energy requirements for controlling different states of the system. Notably, the controller barrier gains exhibit significant variations, with the first barrier requiring substantial energy to restore the WMR to its initial position and velocity. This underscores the dynamic nature of the control system and its ability to adapt to complex scenarios.

The use of control barrier function (CBF)-based algorithms represents a significant advancement over conventional approaches, offering efficient, real-time obstacle avoidance capabilities. These algorithms are particularly advantageous in dynamic environments, where traditional methods often struggle to maintain safety and performance. By integrating CBFs into control systems, the WMR achieves globally asymptotically stable tracking, ensuring reliable and safe trajectories even in the presence of external disturbances. This stability is critical for applications in unpredictable environments, such as agricultural fields, where obstacles and terrain conditions can change rapidly.

Moreover, CBFs have demonstrated considerable potential in hybrid system verification, a critical aspect of ensuring that robotic systems adhere to predefined safety standards throughout their operation. Hybrid systems, which combine continuous and discrete dynamics, are inherently complex and challenging to verify. However, CBFs provide a systematic framework for enforcing safety constraints across both types of dynamics, ensuring that the system remains within safe operational bounds at all times. This capability is particularly valuable in safety-critical applications, where even minor deviations from safe behavior can lead to catastrophic outcomes.

The ability of CBF-based algorithms to handle nonlinearities and disturbances makes them highly suitable for real-world applications. For instance, in the case of the WMR navigating double obstacles, the nonlinear recovery of velocity and the linear restoration of position demonstrate the system's robustness and adaptability. The rapid decline in control effort for velocity regulation, coupled with the gradual reduction for position stabilization, reflects the efficient allocation of energy to achieve optimal performance. Additionally, the variations in controller barrier gains highlight the system's ability to dynamically adjust its control strategies based on the specific requirements of the scenario.

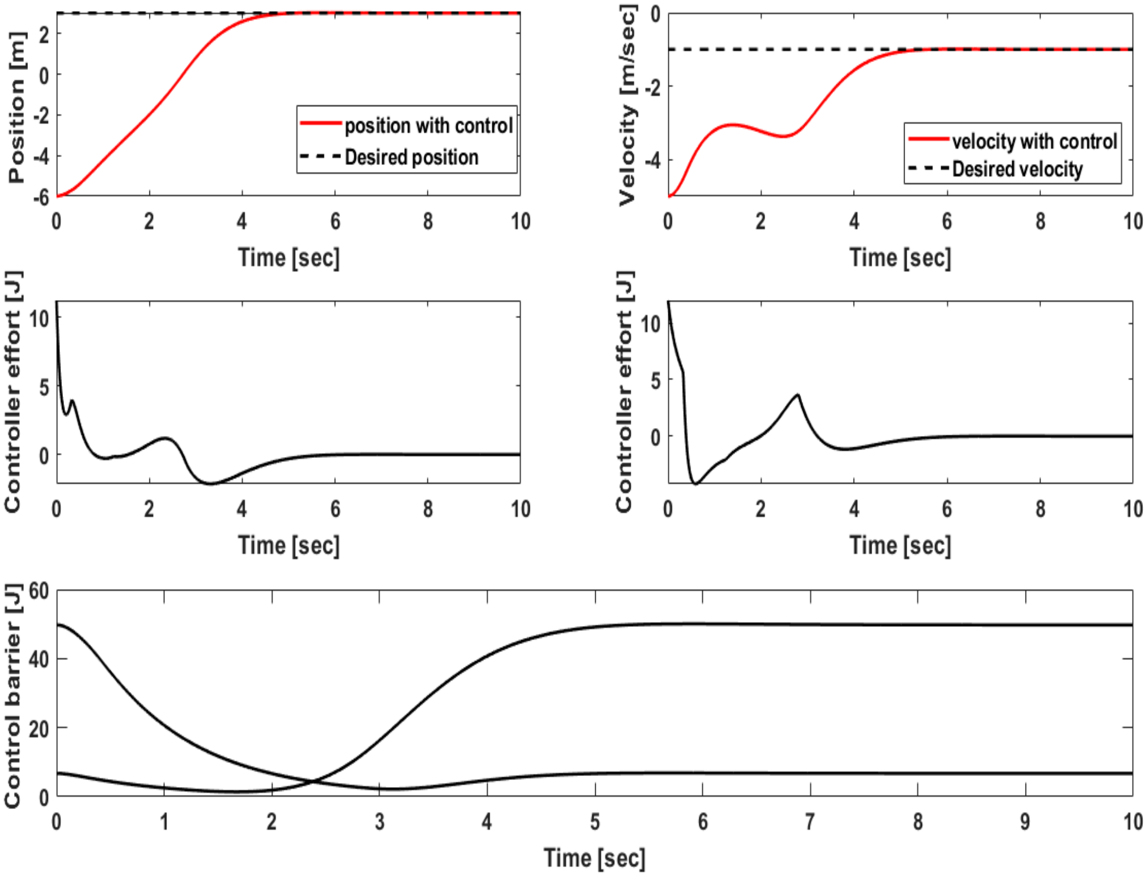

Figure 10. Wheeled mobile collision avoidance at double obstacle states response.

Table 1 highlights the significant performance improvements achieved by the control barrier function quadratic programming (CBF-QP) approach compared to the rapidly exploring random tree (RRT) method for wheeled mobile robots (WMRs). For WMR position, the CBF-QP approach demonstrates improvements of 11% in settling time, 34% in rise time, and 75% in steady-state error. Similarly, for WMR velocity, the improvements are even more pronounced, with 27% in settling time, 37% in rise time, and 99% in steady-state error. Furthermore, the control gain metrics show substantial enhancements, with 50% in settling time, 59% in rise time, and 93% in steady-state error when using CBF-QP over RRT. These results underscore the superior efficiency and precision of the CBF-QP approach in optimizing the performance of WMRs.

Control barrier functions (CBFs) are widely recognized for their ability to ensure obstacle avoidance and stabilize dynamic systems by defining rigorous safety constraints in motion planning. CBFs operate within a quadratic programming framework, where they enforce safety by ensuring that the system remains within predefined safe limits. This approach is particularly effective in dynamic environments, where real-time adaptability and safety are paramount. By integrating CBFs into control algorithms, robotic systems can navigate complex scenarios while maintaining robust safety guarantees, even in the presence of disturbances or unexpected obstacles.

Table 1. Comparisons of CBF and RRT.

Specification of WMR	WMR with CBF-QP			WMR with Rapidly Exploring Random Tree (RRT) [26]			% Change in CBF-QP over RRT		
	Settling Time (s)	Rise Time (s)	Error	Settling Time (s)	Rise Time (s)	Error	Settling Time (s)	Rise Time (s)	Error
Position (m)	0.98	0.67	0.005	1.1	1.02	0.02	11%	34%	75%
Velocity (m/s)	0.88	0.66	0.003	1.2	1.05	0.5	27%	37%	99%
Control gain (J)	10	9	0.05	20	22	0.7	50%	59%	93%

In contrast, RRTs are sampling-based motion planning algorithms that excel in exploring and identifying feasible paths in high-dimensional and complex environments. RRTs are designed to efficiently discover collision-free paths by randomly sampling the configuration space and incrementally building a tree of possible trajectories. While RRTs are highly effective for path planning in static or known environments, they lack the inherent ability to enforce real-time safety constraints, making them less suitable for dynamic or uncertain scenarios.

The key distinction between CBF-QP and RRT lies in their primary objectives. CBF-QP prioritizes safety and stability by rigorously enforcing constraints that ensure the system operates within safe bounds. This makes it particularly well suited for autonomous systems operating in dynamic environments, where safety is a critical concern. On the other hand, RRT focuses on path discovery and exploration, making it ideal for applications where the primary challenge is navigating complex, high-dimensional spaces.

The performance improvements demonstrated by CBF-QP, as shown in Table 1, highlight its advantages over RRT in terms of settling time, rise time, and steady-state error. These metrics reflect the system's ability to achieve faster and more accurate responses while maintaining stability and safety. The significant reduction in steady-state error, in particular, underscores the precision of the CBF-QP approach in achieving desired trajectories and avoiding deviations caused by external disturbances.

Generally, the CBF-QP approach offers a robust and efficient solution for enhancing the performance of WMRs in dynamic environments. By leveraging the strengths of CBFs in enforcing safety constraints and quadratic programming in optimizing control inputs, this approach outperforms traditional methods like RRT in critical performance metrics. While RRT remains a powerful tool for path planning in complex environments, CBF-QP provides a more comprehensive framework for ensuring safety and stability in real-time applications, making it a preferred choice for autonomous systems operating in dynamic and uncertain settings.

Table 2 highlights the significant performance improvements achieved by the control barrier function quadratic programming (CBF-QP) approach compared to adaptive model predictive control (AMPC) for wheeled mobile robots (WMRs). The results demonstrate substantial enhancements across key metrics: the WMR's position and velocity improved by 42% and 54%, respectively, while the control effort gain saw a remarkable improvement of 90%. Furthermore, the rise time for position improved by 58%, for velocity by 61%, and for control effort gain by 92%. Additionally, the steady-state error for position improved by 98%, for velocity by 99%, and for control effort gain by 94% when using CBF-QP over AMPC. These improvements underscore the superior efficiency and precision of the CBF-QP approach in optimizing the performance of WMRs.

Table 2. Comparisons of CBF and adaptive model productive control (AMPC).

Specification of WMR	WMR with CBF-QP			WMR with Adaptive Model Productive Control (AMPC) [22]			% Change in CBF-QP over Adaptive Model Productive Control (AMPC)		
	Settling Time (s)	Rise Time (s)	Error	Settling Time (s)	Rise Time (s)	Error	Settling Time (s)	Rise Time (s)	Error
Position (m)	0.98	0.67	0.005	1.7	1.6	0.4	42%	58%	98%
Velocity (m/s)	0.88	0.66	0.003	1.9	1.7	0.3	54%	61%	99%
Control gain (J)	10	9	0.05	100	110	0.9	90%	92%	94%

The exceptional performance of CBF-QP can be attributed to its suitability for real-time, safety-critical applications, such as obstacle avoidance in dynamic environments. By leveraging control barrier functions (CBFs), the CBF-QP approach enforces rigorous safety constraints, ensuring that the system operates within predefined safe limits. This is particularly advantageous in scenarios where real-time adaptability and safety are paramount, such as agricultural fields or autonomous navigation in unpredictable terrains. The ability of CBF-QP to provide real-time safety guarantees while minimizing control effort makes it an ideal choice for applications requiring robust and efficient collision avoidance.

In contrast, AMPC is better suited for applications that prioritize optimal long-term planning and adaptability to changing conditions. AMPC optimizes control inputs over a finite horizon, providing globally optimal or near-optimal solutions for trajectory planning. This makes it highly effective in scenarios where computational resources are available, and the primary challenge is to achieve optimal performance over extended periods. However, the computational complexity of AMPC can be a limiting factor in real-time applications, particularly those with strict timing requirements or limited processing capabilities.

The choice between CBF-QP and AMPC ultimately depends on the specific requirements of the application. For safety-critical tasks in dynamic environments, such as obstacle avoidance or real-time navigation, CBF-QP offers a more efficient and reliable solution. On the other hand, for applications requiring long-term planning and adaptability to changing conditions, AMPC provides a robust framework for achieving optimal performance. In some cases, a hybrid approach that combines the strengths of both methods could be highly beneficial. For instance, integrating CBF-QP for real-time safety enforcement with AMPC for long-term trajectory optimization could yield a comprehensive solution that addresses both safety and optimality.

The performance improvements demonstrated by CBF-QP, as highlighted in Table 2, reflect its ability to achieve faster response times, higher precision, and reduced energy consumption compared to AMPC. These advantages are particularly critical in applications where real-time decision-making and safety are paramount. By providing a systematic framework for enforcing safety constraints and optimizing control inputs, CBF-QP enhances the reliability and efficiency of WMRs in dynamic and complex environments.

In summary, the CBF-QP approach offers a robust and efficient solution for enhancing the performance of WMRs in safety-critical applications, while AMPC excels in scenarios requiring long-term planning and adaptability. The choice between these methods should be guided by the specific requirements of the application, such as the need for real-time safety, computational efficiency, or optimal performance. In some cases, a hybrid approach that leverages the strengths of both CBF-QP and AMPC could provide a comprehensive solution, enabling WMRs to operate safely and efficiently in a wide range of environments.

5. Conclusions

This paper presents a collision avoidance system for smart agriculture using a Control barrier function (CBF)-based approach. The system enables wheeled mobile robots (WMRs) to navigate agricultural fields safely by monitoring speed, distance, and potential hazards, thereby reducing the risk of collisions. Collision avoidance systems (CASs) are critical in agricultural environments, where the presence of static obstacles, dynamic conditions, and multiple agents necessitates robust safety mechanisms. By automatically applying braking mechanisms when collisions become unavoidable, these systems minimize the impact of accidents, reducing injuries, damage to WMRs, and crop losses.

This study focuses on enhancing the safety and collision mitigation capabilities of smart agriculture systems through the integration of a CBF-based controller into the WMR framework. The multi-agent collision avoidance problem is addressed within a centralized framework, where each agent has continuous access to the positions of all other group members. This setup considers a set of N WMRs operating on the Euclidean plane, navigating around static obstacles. Each WMR is modeled as a unicycle with a passive wheel, a common representation for mobile robots due to its simplicity and applicability to real-world scenarios.

Designing a CBF that ensures the full forward invariance of the safe set is a challenging task. However, constructing a CBF that partially recovers the safe set is more feasible and practical for real-world applications. The control algorithms utilize feedback signals to determine whether it is safe to execute desired control actions. These signals are analyzed for safety using CBF quadratic programming (CBF-QP) and then sent to a data buffer for model-based rollout with input selection. This approach ensures that the system operates within safe bounds while maintaining efficiency and responsiveness.

In agricultural environments, collision avoidance relies on WMRs leveraging visual signals and situational awareness to navigate through single and double conflicting conditions. Initially, the control barriers are reduced and periodically adjusted to initial conditions before gradually increasing until equilibrium is achieved. Over time, both the control barriers and the control effort decrease, but at different rates: the control barriers decrease moderately, while the control effort diminishes rapidly. This differential reduction ensures that the system maintains stability and safety while minimizing energy consumption.

Control barrier functions (CBFs) are mathematical tools in control theory that enforce safety requirements during system operation, analogous to how Lyapunov functions ensure stability. By providing a systematic framework to guarantee that a system operates within predefined safe bounds, CBFs significantly enhance the safety and autonomy of robotic systems. This is particularly important in dynamic and unpredictable environments, such as agricultural fields, where obstacles and terrain conditions can change rapidly.

Performance comparisons demonstrate that the CBF-QP approach outperforms both rapidly exploring random trees (RRT) and adaptive model predictive control (AMPC). For WMR position, CBF-QP improves settling time by 11%, rise time by 34%, and steady-state error by 75%. For WMR velocity, the improvements are even more pronounced, with 27% in settling time, 37% in rise time, and 99% in steady-state error. Compared to AMPC, CBF-QP achieves a 98% improvement in steady-state error for position, 99% for velocity, and 94% for control effort gain. These results highlight the superior efficiency and precision of the CBF-QP approach, making it well suited for collision avoidance in smart agriculture.

The authors recommend further exploration of a hybrid approach that integrates CBF with RRT (CBF-RRT) to enhance motion planning. This combination leverages the safety guarantees of CBFs with the efficient exploration capabilities of RRTs, aiming to generate stable, collision-free trajectories. Such a hybrid approach could address the limitations of individual methods, providing a comprehensive solution for safe and efficient navigation in complex environments.

Generally, this study demonstrates the effectiveness of the CBF-based approach in improving the safety and performance of WMRs in smart agriculture. By integrating CBFs into the control framework, the system achieves robust collision avoidance, enhanced stability, and efficient energy utilization. The proposed method outperforms traditional approaches like RRT and AMPC, making it a promising solution for real-world agricultural applications. The potential for a CBF-RRT hybrid approach further underscores the versatility and adaptability of this framework, paving the way for future advancements in robotic autonomy and safety.

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References

1. Mylvaganam, T.; Sassano, M. Autonomous collision avoidance for wheeled mobile robots using a differential game approach. *Eur. J. Control* **2018**, *40*, 53–61. [CrossRef]
2. Xu, T. Recent advances in Rapidly-exploring Random Tree: A Review. *Heliyon* **2024**, *10*, e32451. Available online: https://www.cell.com/heliyon/fulltext/S2405-8440(24)08482-2 (accessed on 18 November 2024). [CrossRef] [PubMed]
3. Umari, H.; Mukhopadhyay, S. Autonomous robotic exploration based on multiple rapidly-exploring randomized trees. In Proceedings of the 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Vancouver, BC, Canada, 24–28 September 2017; IEEE: Piscataway, NJ, USA, 2017; pp. 1396–1402. Available online: https://ieeexplore.ieee.org/abstract/document/8202319/ (accessed on 18 November 2024).
4. Sairoel, A.; Gebresenbet, G.; Alwan, H.M. *Optimizing the Performance of a Wheeled Mobile Robot for Use in Agriculture*; IntechOpen: London, UK, 2024.
5. Xiao, W.; Belta, C. Control barrier functions for systems with high relative degree. In Proceedings of the 2019 IEEE 58th Conference on Decision and Control (CDC), Nice, France, 11–13 December 2019; IEEE: Piscataway, NJ, USA, 2019; pp. 474–479. Available online: https://ieeexplore.ieee.org/abstract/document/9029455/ (accessed on 18 November 2024).
6. Taylor, A.J.; Ames, A.D. Adaptive safety with control barrier functions. In Proceedings of the 2020 American Control Conference (ACC), Denver, CO, USA, 1–3 July 2020; IEEE: Piscataway, NJ, USA, 2020; pp. 1399–1405. Available online: https://ieeexplore.ieee.org/abstract/document/9147463/ (accessed on 18 November 2024).
7. Sairoel, A.; Gebresenbet, G.; Alwan, H.M. Optimizing the performance of a wheeled mobile robots for use in agriculture using a linear-quadratic regulator. *Robot. Auton. Syst.* **2024**, *174*, 104642.
8. Xu, X.; Tabuada, P.; Grizzle, J.W.; Ames, A.D. Robustness of control barrier functions for safety critical control. *IFAC-PapersOnLine* **2015**, *48*, 54–61. [CrossRef]
9. Robey, A.; Hu, H.; Lindemann, L.; Zhang, H.; Dimarogonas, D.V.; Tu, S.; Matni, N. Learning control barrier functions from expert demonstrations. In Proceedings of the 2020 59th IEEE Conference on Decision and Control (CDC), Jeju, Republic of Korea, 14–18 December 2020; IEEE: Piscataway, NJ, USA, 2020; pp. 3717–3724. Available online: https://ieeexplore.ieee.org/abstract/document/9303785/ (accessed on 18 November 2024).
10. Ames, A.D.; Coogan, S.; Egerstedt, M.; Notomista, G.; Sreenath, K.; Tabuada, P. Control barrier functions: Theory and applications. In Proceedings of the 2019 18th European control conference (ECC), Naples, Italy, 25–28 June 2019; IEEE: Piscataway, NJ, USA, 2019; pp. 3420–3431. Available online: https://ieeexplore.ieee.org/abstract/document/8796030/ (accessed on 18 November 2024).

11. Rauscher, M.; Kimmel, M.; Hirche, S. Constrained robot control using control barrier functions. In Proceedings of the 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Daejeon, Republic of Korea, 9–14 October 2016; IEEE: Piscataway, NJ, USA, 2016; pp. 279–285. Available online: https://ieeexplore.ieee.org/abstract/document/7759067/ (accessed on 18 November 2024).
12. Rodriguez, S.; Tang, X.; Lien, J.-M.; Amato, N.M. An obstacle-based rapidly-exploring random tree. In Proceedings of the Proceedings 2006 IEEE International Conference on Robotics and Automation, 2006. ICRA 2006, Orlando, FL, USA, 15–19 May 2006; IEEE: Piscataway, NJ, USA, 2006; pp. 895–900. Available online: https://ieeexplore.ieee.org/abstract/document/1641823/ (accessed on 18 November 2024).
13. Sairoel, A.; Gebresenbet, G.; Alwan, H.M.; Vladmirovna, K.O. Assessment of smart mechatronics applications in agriculture: A review. *Appl. Sci.* **2023**, *13*, 7315. [CrossRef]
14. Sairoel, A.F.; Gebresenbet, G.; Alwan, H.M. Utilizing an internet of things (IoT) device, intelligent control design, and simulation for an agricultural system. *IoT* **2024**, *5*, 58–78. [CrossRef]
15. Sairoel, A.; Gebresenbet, G.; Alwan, H.M. Modeling of unmanned aerial vehicles for smart agriculture systems using hybrid fuzzy PID controllers. *Appl. Sci.* **2024**, *14*, 3458. [CrossRef]
16. Zagare, F.C. Game Theory: Concepts and Applications. 1984. Available online: https://books.google.com/books?hl=en&lr=&id=YLuwr8HqbBEC&oi=fnd&pg=PA5&dq=game+theory&ots=65TZLWGW59&sig=brTo-hGLow7gBACnmbh9Td_JJfk (accessed on 18 November 2024).
17. Zhai, X.; Chu, X.; Chai, C.S.; Jong, M.S.; Istenic, A.; Spector, M.; Liu, J.B.; Yuan, J.; Li, Y. A Review of Artificial Intelligence (AI) in Education from 2010 to 2020. *Complexity* **2021**, *2021*, 8812542. [CrossRef]
18. Russell, S.; Norvig, P. AI a modern approach. *Learning* **2005**, *2*, 4.
19. Jordan, M.; Perez, A. Optimal Bidirectional Rapidly-Exploring Random Trees. 2013. Available online: https://dspace.mit.edu/bitstream/handle/1721.1/79884/MIT-CSAIL-TR-2013-021.pdf (accessed on 18 November 2024).
20. Sairoel, A.; Gebresenbet, G. Annual comprehensive on hyper-automation technology application in farming. *Smart Agric. Technol.* **2024**, *9*, 100516.
21. Bai, Y.; Zhang, B.; Xu, N.; Zhou, J.; Shi, J.; Diao, Z. Vision-based navigation and guidance for agricultural autonomous vehicles and robots: A review. *Comput. Electron. Agric.* **2023**, *205*, 107584. [CrossRef]
22. Zheng, Y.; Zheng, J.; Shao, K.; Zhao, H.; Xie, H.; Wang, H. Adaptive trajectory tracking control for nonholonomic wheeled mobile robots: A barrier function sliding mode approach. *IEEECAA J. Autom. Sin.* **2024**, *11*, 1007–1021. Available online: https://ieeexplore.ieee.org/abstract/document/10379559/ (accessed on 25 February 2024). [CrossRef]
23. Heirung, T.A.N.; Ydstie, B.E.; Foss, B. Dual adaptive model predictive control. *Automatica* **2017**, *80*, 340–348. [CrossRef]
24. Adetola, V.; DeHaan, D.; Guay, M. Adaptive model predictive control for constrained nonlinear systems. *Syst. Control Lett.* **2009**, *58*, 320–326. [CrossRef]
25. Han, Z.; Narendra, K.S. New concepts in adaptive control using multiple models. *IEEE Trans. Autom. Control* **2011**, *57*, 78–89.
26. LaValle, S.M.; Kuffner, J.J. Rapidly-exploring random trees: Progress and prospects: Steven m. lavalle, iowa state university, a james j. kuffner, jr., university of tokyo, tokyo, japan. In *Algorithmic and Computational Robotics*; Taylor & Francis Group: Abingdon, UK, 2001; pp. 303–307.