



Is forest conservation a socially optimal strategy for increasing forest carbon sequestration?

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ABSTRACT

Previous studies show that the optimal rotation period would be infinitely long when carbon price is sufficiently high, indicating that forest preservation could serve as an optimal strategy for mitigating global warming. This paper examines the impact of the substitution effect of harvested wood products (HWP) and the risk of natural disturbances on the optimality of infinitely long rotation period. Our analysis shows that when the substitution effects of HWP are significant, the optimal rotation remains finite regardless of how high the carbon price is. Conversely, when the substitution effects are minimal, there exists a threshold carbon price beyond which the optimal rotation period becomes infinite. Furthermore, we demonstrate that the risk of natural disturbances can either increase or decrease the likelihood that forest preservation remains the optimal choice for climate change mitigation. A numerical example illustrates that even with conservative assumptions about the substitution effect of HWP, the optimal rotation remains finite, and the risk of forest damage further reduces the optimal rotation.

1. Introduction

The conservation and enhancement of forest carbon stocks are integral to reducing greenhouse gas emissions and achieving the Paris Agreement's goal of limiting global temperature rise to well below 2 °C above pre-industrial levels (UNFCCC, 2015). Many countries include forestry measures in their national climate action plans (UNFCCC, 2024). Recognizing the potential of increasing the carbon sink in the land use, land use change, and forestry (LULUCF) sector, the EU LULUCF regulation (EU, 2023) sets an EU-level net removal target of 310 MtCO₂e by 2030. Measures with additional mitigation potentials include decreased deforestation, increased afforestation, improved forest management, reduced harvesting levels, rewetting of drained soils with a high carbon content, such as peatlands, and improved crop rotation and improved grassland management.

Extending the rotation age is considered a cost-efficient forest management strategy to enhance onsite carbon storage (Ontl et al., 2020; Liski et al., 2001). A widely accepted conclusion in the economics of forest carbon pricing is that the optimal rotation age increases¹ with higher carbon prices (Manley, 2020; Niinimäki et al., 2013; Pukkala, 2011; Stainback and Alavalapati, 2002; van Kooten et al., 1995). This

impact on the rotation age can be further amplified if the carbon price rises over time (Ekholm, 2016). However, the risk of natural disturbances can offset the positive impact of increasing carbon prices on the optimal rotation age (Couture and Reynaud, 2011; Daigneault et al., 2010; Ekholm, 2020; van Kooten et al., 2019).

From a theoretical viewpoint, several studies suggest that the incorporation of carbon benefits could lead to an infinite rotation period. For example, the seminal work by van Kooten et al. (1995) determined that it is optimal never to harvest a forest stand when the carbon benefits surpass a certain threshold. Hoel et al. (2014) extended this model by incorporating multiple forest carbon pools and found that, with interest rates above 1 %, there exists a threshold carbon price beyond which a forest stand should never be harvested.

Empirical studies have also suggested that it might be optimal to manage a forest with an infinite rotation, maintaining continuous cover through periodic selection harvests rather than ceasing harvesting entirely. For example, Assmuth et al. (2018) determined that the optimal rotation length is infinite for boreal spruce stands at a 2 % interest rate and carbon prices of €30/tCO₂, or at 4 % interest rate regardless of the carbon price. Similarly, Assmuth et al. (2021) found that managing mixed stands with continuous cover and frequent selection harvests is

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¹ An exception is the work by Assmuth et al. (2021) and Akao (2011), which found that shorter rotation ages can be optimal under specific conditions—namely, when interest rates are low (Assmuth et al., 2021) and when the rate of carbon release from harvested wood products is slow (Akao, 2011).

optimal at a 3 % interest rate and carbon prices between €0–50/tCO₂.

The recent development of carbon markets has enabled forest offset projects to contribute to reducing greenhouse gas emissions, while also providing economic benefits to forest landowners through payments for carbon sequestration (Li et al., 2021; Sohngen, 2020; Sohngen and Mendelsohn, 2003). Several forest carbon programs require long-term commitments from landowners to ensure forest conservation. For instance, California's compliance carbon offset market program mandates that forest projects must commit to a minimum of 100 years (California Environmental Protection Agency, 2015). Similarly, the New Zealand Emission Trading Scheme's "permanent forestry" option requires that forests cannot be clear-felled for at least 50 years, with the possibility of extending the commitment every 25 years thereafter.² In the U.S., voluntary carbon markets such as the Forest Carbon Works program impose a minimum commitment of 125 years.³ However, the permanence of sequestered carbon may be threatened by natural disturbances such as storms, wildfires, and pest outbreaks (Gren and Zeleke-Aklilu, 2016).

This paper examines the conditions under which the permanent protection of forests is a **socially optimal strategy** for enhancing carbon sequestration in even-aged stands, while considering multiple carbon pools and the substitution effect of harvested wood products (HWP). We also explore the impact of natural disturbances on the optimality of forest preservation. The structure of the paper is as follows: in the next section, we introduce the theoretical framework of the model and derive the optimal conditions for forest protection. We then present numerical simulations of stand growth, carbon pool dynamics, substitution effects of HWP, and the impact of wildfire and storm damage on the stand. The numerical results for the optimal management of the example stand are discussed. Finally, we conclude with a summary of findings and policy implications.

2. Theoretical analysis

2.1. The model

Let us consider the problem of even-aged stand management starting from bare land. A forest stand is established at time $t = 0$, and the growing stock of timber in the forest stand at age t is denoted by $G(t)$, with the instantaneous growth of the growing stock of timber given by $g(t)$. We consider two carbon pools: living biomass and dead organic matter (DOM) which consist of soil organic carbon and litter on forest floor. The carbon stock in harvested wood products (HWP) is also modeled in the calculation of CO₂ emissions that occur following the harvest a forest stand (see Appendix B). Let $\alpha(t)$ denote the amount of carbon in living biomass corresponding to one cubic meter of the growing stock of timber at age t , and $D(t)$ represent the carbon stock in DOM in the stand at age t . Given a carbon price p_c , and a discount rate r , the present value of carbon sequestration benefits of the stand during a period of t years is

$$C_B(t) = \int_0^t p_c [\alpha(z)g(z) + D'(z)] e^{-rz} dz \quad (1)$$

Let λ represent the probability that the forest stand is damaged by a natural event (e.g. wildfire, storm, or insect outbreak) to the extent that it has to be regenerated following a timber salvage harvest. The expected net revenue of a salvage harvest at age t is denoted by $S(t)$. If the stand survives until a predetermined rotation age T , it will be harvested and immediately regenerated. The net revenue of a final felling of the stand at age T is denoted by $V(T)$. Moreover, we assume that factors such as land productivity, management costs, timber and carbon prices as well

as the discount rate remain constant over time. Given a rotation age T , the land expectation value (LEV), which represents the expected net present value (NPV) of growing forest for timber production and CO₂ sequestration during an infinite period, can be calculated as follows

$$LEV(T) = -C + \int_0^T \lambda e^{-\lambda t} [C_B(t) + [S(t) - E_d(t) + LEV(T)] e^{-rt}] dt + C_B(T) e^{-\lambda T} + [V(T) - E_h(T) + LEV(T)] e^{-(r+\lambda)T} \quad (2)$$

where C is the cost of establishing or regenerating the forest stand, $E_d(t)$ and $E_h(T)$ are the cost of net CO₂ emissions resulting from damage to the stand at age t and the net emission cost of an ordinary harvest at the end of the rotation T , respectively. Net emissions refer to the amount of carbon released to the atmosphere from DOM and HWP minus the reduction in emissions due to the production and use of non-wood materials that are replaced by HWP. $E_d(t)$ and $E_h(T)$ represents the "present value" at the time when the stand is damaged or harvested. Therefore, these values are discounted by t and T years, respectively, in Eq. (2) to convert to the present value at time 0.

Solving Eq. (2) for $LEV(T)$ we obtain

$$LEV(T) = \frac{\lambda + r}{r(1 - e^{-(r+\lambda)T})} \left[-C + \int_0^T \lambda e^{-\lambda t} [C_B(t) + [S(t) - E_d(t)] e^{-rt}] dt + C_B(T) e^{-\lambda T} + [V(T) - E_h(T)] e^{-(r+\lambda)T} \right] \quad (3)$$

By including the substitution effect of HWP, the LEV defined in Eq. (3) refers to the present value of the net social benefits of an infinite number of rotations with a rotation age T . The first-order derivative of the land expectation value with respect to the rotation age can be expressed as follows (see the derivation of Eq. (A3) in Appendix A)

$$LEV'(T) = \frac{(\lambda + r)}{r(e^{(r+\lambda)T} - 1)} \left[\lambda [S(T) - E_d(T)] + V'(T) - E'_h(T) + C'_B(T) e^{rT} - (r + \lambda) [V(T) - E_h(T)] - rLEV(T) \right] \quad (4)$$

Assuming it is optimal to harvest and regenerate the stand at a finite rotation age T^* , the optimal rotation age must satisfy the following first-order condition:

$$V'(T^*) + C'_B(T^*) e^{rT^*} = r[V(T^*) - E_h(T^*)] + rLEV(T^*) + \lambda[V(T^*) - E_h(T^*) - S(T^*) + E_d(T^*)] + E'_h(T^*) \quad (5)$$

The left-hand side of Eq. (5) represents the increment in the net revenue from the current rotation following a marginal increase in the rotation age. The right-hand side of Eq. (5) is the marginal cost of extending the rotation age. The sum of the first two terms captures the opportunity cost of occupying the capital (trees and land), the third term represents the expected loss due to the risk of damage to the stand from natural disturbances, and the last term reflects the increase in emission costs.

The main purpose of this paper is to determine whether it is optimal to choose an infinitely long rotation age (i.e. $T^* \rightarrow \infty$). A necessary condition for the optimal rotation to be infinitely long is that the first-order derivative of the land expectation value is greater than zero at any finite rotation age. Let $\bar{T}$ denote the age when the stand reaches a steady state (disregarding the risk of damage). Thus, for any rotation age $T \geq \bar{T}$, $V(T) = V(\bar{T})$, $C_B(T) = C_B(\bar{T})$, $S(T) = S(\bar{T})$, $E_d(T) = E_d(\bar{T})$, $E_h(T) = E_h(\bar{T})$, $V'(T) = E'_h(T) = C'_B(T) = 0$, and the first-order derivative of $LEV(T)$ is (see Appendix A)

² <https://www.mpi.govt.nz/forestry/forestry-in-the-emissions-trading-scheme/permanent-forests-in-the-ets/>

³ <https://forestcarbonworks.org/how-projects-work/>

$$LEV'(T) = \frac{(\lambda+r)^2 e^{-(r+\lambda)T}}{r(1-e^{-(r+\lambda)T})^2} \left[\int_0^{\bar{T}} \lambda [S(\bar{T}) - E_d(\bar{T}) - S(t) + E_d(t)] e^{-(r+\lambda)t} dt \right. \\ \left. - \int_0^{\bar{T}} \lambda e^{-\lambda t} C_B(t) dt - C_B(\bar{T}) e^{-\lambda \bar{T}} - V(\bar{T}) + C + E_h(\bar{T}) \right] \quad (6)$$

2.2. The risk-free case

In the risk-free case, i.e. when $\lambda = 0$, Eq. (6) reduces to

$$LEV'(T) = \frac{re^{-rT}}{(1-e^{-rT})^2} [C - C_B(\bar{T}) - V(\bar{T}) + E_h(\bar{T})] \quad (7)$$

Let us define a harvest emission factor β_h such that

$$\beta_h = \frac{E_h(\bar{T})}{p_c [\alpha(\bar{T})G(\bar{T}) + D(\bar{T})]}$$

Using the harvest emission factor, the cost of CO₂ emissions following a final felling of the stand at a rotation age $T \geq \bar{T}$ can be expressed as

$$E_h(\bar{T}) = p_c \beta_h [\alpha(\bar{T})G(\bar{T}) + D(\bar{T})] = \int_0^{\bar{T}} p_c \beta_h [\alpha(z)g(z) + D'(z)] dz \quad (8)$$

Substituting Eqs. (1) and (8) into (7) yields

$$LEV'(T) = \frac{(\lambda+r)^2 e^{-(r+\lambda)T}}{r(1-e^{-(r+\lambda)T})^2} \int_0^{\bar{T}} \lambda \left[S(\bar{T}) - S(t) - \int_t^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_d(z) - e^{r(t-z)}) dz \right] e^{-(r+\lambda)t} dt \quad (13)$$

$$LEV'(T) = \frac{re^{-rT}}{(1-e^{-rT})^2} \left[C - V(\bar{T}) + \int_0^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz \right] \quad (9)$$

Given a discount rate, there exists a value of β_h such that

$$\int_0^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz = 0 \quad (10)$$

for all carbon prices $p_c > 0$.⁴ Denote this threshold value of β_h by $\bar{\beta}_h$. Moreover, assume that $V(\bar{T}) > C$. When $\beta_h \leq \bar{\beta}_h$, $LEV'(T) < 0$ for any $T \geq \bar{T}$, implying that the optimal rotation is shorter than $\bar{T}$. When $\beta_h > \bar{\beta}_h$, $\int_0^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz > 0$ and $LEV'(T)$ could be smaller than, equal to, or greater than 0, depending on the carbon price. Let $\bar{p}_c$ denote the carbon price at which $LEV'(T) = 0$ for any $T \geq \bar{T}$. That is,

$$\bar{p}_c = \frac{V(\bar{T}) - C}{\int_0^{\bar{T}} [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz} \quad (11)$$

From Eqs. (9) and (11), we can infer that $LEV'(T) > 0$ when $p_c > \bar{p}_c$. Under these conditions, postponing the harvest of the stand at any age

$T \geq \bar{T}$ would lead to an increase in the LEV, indicating that it is optimal never to harvest the stand. Therefore, under the condition that $V(\bar{T}) > C$, the optimal rotation is infinite when the harvest emission factor is large and the carbon price high. Under normal circumstances, the carbon stocks in living biomass and in DOM increase monotonously when stand age increases, before the stand reaches the steady state. This means that $\alpha(z)g(z) > 0$ and $D'(z) > 0$ for $0 \leq z < \bar{T}$, and thus the denominator of the right-hand-side of Eq. (11) increases when β_h increases. Thus, a larger value of β_h would imply that a lower carbon price is required to incentivize the decision to never harvest the forest stand.⁵

2.3. The effect of damage risk

In the presence of forest damage risk (i.e. when $\lambda > 0$), the cost of CO₂ emissions due to damage to the forest stand can be modeled in the same fashion as Eq. (8). Thus

$$E_d(t) = p_c \beta_d(t) [\alpha(t)G(t) + D(t)] = \int_0^t p_c \beta_d(t) [\alpha(z)g(z) + D'(z)] dz \quad (12)$$

where $\beta_d(t)$ is the share of carbon emission cost when the stand is damaged relative to the value of carbon stock accumulated in the stand with $\beta_h \leq \beta_d(t) \leq 1$. To explore how forest damage risk would affect the likelihood that an infinite rotation is socially optimal, let us examine the first-order derivative of $LEV(T)$ at a rotation age $T > \bar{T}$ when $\beta_h > \bar{\beta}_h$ and $p_c = \bar{p}_c$. In this case, $LEV'(T) = 0$ if $\lambda = 0$, which means (see Eq. (7)) that $C - V(\bar{T}) + E_h(\bar{T}) = C_B(\bar{T})$. Substituting this into Eq. (6), we obtain the following result (see Appendix A)

Eq. (13) indicates that if $S(t) = 0$ and $\beta_d(t) = 1$ for $t \geq 0$, then $LEV'(T) < 0$ when $\lambda > 0$, while $LEV'(T) = 0$ when $\lambda = 0$. This result suggests that when $\beta_h > \bar{\beta}_h$ and $p_c = \bar{p}_c$, it is optimal never to harvest the forest if there is no risk of forest damage, but it is optimal to harvest the stand before it reaches age $\bar{T}$ if there is a risk of forest damage. In other words, a higher carbon price is required for the optimal rotation to be infinitely long in the presence of forest damage risk. On the other hand, if $S(t)$ and β_d are close to $V(t)$ and β_h , respectively, it might be possible that, at any age $T > \bar{T}$, $LEV'(T) > 0$ when $\lambda > 0$ while $LEV'(T) = 0$ when $\lambda = 0$, which means that the risk of forest damage increases the likelihood that permanent protection of a forest stand becomes socially optimal.

From Eq. (9) and (11) we know that when $\beta_h > \bar{\beta}_h$ and $p_c = \bar{p}_c$, $LEV'(T) = 0$ at any $T \geq \bar{T}$ in the risk-free case, which means $V(\bar{T}) - E_h(\bar{T}) + C_B(\bar{T}) - C = 0$ (see Eq. (7)). Thus, given a finite rotation age $T \geq \bar{T}$, the NPV of a T -year old stand is

⁵ If $V(\bar{T}) \leq C$, then $LEV'(T) \geq 0$ for any $T \geq \bar{T}$ when $\beta_h \geq \bar{\beta}_h$, implying that the optimal rotation age is infinite for any non-negative carbon price. However, when $\beta_h < \bar{\beta}_h$, $LEV'(T) < 0$ if carbon price is higher than the threshold price defined in Eq. (11), indicating that it is optimal to choose a finite rotation age when the harvest emission factor is small and carbon price is high. Under normal circumstances, the net revenue from timber harvest at a high stand age (when $T \geq \bar{T}$) is significantly larger than the regeneration cost. Therefore, the case when $V(\bar{T}) \leq C$ is unlikely and will not be further considered in this analysis.

⁴ This is because $\int_0^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz < 0$ when $\beta_h = 0$ and $\int_0^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_h - e^{-rz}) dz > 0$ when $\beta_h = 1$.

$$V(\bar{T}) - E_h(\bar{T}) + \frac{-C + C_B(\bar{T}) + [V(\bar{T}) - E_h(\bar{T})]e^{-rT}}{(1 - e^{-rT})} \\ = \frac{-C + C_B(\bar{T}) + V(\bar{T}) - E_h(\bar{T})}{(1 - e^{-rT})} = 0.$$

If the stand is preserved forever, its NPV is also equal to zero, meaning that, preserving the stand is an equally good option as harvesting and regenerating the stand after it has reached a steady state.⁶ Other things being equal, the presence of forest damage risk makes it unprofitable to harvest and regenerate an existing stand at an age $T \geq \bar{T}$ when $\beta_h > \bar{\beta}_h$ and $p_c = \bar{p}_c$, because the risk of damage reduces the expected NPV of future rotations. At the same time, the presence of forest damage risk implies that, if the stand is not harvested and regenerated now, it might eventually be impacted by a natural event in the future, which would necessitate regeneration. If the profit of salvage harvesting $[S(\bar{T}) - E_d(\bar{T})]$ is the same as that of an ordinary harvest $[V(\bar{T}) - E_h(\bar{T})]$, then preserving the stand is equivalent to postponing an unprofitable operation as long as possible, which is beneficial. On the other hand, if the profit of salvage harvesting is significantly lower, and the emission cost is higher than that of an ordinary harvest, then it would be better to harvest and regenerate the stand now to avoid an even larger loss that would occur if the stand is damaged at a future time. Therefore, the risk of forest damage can either increase or decrease the likelihood that the optimal rotation age is infinitely long, depending on the nature of the risk.

3. Numerical simulation

3.1. A discrete-time model

The simulation is made in discrete time, and the model used to conduct the simulation is presented below. Let ρ represent the probability that a stand is damaged during one year. The probability that a stand survives to age t is $(1 - \rho)^t$, while the probability that a stand is damaged between ages t and $t + 1$ is $\rho(1 - \rho)^t$. In analogy to Eq. (2), the LEV is modeled in the following way:

$$LEV(T) = -C + \sum_{t=0}^{T-1} \rho(1 - \rho)^t [C_B(t) + [S(t) - E_d(t) + LEV(T)]e^{-r(t+1)}] \\ + \sum_{t=1}^{T-1} (1 - \rho)^t [R(t) - E_{th}(h_t)]e^{-rt} \\ + (1 - \rho)^T C_B(T) + (1 - \rho)^T [V(T) - E_h(T) + LEV(T)]e^{-rT}$$

from which we develop the following LEV function:

$$LEV(T) = \frac{1 - \delta}{(1 - \delta^T)(1 - e^{-r})} \left\{ -C + C_B(T)(1 - \rho)^T \right. \\ \left. + [V(T) - E_h(T)]\delta^T + \sum_{t=1}^{T-1} [R(t) - E_{th}(h_t)]\delta^t \right\} \\ + \sum_{t=0}^{T-1} \rho [C_B(t)(1 - \rho)^t + [S(t) - E_d(t)]\delta^t e^{-r}] \quad (14)$$

where $\delta = (1 - \rho)e^{-r}$, $h(t)$ is the thinning removal at age t , $R(t)$ is the net revenue of thinning, and $E_t(h_t)$ is the sum of discounted costs of CO₂ emissions from the biomass removed from the stand through thinning at age t . In the simulation, we use Eq. (14) to calculate the LEV associated

Table 1

Management program and costs for the example stand.

Year	Activity	Thinning intensity	cost
0	Regeneration		8200 SEK/ha
15	Pre-commercial thinning		3000 SEK/ha
45	First thinning	52.4 m ³ /ha (40 % of basal area)	2000 SEK/ha + 180 SEK/m ³
70	Second thinning	65.5 m ³ /ha (35 % of basal area)	2000 SEK/ha + 160 SEK/m ³

with different rotation ages, and thereby to determine the optimal rotation age.

3.2. Stand growth, timber yield, and harvest revenue

We consider a Scots pine stand in northern Sweden with an average site quality, where the site index is 20 m (base age 100 years). The silvicultural program and management costs for the stand are presented in Table 1. The final felling cost is 2000 SEK/ha (fixed) plus 100 SEK/m³ (variable).

Stand growth is simulated using the growth model applied in the Beståndsmetoden for forest valuation (Lantbruksstyrelsen and Lantmäteriverket, 1988a). The growing stock of timber at age t in an unthinned stand is modeled as

$$G^0(t) = MAI * VT * 1.6416 * (1 - 6.3582^{-t/VT})^{2.8967} \quad (15)$$

where $MAI = 4.0$ m³/ha/year is the maximum mean annual volume increment and $VT = 95$ years is the rotation age which maximizes the mean annual volume increment. The current annual increment of an unthinned stand is

$$g(t) = [G^0(t) - G^0(t - 1)] \quad (16)$$

After the stand has been thinned, the current annual growth $g(t)$ and the growing stock of timber $G(t)$ are predicted using the following models

$$g(t) = [G^0(t) - G^0(t - 1)] \left(\frac{G(t - 1)}{G^0(t - 1)} \right)^{0.5} \quad (17)$$

$$G(t) = G(t - 1) + g(t) - h(t) \quad (18)$$

where $h(t)$ is the thinning removal at age t . To determine whether it is optimal to choose an infinitely long rotation age, we assume that the stand reaches a steady state at an age beyond which the projected growth of the stand is sufficiently small.

Three forest products – sawlog, pulpwood, and biofuel – are produced at thinning and final felling. Biofuel consists of bark and, in some cases, a certain percentage of logging residues (tree top and branch). For convenience, biofuel will be referred to as one timber assortment in the analysis. The yields of sawlog, pulpwood, and bark are estimated using the method applied in Beståndsmetoden (Lantbruksstyrelsen and Lantmäteriverket, 1988b). The price of sawlog is 550 SEK/m³, the price of pulpwood is 400 SEK/m³, and the price of biofuel is 420 SEK/t (Swedish Energy Agency, 2023). Let $y_i(t)$ denote the yield of timber assortment i ($i = 1$ for sawlog, $i = 2$ for pulpwood and $i = 3$ for biofuel) from the harvest of one cubic meter of growing stock of timber at stand age t . The net revenues of thinning and final felling are respectively

$$R(t) = \sum_{i=1}^3 p_i * y_i(t) * h(t) - [c_0^{th} + c_1^{th} * h(t)]$$

$$V(T) = \sum_{i=1}^3 p_i * y_i(T) * G(T) - [c_0^f + c_1^f * G(T)]$$

where p_i is the price of product i , c_0^{th} and c_1^{th} are the fixed costs for

⁶ In this paper, we focus only on the timber and carbon sequestration benefits. Taking into account other environmental benefits, such as biodiversity, it would be better to preserve the stand under the given conditions. Preserving the stand would yield greater environmental benefits over time compared to harvesting and regenerating it.

Table 2
Input-output table.

Raw material	wood product			
	Biofuel (tonne)	Pulp and paper product (tonne)	Sawnwood (m ³)	Waste (tonne)
Sawlog (1 m ³)	0.12	0.06	0.5	0.03
Pulpwood (1 m ³)	0.16	0.23	–	0.03
Biomass (1 ton)	0.95	–	–	0.05

thinning and final felling, and c_1^h and c_1^f are the variable costs of thinning and final felling, respectively.

3.3. Carbon sequestration and emission

As discussed in Section 2, the simulations consider carbon sequestration (carbon removal from the atmosphere) and emission resulting from timber harvest as separate components. In discrete time, the present value of carbon sequestration benefits of the stand over a period of t years is represented by a modified version of Eq. (1):

$$C_B(t) = \sum_{z=1}^t p_c [\alpha(z)g(z) + \Delta D(z)] e^{-rz} \quad (19)$$

where $\Delta D(z)$ is the change of the carbon stock in DOM in the forest between years $z-1$ and z before any harvest operation (thinning or final felling) occurs. Assuming a biomass expansion factor of 0.71 t (dry mass of total living biomass) per m³ of stem volume (Lehtonen et al., 2004), the amount of carbon in living biomass corresponding to one m³ of growing stock of timber at any age z is.

$$\alpha(z) = 0.71 * 0.5 * 44/12 = 1.3 \text{ tCO}_2\text{eq/m}^3.$$

Assuming that the DOM decays at a constant rate of θ_1 , and ignoring logging residues, the annual change of the carbon stock in DOM in the forest can be expressed as

$$\Delta D(z) = D_g(z) - \theta_1 D(z-1) \quad (20)$$

The production of DOM (tCO₂/ha) during year z , $D_g(z)$, is estimated using the following function (Starr et al., 2005)

$$D_g(z) = 1.1526 + 0.077 * B(z-1)$$

where $B(z-1)$ is the basal area (m²/ha) of the stand at age $z-1$. The DOM decomposition rate used in the simulation is $\theta_1 = 0.02$. Assuming that the harvest operation in any year will be carried out at the end of the year, the total amount of carbon in DOM at the end of year z (after an eventual harvest has been made) is

$$D(z) = D(z-1) + \Delta D(z) + L(h_z) \quad (21)$$

where $L(h_z)$ is the amount of logging residue (measured in terms of CO₂eq) left in the forest. Denoting by w_i the density of timber assortment i and assuming that the carbon content of biomass is 50 %, the logging residue left in the forest at thinning is

$$L(h_z) = \alpha(z) * h_z - \left[\sum_{i=1}^3 w_i * y_i(z) * h(z) \right] * 0.5 * 44/12 \quad (22)$$

We assume that the densities of sawlog and pulpwood are $w_1 = w_2 = 0.42 \text{ ton/m}^3$. The yield of biofuel is estimated in terms of dry mass and its density is $w_3 = 1$. The amount of carbon in DOM at the beginning of a rotation does not affect the optimal rotation age (Holtsmark et al., 2013) and, therefore, we assume that $D(0) = 0$.

The cost of carbon emission from the decay of DOM before the stand is harvested and regenerated are subtracted from the carbon sequestration benefits using Eqs. (19–21). After the stand is harvested at a

rotation age T , the sum of the total CO₂ emission costs, discounted to time T , due to decay of the accumulated DOM and the logging residue left in the forest is (the full derivation of emission cost functions is presented in Appendix B)

$$EC_{dom} = p_c [D(T) + L(T)] \frac{\theta_1 e^{-r}}{1 - (1 - \theta_1) e^{-r}} \quad (23)$$

Similar to Eq. (21), the logging residues (measured in terms of CO₂eq) left in the forest at final felling is

$$L(T) = \alpha(T) * G(T) - \left[\sum_{i=1}^3 w_i * y_i(T) * G(T) \right] * 0.5 * 44/12 \quad (24)$$

In this example, we consider three categories of wood products – biofuel, pulp and paper, and sawnwood. The outputs for each category per unit of timber assortment (see Table 2) were estimated using data from the Swedish forest industry, specifically roundwood consumption statistics from Biometria (2023) and wood product production figures from Forest Industries (2024) for the years 2018–2022.

For both thinning and final felling the cost of carbon emission associated with one cubic meter of the growing stock of timber (excluding emissions from logging residues) is

$$E_R(t) = \sum_{i=1}^3 y_i(t) \left[p_c Em_h + \sum_{j=1}^3 a_{ij} EC_j + \alpha_{iw} EC_w \right] \quad (25)$$

where Em_h is the emissions from timber harvest operations and road transportation of timber, EC_j is the present value of carbon emission costs⁷ of producing and using one unit of product j , and α_{iw} is the amount of production waste from processing one unit of timber assortment i , and EC_w is the CO₂ emission cost from one unit of the production waste.

The present value of the CO₂ emission cost of biofuel is (see Appendix B)

$$EC_1 = -p_c b_1 SF_f \quad (26)$$

where b_1 is the amount of CO₂ equivalent to the carbon content of one unit of biofuel, and SF_f is the substitution factor of biofuel. Let R_2 denote the recovery rate of the paper products at the end of their lifetime. Assume that γ percent of the recycled paper product is used for paper production and the rest as biofuel. The present value of carbon emissions from one unit of paper product is thus

$$EC_2 = p_c b_2 \frac{r + k_2}{r + k_2 - R_2 \gamma k_2} \left[\frac{k_2}{r + k_2} ((1 - R_2) d_i - R_2 (1 - \gamma) SF_f) - SF_p \right] \quad (27)$$

where b_2 is the amount of CO₂ equivalent to the carbon content of one tonne of paper products, $d_i = \frac{(1 - e^{-k_i}) e^{-r}}{1 - e^{-(r+k_i)}}$, k_i is the decay rate of paper product disposed of in the landfill, k_2 is the decay rate of paper products, and SF_p is the substitution factor of paper products. Let R_3 denote the recovery rate of sawnwood and assume that all recovered sawnwood is used as biofuel. The present value of the costs of carbon emissions in all future years from one unit of sawnwood produced at time 0 is

$$EC_3 = p_c b_3 \left(\frac{k_3}{r + k_3} ((1 - R_3) d_i - R_3 SF_f) - SF_s \right) \quad (28)$$

where b_3 and SF_p are the amount of CO₂ equivalent to the carbon content of one m³ of sawnwood and is the substitution factor of sawnwood, respectively.

Finally, the CO₂ emission cost from one unit of production waste is

⁷ In this section, the present value of CO₂ emission cost refers to the sum of CO₂ emission costs incurred at different times, discounted to the time when the product is produced.

Table 3

Parameters used to calculate carbon emission costs of harvested wood products.

	Pulp and paper product (tonne)	Sawnwood (m ³)	Biofuel (tonne)	Waste (tonne)
Carbon content (tCO ₂ per unit)	1.83	0.77 ^a	1.83	1.83
Substitution factor (tCO ₂ /tCO ₂)	0.55 ^b	1.14 ^b	0.35	0
Decay rate	Ln(2)/3	Ln(2)/35		Ln(2)/35
Recycling rate	90 % ($\gamma = 0.6$)	80 %		

^a Assume that wood density is 0.42 t/m³.^b Substitution factor at the production stage. We assume that the substitution factor at the production stage is equal to the total substitution factor minus the substitution factor at the end of product life.

$$EC_w = p_c b_w d_i \quad (29)$$

Eqs. (24)–(28) enable us to calculate the cost of carbon emissions from timber and biomass removed from the forest after the harvest of one cubic meter of the growing stock of timber $E_R(t)$. The total emission from timber harvest operation and road transportation of timber is assumed to be $Em_h = 0.0124 \text{ tCO}_2/\text{m}^3$ (Björheden, 2019). The substitution factors of wood products are adopted from Skytt et al. (2021). Table 3 presents the other parameters used in to calculate the emissions costs of different wood products and production waste.

The present value of carbon emission costs due to thinning at age t is

$$E_{th}(h_t) = E_R(t)h(t) \quad (30)$$

where $h(t)$ is the thinning removal at age t . The present value of carbon emission costs due to final felling at age T is

$$E_h(T) = E_R(T)G(T) + EC_{dom} \quad (31)$$

where EC_{dom} is the present value of the emission costs due to the decay of the accumulated DOM and the logging residues left in the forest at the final felling (see Eq. (22)). After having calculated the present value of carbon emission costs due to final felling, $E_h(T)$, the actual harvest emission factor is determined as

$$\beta_h = \frac{E_h(T)}{P_c[\alpha(T)G(T) + D(T)]} \quad (32)$$

The threshold value of the harvest emission $\bar{\beta}_h$ is determined by the numerically solving the following equation:

$$\sum_{z=1}^T p_c[\alpha(z)g(z) + \Delta D(z)](\bar{\beta}_h - e^{-rz}) = 0 \quad (33)$$

When $\beta_h \geq \bar{\beta}_h$, the lowest carbon price above which the optimal solution is never to harvest the forest stand is

$$\bar{p}_c = \frac{V(\bar{T}) - C}{\sum_{z=1}^{\bar{T}} [\alpha(z)g(z) + \Delta D(z)](\bar{\beta}_h - e^{-rz})} \quad (34)$$

3.4. Salvage harvest revenue and emissions costs

The net revenue of salvage harvest is calculated similarly to that of final felling, except that the harvest volume, the distribution of timber assortment and harvesting cost typically differ between the two types of harvests. If the stand is damaged at an age t ($0 < t \leq T$), the net revenue of salvage harvest is

$$S(t) = \sum_{i=1}^3 p_i y_i^s(t) G^s(t) - [c_0^s + c_1^s G^s(t)] \quad (35)$$

where p_i is the price of timber assortment i , $y_i^s(t)$ denotes the share of timber assortment i , $G^s(t)$ is the total amount of timber to be harvested, and c_0^s and c_1^s are the fixed and variable costs of salvage harvesting,

respectively. To construct a general formula for the cost of carbon emissions related to forest damage and subsequent salvage harvest, we assume that δ_d percent of the carbon in the accumulated DOM and δ_l percent of the carbon in the living biomass are released to the atmosphere immediately when the damage occurs. Adding the cost of the immediate emissions to the present value of emission costs from HWP and the decay of DOM yields

$$E_d(t) = P_c^*[\delta_d D(t) + \delta_l \alpha(t)G(t)] + E_R^s(t)G^s(t) + EC_{dom}^s \quad (36)$$

where $E_d(t)$ represents the present value of carbon emission costs due to stand damage and salvage harvest at age t , $E_R^s(t)$ is the cost of carbon emissions from timber and biomass removed from the forest after the harvest of one cubic meter of the timber, which is calculated using Eqs. (24)–(28) and the yields of different timber assortments $y_j^s(t)$ from salvage harvest, and EC_{dom}^s represents the present value of carbon emission costs from decay of the remaining DOM and logging residues after salvage harvest, and is calculated as

$$EC_{dom}^s = P_c[(1 - \delta_d)D(t) + L^s(t)] \frac{\theta_1 e^{-r}}{1 - (1 - \theta_1)e^{-r}} \quad (37)$$

where $L^s(t) = (1 - \delta_l)\alpha(t)G(t) - [\sum_{i=1}^3 w_i y_i^s(t)G^s(t)] * 0.5 * 44/12$ and represents the logging residues left in the forest after salvage harvest.

In the simulation we considered two types of damage - storm felling and wildfire. Table 4 presents the effects of the damage on timber yield, harvest costs, and immediate carbon emissions.

3.5. Carbon price and interest rate

We determined the optimal rotation age in the baseline case described in sections 3.2–3.4 using a carbon price of 1000 SEK/tCO₂ for different interest rates ranging from 1 to 5 %. Then we considered three lower levels of the substitution effect of HWP and determined the threshold carbon price above which the optimal rotation will be infinitely long. Moreover, we used two carbon prices (1500 and 2000 SEK/tCO₂) to illustrate that, even if the optimal rotation is infinitely long, the marginal effect of increasing the rotation age is diminishing when the rotation age is very high.

3.6. Results

Using the state of the stand at the age of 200 years as an approximation of the steady state of the stand, we calculated the threshold value of the harvest emission factor defined in Eq. (10) and the actual harvest emission factor according to Eq. (32). The results, shown in Table 5, illustrate the actual harvest emission factor, which represents the proportion of carbon stored in living biomass and dead organic matter (DOM) that corresponds, in present value terms, to the net emissions

Table 4

Parameters used to calculate the net revenues of salvage harvest and carbon emission cost when a stand is damaged by storm or fire.

symbol	definition	storm	fire
$y_1^s(t)$	yield of sawlog	60 % of final felling	0
$y_2^s(t)$	yield of pulpwood	80 % of final felling	0
$y_3^s(t)$	yield of biofuel	$1 - y_1^s(t) - y_2^s(t)$	1
c_0^s	fixed harvest cost	3000	3 000
c_1^s	variable harvest cost	180	160
$G^s(t)$	the total amount of timber to be harvested	90 % of final felling	50 % of final felling
δ_d	damage caused DOM carbon emission	0	15 %
δ_l	damage caused emission from living biomass	0	30 %

Table 5

The threshold harvest emission factor under which the optimal rotation age is finite, and the actual harvest emission factor corresponding to different interest rates.

Interest rate	Threshold harvest emission factor	Actual harvest emission factor
1.0 %	0.4566	0.1855
1.5 %	0.3394	0.1177
2.0 %	0.2642	0.0674
2.5 %	0.2129	0.0287
3.0 %	0.1762	−0.0020
4.0 %	0.1275	−0.0474
5.0 %	0.0971	−0.0792

Table 6

The Faustmann rotation (T_F), the carbon benefits maximization rotation (T_C), and the rotation age which maximizes the sum of timber and carbon benefits (T^*). Carbon price = 1000 SEK/tCO₂. Rotation age in years.

Interest rate	T_F	T_C	T^*
1.0 %	97	111	106
1.5 %	90	109	102
2.0 %	84	106	98
2.5 %	80	102	94
3.0 %	76	99	90
4.0 %	71	92	84
5.0 %	71	87	79

gradually released after the stand is harvested. This factor reflects the carbon cost of harvest emissions relative to the total carbon stored in the stand. For example, the emission factor of 0.0287 implies that, at a 2.5 % interest rate, the present value of net emission costs resulting from the harvest of the stand is equivalent to the cost of an immediate emission of 2.87 % of the carbon accumulated in the stand. The harvest emission factor decreases with rising interest rates, as a higher interest rate leads to a lower present value of the emission costs. At an interest rate of 3 % or higher, the harvest emission factor becomes negative. According to Eq. (10), the threshold value for the harvest emission factor remains strictly positive as long as the interest rate is greater than zero, however. Therefore, under conditions of certainty and assuming a positive interest rate, the optimal rotation age is always finite when the substitution effect of HWP is taken into account.

Gong and Kriström (1999) proved that, when considering both timber production and carbon sequestration benefits, the optimal rotation age falls within the interval (T_F, T_C), where T_F is the classical Faustmann rotation age and T_C is the rotation age which maximizes the present value of net carbon sequestration benefits of an infinite number of rotations, $[C_B(T) - E_h(T)e^{-rT}]/(1 - e^{-rT})$. The present value of net carbon sequestration benefits is proportional to carbon price, meaning that T_C remains unaffected by changes in carbon price. For the example problem, T_C ranges between 87 and 111 years, depending on the interest rate (see Table 6). Assume that carbon price is 1000 SEK/t CO₂, the optimal rotation age is 79–106 years. As the carbon price increases, the optimal rotation age converges to T_C .

The theoretical analysis in Section 2 revealed that the harvest emission factor plays a decisive role in determining whether the optimal rotation could be infinitely long. The harvest emission factor is influenced by the substitution effects of HWP, as discussed in Section 3.3. Sensitivity analysis shows that if the substitution factors of the HWP are equal to or higher than 40 % of the values presented in Table 3, the optimal rotation age is finite, regardless of how high the carbon price is. Conversely, if the substitution factors are below 40 % of the values in Table 3, the optimal rotation age will be infinitely long, provided that the carbon price is sufficiently high. Table 7 presents the threshold carbon price above which the optimal rotation is infinitely long. According to Eq. (11), this threshold carbon price for the optimal rotation to be infinitely long is dependent on the interest rate and the substitution effect of HWP. An increase in the interest rate reduces the term e^{-rT} in

Table 7

The lowest carbon price (SEK/tCO₂) for the optimal rotation to be infinitely long.

Interest rate	Substitution factors ^a			Substitution ignored ^b
	30 %	10%	0%	
1.0%	+ ∞	9579.55	3019.21	492.93
1.5 %	+ ∞	2680.78	1687.60	461.18
2.0%	12,653.46	1974.05	1388.22	454.07
2.5 %	6626.67	1756.92	1284.82	457.43
3.0%	5459.20	1683.85	1251.20	465.73
4.0%	5221.35	1693.00	1265.44	488.28
5.0%	5904.57	1785.28	1323.59	513.11

^a Substitution factors in percentage of the default values given in Table 3.

^b The substitution of HWP for non-wood products is ignored in the calculation of the forest harvest emission cost.

Eq. (11), which has a positive effect on the threshold carbon price. However, a higher interest rate results in a lower value of the harvest emission factor β_h , which negatively affects the threshold carbon price. The results in Table 7 show that the positive effect dominates when the interest rate deviates from 3 %. For the example problem, the threshold carbon price continues to decrease when the interest rate increases from 5 %. Additionally, the threshold carbon price decreases when the substitution effects of HWP decrease. Smaller substitution factors lead to larger harvest emission factor (see Eqs. 25–28 and Eq. 32), which in turn results in a lower threshold carbon price (see Eq. 11).

The case where the substitution factors are equal to zero implies that the amount of GHG emissions from HWP is equal to the emission from the non-wood products that are substituted, implying that the production and use of HWP does not affect the total emission of GHGs.⁸ The last column in Table 7 presents the minimum carbon price for the optimal rotation to be infinitely long, when the substitution of HWP for non-wood products is ignored in the calculation of the forest harvest emission cost. In this case, the use of HWP would lead to an increase in the total amount of GHG emissions.

When the substitution factors are low and the carbon price is high, the optimal rotation can indeed be infinitely long. However, the LEV curve for very high rotation age becomes quite flat, with only a small difference between the LEV of harvesting the stand once it has reached the steady state and never harvesting the stand (Fig. 1).

Table 8 presents the optimal rotation ages under storm and fire risk, respectively. The results show that incorporating these risks into the decision model results in a shorter optimal rotation. A higher risk leads to a shorter optimal rotation. With the same probability of occurrence, fire risk has a larger impact on the optimal rotation age than storm risk.

In section 2.3, we discussed how the presence of damage risks to the stand could, in some cases, lead to a longer optimal rotation. One such case occurs when the substitution factors are zero, the salvage rate is 80 % of the yield at a normal final harvest for sawlog, pulpwood, and total harvest, the interest rate is 3 %, and the carbon price is 1250 SEK/tCO₂. Under these conditions, the optimal rotation age for the example stand is 200 years in a risk-free scenario. However, when there is a probability of damage to the stand, the optimal rotation can become infinitely long if the probability of damage lies within the range [0.0003, 0.03].

The presence of storm or fire risk reduces the LEV, as the NPV of both

⁸ The substitution factor is defined as $SF = \frac{GHG_{non-wood} - GHG_{wood}}{WU_{wood} - WU_{non-wood}}$, where $GHG_{non-wood}$ and GHG_{wood} are the GHG emissions resulting from the use of wood product and the non-wood alternative, respectively, and WU_{wood} and $WU_{non-wood}$ are the amounts of wood used in the wood and non-wood alternatives (Sathre and O'Connor, 2010). $SF = 0$ means that $GHG_{non-wood}$ is equal to GHG_{wood} .

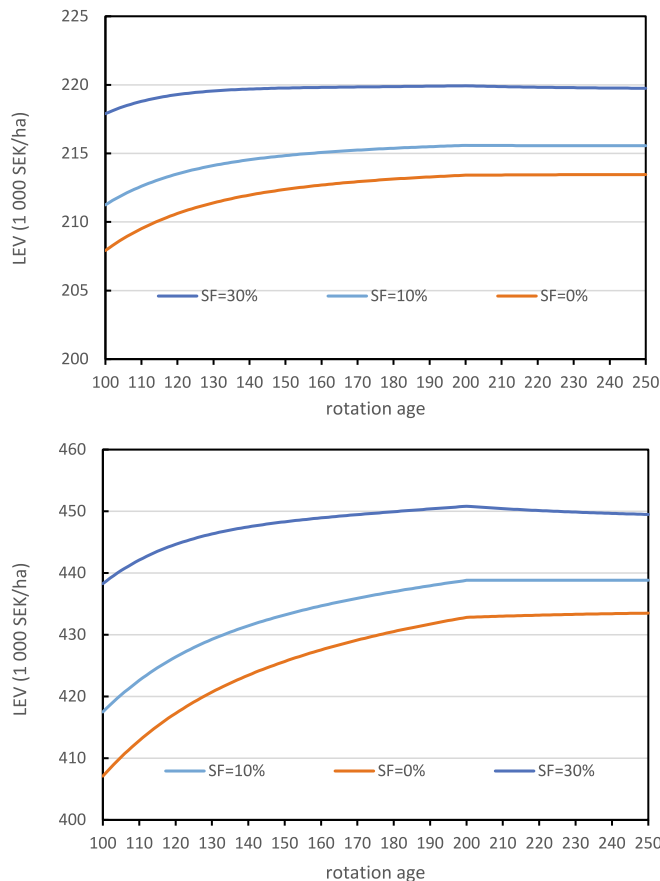


Fig. 1. The LEV with varying rotation ages and substitution factors of HWP. Top: interest rate = 3 % and carbon price = 1500 SEK/t CO₂. Bottom: interest rate = 2 % and carbon price = 2000 SEK/t CO₂. SF indicates the substitution factors in percentage of the values given in Table 3.

Table 8

The optimal rotation age (years) under storm or fire risk (carbon price = 1000 SEK/t CO₂, substitution factors as given in Table 3).

Interest rate	Storm risk		Fire risk	
	$\rho = 0.002$	$\rho = 0.005$	$\rho = 0.002$	$\rho = 0.005$
1.0%	105	104	102	97
1.5 %	101	100	99	94
2.0%	97	96	95	91
2.5 %	93	92	91	87
3.0%	90	89	88	84
4.0%	83	83	82	79
5.0%	78	78	77	75

timber production benefits and carbon sequestration benefits decreases when the risk of storm or fire damage increases (see Table 9).⁹ Like its effect on the optimal rotation age, fire risk has a larger negative impact on the LEV compared to storm risk.

4. Discussion and conclusions

Understanding the socially optimal rotation age is crucial for designing cost-effective policy aimed at increasing forest carbon sequestration. This paper examines the question whether the optimal

Table 9

The impact of storm and fire risk on the LEV (carbon price = 1000 SEK/tCO₂, interest rate = 2.5 %, substitution factors as given in Table 3).

	Risk free case	Storm risk		Fire risk	
		$\rho = 0.002$	$\rho = 0.005$	$\rho = 0.002$	$\rho = 0.005$
NPV of timber (SEK/ha)	1272	−23	−1953	−526	−3102
NPV of carbon (SEK/ha)	199,143	194,986	188,952	192,597	183,420
LEV (SEK/ha)	200,415	194,963	186,999	192,071	180,318

rotation is infinitely long, which is central to determining whether expanding forest reserve is a desirable strategy for enhancing forest carbon sequestration. Our analysis shows that, when considering only the benefits of timber production and carbon sequestration, the socially optimal rotation age is finite for forest stands of average productivity in northern Sweden. This implies that permanently protecting forests is not a cost-effective strategy for increasing the forest sector's contribution to climate change mitigation.

The impact of other forest benefits, such as recreational value and biodiversity, on the optimal rotation depends on how these benefits evolve with stand age. It is likely that the total benefits from ecosystem services other than timber production and carbon sequestration increases as the stand ages. Therefore, including the full range of benefits of ecosystem services provided by forests could increase the likelihood that the optimal rotation age is infinitely long. In the simulations, we assumed a harvest rate for logging residues of zero, meaning that all logging residues are left in the forest to decay after harvest. However, some of the logging residue could be harvested and used as biofuel. The substitution effect of biofuel would reduce the net emission from logging residues, making it even less likely that the optimal rotation age is infinitely long. The optimal rotation age is highly sensitive to the substitution effects and decay rates of HWP. Larger substitution effects and lower decay rates of HWP result in smaller carbon emissions costs from the harvest and use of timber, which leads to a shorter optimal rotation age. The default values for the substitution factors used in the simulations represent the average substitution effects derived from a wide range of studies (Skytt et al., 2021). Our results show that, with the average substitution effects of HWP, the harvest and use of timber leads to a reduction of discounted net emissions when the interest rate is 3 % or higher, with a reduction of less than 0.185 t CO₂-eq per cubic meter of timber for interest rates between 3 %–5 %. However, when the interest rate is lower than 3 %, timber harvest leads to an increase in the discounted net emission of CO₂. In comparison, Lundmark et al. (2014) reported an average CO₂ emission reduction effect of 0.466 t CO₂-eq per cubic meter of timber harvested in Sweden. Braun et al. (2016) estimated that the use of timber produced in Austria resulted in a reduction of net emissions by 0.64 t CO₂-eq per cubic meter wood use. It appears that the CO₂ emission reduction effect of timber harvest and utilization was underestimated in this study, likely due to assumptions about the half-life of different HWP, the harvest rate of logging residues, the decay rate of dead organic matter in the forest. Intuitively, a larger CO₂ emission reduction effect of timber harvest and utilization would imply a shorter optimal rotation period.

There is significant variation in the substitution effects of HWP across different studies. Our simulations show that, even with relatively conservative estimates of these substitution effects, the optimal rotation age is most likely finite. For example, even if the substitution factors are only 30 % of the average values, the optimal rotation remains finite when carbon price is below 5221 SEK/tCO₂ (about 460 Euro/ tCO₂).

The analytical examination of the effect of forest damage risk considered a marginal case where multiple rotation ages exist – specifically, it is optimal to harvest the stand at any time after the stand has reached the steady state – under conditions of certainty. In this case, the

⁹ Table 9 presents the result when the interest rate is 2.5 %. Storm and fire risks affect the NPVs of timber and carbon benefits as well as the LEV with other interest rates in the same way as shown in Table 9.

inclusion of forest damage risk could either make the optimal rotation finite or infinite, depending on the nature of the risk event. In more general cases, the effect of forest damage on the optimal rotation age cannot be determined through theoretical analysis. However, the simulation results suggest that, if the optimal rotation under conditions of certainty is finite, then forest damage risk generally shortens the optimal rotation.

Based on the results from this study and discussion above, we draw several important conclusions. Firstly, the substitution effects of HWP play a crucial role in determining whether the optimal rotation age could be infinitely long. When the substitution effects of HWP are large, the optimal rotation age remains finite, regardless of how high the carbon price is. However, if the substitution effects are small, there exists a threshold carbon price above which the optimal rotation becomes infinitely long. Furthermore, given the existing estimations of the substitution effect of HWP, the optimal rotation for Scots pine stands with average site quality is finite. In addition, the risk of forest damage further reduces the optimal rotation age.

The analysis in this paper assumes that the price of carbon is constant over time. The carbon price used in a socio-economic analysis should correspond to the social cost of carbon, which reflects the cost to society of emitting one unit of carbon dioxide into the atmosphere, or the marginal societal net benefit of reducing emissions of carbon dioxide.

Appendix A. First-order condition

Given the land expectation value function

$$LEV(T) = \frac{\lambda + r}{r(1 - e^{-(r+\lambda)T})} \left[-C + \int_0^T \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt + C_B(T)e^{-\lambda T} + [V(T) - E_h(T)]e^{-(r+\lambda)T} \right] \quad (A1)$$

The first-order derivative of $LEV(T)$ with respect to rotation age T is

$$LEV'(T) = \frac{\lambda + r}{r(1 - e^{-(r+\lambda)T})} \left[\begin{aligned} &\lambda e^{-\lambda T} \{C_B(T) + [S(T) - E_d(T)]e^{-rT}\} \\ &+ C_B'(T)e^{-\lambda T} + [V'(T) - E_h'(T)]e^{-(r+\lambda)T} \\ &- \lambda C_B(T)e^{-\lambda T} - (r + \lambda)[V(T) - E_h(T)]e^{-(r+\lambda)T} \end{aligned} \right] - \frac{(\lambda + r)^2 e^{-(r+\lambda)T}}{r(1 - e^{-(r+\lambda)T})^2} \left[-C + \int_0^T \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt + C_B(T)e^{-\lambda T} + [V(T) - E_h(T)]e^{-(r+\lambda)T} \right] \quad (A2)$$

Substitute Eq. (A1) into (A2) and after rearrangement we obtain

$$LEV'(T) = \frac{(\lambda + r)}{r(e^{(r+\lambda)T} - 1)} \left[\begin{aligned} &\lambda[S(T) - E_d(T)] + V'(T) - E_h'(T) \\ &+ C_B'(T)e^{rT} - (r + \lambda)[V(T) - E_h(T)] - rLEV(T) \end{aligned} \right] \quad (A3)$$

Suppose that $LEV(T)$ reaches a maximum at a finite rotation age T^* . The first-order derivative of $LEV(T)$ with respect to T should be equal to zero when $T = T^*$, which means that

$$V'(T) - E_h'(T) + C_B'(T)e^{rT} = r[V(T) - E_h(T)] + rLEV(T) + \lambda[V(T) - E_h(T) - S(T) + E_d(T)] \quad (A4)$$

By combining the terms on the right-hand side of Eq. (A2), the first-order derivative of $LEV(T)$ with respect to T can be expressed as

$$LEV'(T) = B(T) \left[\begin{aligned} &\frac{(1 - e^{-(r+\lambda)T})}{\lambda + r} \{ \lambda[S(T) - E_d(T)] + V'(T) - E_h'(T) + C_B'(T)e^{rT} \} \\ &+ C - \int_0^T \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt \\ &- C_B(T)e^{-\lambda T} - [V(T) - E_h(T)]e^{-(r+\lambda)T} \end{aligned} \right] \quad (A5)$$

where $B(T) = \frac{(\lambda + r)^2 e^{-(r+\lambda)T}}{r(1 - e^{-(r+\lambda)T})^2}$. Let $\bar{T}$ denote the age when the stand reaches a steady state (disregarding the risk of damage). For any rotation age $T \geq \bar{T}$, $V'(T) = E_h'(T) = g(T) = D'(T) = 0$, $V(T) = V(\bar{T})$, $S(T) = S(\bar{T})$, $C_B(T) = C_B(\bar{T})$, $E_d(T) = E_d(\bar{T})$, and $E_h(T) = E_h(\bar{T})$. When $T \geq \bar{T}$,

$$\int_0^T \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt = \int_0^{\bar{T}} \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt + \int_{\bar{T}}^T \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt$$

Estimates of the social cost of carbon have increased since 2010 (Tol, 2023) and will continue to increase (US Environmental Protection Agency, 2023). Ekholm (2016) showed that rising carbon prices over time would lead to increasingly longer optimal rotations. The study by Ekholm (2016) examined the decision problem from a forest owner's perspective - the forest owner gets paid for the uptake of CO₂ and is taxed for the carbon released back to the atmosphere following the harvest of a forest stand. The impact of rising carbon prices over time on socially optimal rotation ages requires further study.

CRedit authorship contribution statement

Peichen Gong: Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization. **Andres Susaeta:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

$$\begin{aligned}
&= \int_0^{\bar{T}} \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt + C_B(\bar{T})[e^{-\lambda \bar{T}} - e^{-\lambda T}] \\
&+ [S(\bar{T}) - E_d(\bar{T})] \frac{\lambda}{r + \lambda} [e^{-(r+\lambda)\bar{T}} - e^{-(r+\lambda)T}]
\end{aligned} \tag{A6}$$

Therefore, when $T \geq \bar{T}$

$$\begin{aligned}
LEV(T) &= B(T) \left[\frac{\lambda}{\lambda + r} [S(\bar{T}) - E_d(\bar{T})] [1 - e^{-(r+\lambda)\bar{T}}] - \int_0^{\bar{T}} \lambda e^{-\lambda t} \{C_B(t) + [S(t) - E_d(t)]e^{-rt}\} dt - C_B(\bar{T})e^{-\lambda \bar{T}} - V(\bar{T}) + C + E_h(\bar{T}) \right] \\
&= B(T) \left[\int_0^{\bar{T}} \lambda [S(\bar{T}) - E_d(\bar{T}) - S(t) + E_d(t)] e^{-(r+\lambda)t} dt - \int_0^{\bar{T}} \lambda e^{-\lambda t} C_B(t) dt - C_B(\bar{T})e^{-\lambda \bar{T}} - V(\bar{T}) + C + E_h(\bar{T}) \right]
\end{aligned} \tag{A7}$$

When $\beta > \bar{\beta}$ and $p_c = \bar{p}_c$, $C - V(\bar{T}) + E_h(\bar{T}) = C_B(\bar{T})$ and hence

$$\begin{aligned}
LEV(T) &= B(T) \left[\int_0^{\bar{T}} \lambda [S(\bar{T}) - E_d(\bar{T}) - S(t) + E_d(t)] e^{-(r+\lambda)t} dt \right. \\
&\quad \left. - \int_0^{\bar{T}} \lambda e^{-\lambda t} C_B(t) dt + C_B(\bar{T})(1 - e^{-\lambda \bar{T}}) \right] \\
&= B(T) \left[\int_0^{\bar{T}} \lambda [S(\bar{T}) - E_d(\bar{T}) - S(t) + E_d(t)] e^{-(r+\lambda)t} dt \right. \\
&\quad \left. + \int_0^{\bar{T}} \lambda e^{-\lambda t} [C_B(\bar{T}) - C_B(t)] dt \right]
\end{aligned} \tag{A8}$$

Substitute the following expressions of $C_B(t)$ and $E_d(t)$ into (A8)

$$C_B(t) = \int_0^t p_c [\alpha(z)g(z) + D'(z)] e^{-rz} dz$$

$$E_d(t) = p_c \beta_d(t) [\alpha(t)G(t) + D(t)] = \int_0^t p_c \beta_d(t) [\alpha(z)g(z) + D'(z)] dz$$

and after simplification we obtain

$$LEV(T) = B(T) \lambda \left[\int_0^{\bar{T}} [S(\bar{T}) - S(t) - \int_t^{\bar{T}} p_c [\alpha(z)g(z) + D'(z)] (\beta_d(z) - e^{r(t-z)}) dz] e^{-(r+\lambda)t} dt \right] \tag{A9}$$

Appendix B. The cost of carbon emissions

When calculating the cost of carbon emissions following a salvage or final felling of a forest stand, we consider the emissions due to decay of the accumulated DOM in the stand and emissions from producing and using harvested wood products (HWP), as well as the substitution effect of the HWP. First, let us consider the emission cost of a final felling. Let $L(T)$ denote the amount of logging residues left in the forest when the stand is harvest at a rotation age T . Directly after the harvest takes place, the total amount of DOM accumulated in the stand is $D(T) + L(T)$ tonnes CO_{2-eq}. Assuming a constant annual decay rate θ_1 , the emission in year z after the harvest is $[D(T) + L(T)]\theta_1(1 - \theta_1)^{(z-1)}$. The present value of carbon emission costs¹⁰ due to decay of the accumulated DOM and the logging residues left in the forest is

$$EC_{dom} = p_c [D(T) + L(T)] \sum_{z=1}^{\infty} \left(\theta_1 (1 - \theta_1)^{(z-1)} e^{-rz} \right) = p_c [D(T) + L(T)] \frac{\theta_1 e^{-r}}{1 - (1 - \theta_1)e^{-r}} \tag{B1}$$

where p_c is carbon price and r is the discount rate.

Let a_{ij} denote the output of HWP j from one unit of timber assortment i removed from the forest.¹¹ The cost of carbon emissions due to removal of biomass at final felling is

$$E_R(T) = \sum_{i=1}^n y_i(T) G(T) \left(\sum_{j=1}^n a_{ij} EC_j + \alpha_{iw} EC_w \right) \tag{B2}$$

where $y_i(T)$ is the yield of timber assortment i from the harvest of one cubic meter of growing stock of timber at age T , n is the total number of HWP categories, EC_j is the present value of emission cost of producing and using one unit of product j , α_{iw} is the amount of production waste from processing

¹⁰ In this section, the present value of carbon emission cost refers to the sum of emission costs incurred at different times discounted to the time when a forest stand is harvested or when a HWP is produced.

¹¹ We consider biofuel as one timber assortment as well as a HWP.

one unit of timber assortment i , and EC_w is the present value of carbon emission cost from one unit of the production waste. The present value of carbon emission costs due to final felling is

$$E_h(T) = E_R(T) + EC_{dom} \quad (B3)$$

To calculate the present value of carbon emission costs due to a final felling, we need to calculate the present value of carbon emission costs of producing and using one unit of each HWP category. The time profile of carbon emission differs among different wood products. Biofuel from the forest usually has short storage time. Disregarding the carbon storage in biofuel before it is used, the present value of carbon emission cost of biofuel is

$$EC_b = p_c(b_f - S_f) = p_c b_f \frac{(b_f - S_f)}{b_f} = -p_c b_f S_f \quad (B4)$$

where b_f is the amount of CO₂ equivalent to the carbon content of one unit of biofuel, S_f is the reduction of carbon emission from fossil fuels substituted by using one unit of biofuel, and S_f is the substitution factor of biofuel.

When calculating the cost of carbon emissions from other HWP, we focus on the production stage and the end of the service lifetime of the products (Leskinen et al., 2018). Consider one unit of product j produced at time 0, the carbon emission cost at the production stage is

$$EC_j^{prod} = p_c(em_{wood} - em_{non-wood}) = p_c b_j \frac{(em_{wood} - em_{non-wood})}{b_j} = -p_c b_j S_f \quad (B5)$$

Where em_{wood} is the emissions from producing one unit of wood product j , $em_{non-wood}$ is the reduction of emissions from non-wood materials substituted by wood, b_j is the amount of CO₂ equivalent to the carbon mass in one unit of wood product j , and S_f is the production-stage substitution factor of wood product j .

Let R_j denote the recovery rate of the product at the end of its lifetime, and assume that the unrecovered part, $(1 - R_j)$, is disposed in a landfill. The cost of carbon emissions from the wood product disposed in a landfill, discounted to the time of disposal, is equal to

$$p_c b_j (1 - R_j) \sum_{t=1}^{\infty} e^{-(t-1)k_l} (1 - e^{-k_l}) e^{-tr} = p_c b_j (1 - R_j) \frac{(1 - e^{-k_l}) e^{-r}}{1 - e^{-(r+k_l)}} \quad (B6)$$

where b_j is the amount of CO₂ equivalent to the carbon content of one unit of the product, and k_l is the decay rate of forest products disposed in landfills.

The emission cost of the recovered part of the worn-out product depends on how the recovered product is used. In general, we assume that some percentage (γ_j) of the recovered product is used to produce the same type of wood product and the rest as biofuel. The cost of carbon emission from the product produced through recycling is

$$R_j \gamma_j EC_j \quad (B7)$$

Based on Eq. (B4), the cost of carbon emission from the recovered product which is used as biofuel is

$$-p_c b_j R_j (1 - \gamma_j) S_f \quad (B8)$$

Assume that the lifetime of the product has an exponential distribution with a rate parameter k_j ($k_j = \ln(2)/HL_j$, where HL_j is the half-life of wood product j). The expected present value of the carbon emission costs at the end of life from one unit of wood product j produced at time 0 is

$$\begin{aligned} EC_j^{eol} &= \int_0^{\infty} [p_c b_j (1 - R_j) d_l - p_c b_j R_j (1 - \gamma_j) S_f + R_j \gamma_j EC_j] k_j e^{-k_j z} e^{-rz} dz \\ &= P_c b_j \left((1 - R_j) d_l - R_j (1 - \gamma_j) S_f \right) \frac{k_j}{r + k_j} + R_j \gamma_j EC_j \frac{k_j}{r + k_j} \end{aligned} \quad (B9)$$

where $d_l = \frac{(1 - e^{-k_l}) e^{-r}}{1 - e^{-(r+k_l)}}$.

Based on Eqs. (B5) and (B9), the present value of carbon emissions from one unit of wood product j is

$$EC_j = EC_j^{prod} + EC_j^{eol} = p_c b_j \left[\left((1 - R_j) d_l - R_j (1 - \gamma_j) S_f \right) \frac{k_j}{r + k_j} - S_f \right] \frac{r + k_j}{r + k_j - R_j \gamma_j k_j} \quad (B10)$$

Finally, the CO₂ emission cost from one unit of the production waste is,

$$EC_w = P_c b_w d_l \quad (B11)$$

Where b_w denote the carbon content of one unit of production waste.

Author statement

We acknowledge that the material presented in this manuscript has not been previously published, nor is it simultaneously under consideration by any other journal.

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Data availability

Data will be made available on request.

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