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ARTICLE

Optimal timing and rate of nitrogen fertilizer use: An integrated network technology approach

Tihomir Ancev[1] | **Moriah Bostian**[2,3]

[1]Department of Economics, University of Sydney, Sydney, New South Wales, Australia

[2]Department of Economics, Lewis & Clark College, Portland, Oregon, USA

[3]Centre for Environmental and Resource Economics (CERE), Department of Forest Economics, Swedish University of Agricultural Sciences (SLU), Umeå, Sweden

Correspondence
Moriah Bostian, Department of Economics, Lewis & Clark College, Portland, OR, USA.
Email: mbbostian@lclark.edu

Abstract

Both the rate and timing of crop fertilizer application play an important role in agricultural productivity. However, inefficient fertilizer use can significantly increase production costs, water pollution, and greenhouse gas emissions. To capture both the spatial heterogeneity and dynamic nature of this problem, we develop a multi-stage network production model, which links the sequential stages of crop growth within the overall crop production process. We use this framework to estimate nitrogen fertilizer application inefficiency and to determine the optimal rate and timing of fertilizer application. We apply this framework to wheat production at the field scale, using an agronomic simulation model calibrated to experimental data from Australia. Our results indicate that it is optimal to reduce the overall fertilizer application rate and to move away from the current practice of front-loading fertilizer in the initial stages of crop growth toward the intermediate stages. This can be achieved without compromising yields while also reducing nutrient losses.

KEYWORDS

data envelopment analysis, denitrification-decomposition (DNDC) model, network production technology, nitrogen use efficiency

JEL CLASSIFICATION

Q12, Q15, C61

1 | INTRODUCTION

Nitrogen (N) fertilizer remains essential to agricultural production. However, N runoff from excess N use continues to play a leading role in global water quality problems, including inland area eutrophication, coastal area hypoxia, and biodiversity loss (FAO, 2022). Nitrous oxide emissions from N fertilizer are also one of the main forms of agricultural greenhouse gas (GHG) emissions. This highlights the importance of improved N use efficiency (NUE), from both a productivity and an environmental perspective.

N loss to the environment results from excess N fertilizer application, but it also depends on the timing of application relative to the stages of crop growth and nutrient uptake. Environmental factors, such as soil and climate conditions, also define the process of N loss (Sinha et al., 2017). Improving NUE will require a better understanding of both *how much* and *when* to apply N fertilizer to crops.

Recent advances in modeling production processes as a network of separate, but connected stages of production (Bostian, Färe, Grosskopf, & Lundgren, 2018; Kao, 2017) lend themselves well to the analysis of dynamic efficiency and optimal timing of input use in staged production activities, such as agriculture. This network approach allows for better dynamic representation of the production technology than more common single-stage methods, which fail to distinguish the timing of input use and crop growth. Drawing on this approach, we develop a multi-stage network model to estimate dynamic efficiency for crop production and the optimal timing of N use across the separate stages of crop growth, considering both production output and environmental objectives. To our knowledge, we are the first to apply this framework at the agricultural field level.

Previous studies have applied network production models for agriculture at more aggregated levels, including at the regional level (Kazemi-Matin & Azizi, 2016), across the supply chain (Liu et al., 2012), and for the sector as a whole (Prieto & Zofio, 2007). In contrast, we consider the explicit stages of production directly linked to the stages of crop growth, in order to estimate both the optimal quantity and timing of N use over the crop production cycle. The dynamic nature of this problem is also relevant for other multi-stage production processes, as well as for managerial decisions related to investment in and growth of production units.

Our micro-level approach has several advantages for management and policy. First, it allows us to draw findings and recommendations for optimal N application rates at each crop production stage and for the specific field in question. Second, it allows us to estimate the production technology at each stage and to better assess potential increases to overall crop yields. Third, this framework allows us to assess potential reductions to N losses into the environment due to over-application at different stages of the crop production cycle and under specific environmental conditions. This more detailed assessment can be used to inform management practices aimed at reducing N pollution from agriculture along both spatial and temporal dimensions.

We integrate a locally calibrated bio-physical simulation model of crop growth with an economic network production model to estimate the optimal rate and timing of N use for improved NUE. We apply this framework to wheat production.

2 | LITERATURE REVIEW

2.1 | Fertilizer application rates and timing

Much of the focus in the agricultural economics literature around fertilizer application pertains to the determinants of N use by farmers and finding optimal levels of fertilizer input (Tembo et al., 2008; Tumisiime et al., 2011). This mainly considers fertilizer application as one of many production inputs, without explicitly considering how efficiently this input is transformed into yield, or the biophysical processes determining the fate and transport of excess nutrients to surrounding water systems.

Several studies investigate the potential use of excess fertilizer application as a form of insurance, in response to the uncertain marginal productivity of fertilizer inputs and more general production risk. Babcock (1992) finds that farmers use fertilizer as a risk-reducing strategy and incorporate this into their expected profit maximization calculations. Babcock and Blackmer (1994) test ex-post whether higher ex-ante optimal N fertilizer application rates are justified if the growing conditions turn out to be favorable. They do not find statistical support for this but instead find that higher rates are optimal when more site-specific improvements in growing conditions are considered. Isik and Khanna (2002) analyze the effect of yield uncertainty on the perceived benefit of site-specific technology and show that site-specific technology is beneficial.

2.2 | Nitrogen use efficiency (NUE)

Farmers commonly apply fertilizer in excess of agronomic recommendations (Berbel & Martínez-Dalmau, 2021; Del Rossi et al., 2023; Paulson & Babcock, 2010; Sellers et al., 2020; Sheriff, 2005; X. Zhang et al., 2015). NUE and the associated N loss to the environment have been studied extensively (Ancev et al., 2023; Barnhart et al., 2016; Lamkowsky et al., 2021; Rabotyagov et al., 2010, 2014; Reinhard et al., 1999, 2000; Reinhard & Thijssen, 2000; Robertson et al., 2009, 2014), both at the farm level and by using integrated economic production and biophysical models at the watershed or other larger geographic scale. In general, these studies do not explicitly model the separate stages of crop production or timing of N application, but instead consider total rates of N use in conjunction with other management practices, such as tillage and crop rotation.

Several recent studies review the effectiveness of the 4R approach (right source, right rate, right timing, and right place) for N management (Bergtold et al., 2019; Hou et al., 2017; Yanni et al., 2018). Fixen (2020) overviews the growing emphasis on environmental stewardship within the 4R N framework for N fertilizer. In one closely related study, De Laporte et al. (2021) extend the 4R N framework for NUE to consider rate, timing, and application methods jointly. While they do not explicitly optimize N use, as we do here, they do consider a variety of N rates, as well as alternative N timing scenarios. They find that while overall N rate reduction plays the greater role in reducing N emissions and increasing farm profits, changing to fertilizer injection methods and changing the application timing to later growing stages also reduces N emissions and increases farm profits.

In another closely related study, Kazemi-Matin and Azizi (2016) investigate NUE using a static network production model at the regional level, but without including environmental objectives or optimizing N fertilizer application. To our knowledge, we are the first to examine NUE in a dynamic network production model at the smaller field scale. In addition, we extend this network production model to include both crop production and environmental objectives, as well as environmental growing conditions, similar to De Laporte et al. (2021).

2.3 | Network or multi-stage production technologies

There now exists a growing literature related to the use of network or multi-stage production models that provide more explicit structural representation, particularly in the context of pollution-generating technologies. This includes work to separate the production technology into separate subtechnologies for intended production and pollution generation as a byproduct of pollution-generating inputs (Dakpo & Lansink, 2019; Førsund, 2018a, 2018b; Frisch, 1965; Murty et al., 2012; Murty & Russell, 2018; Wang et al., 2023); work to incorporate circular agriculture principles to the by-production framework (Wang et al., 2023); work to separate the pollution-generating technology into separate subtechnologies for production, pollution, and subsequent abatement (Färe et al., 2013; Hampf, 2014); work to model the optimal reallocation of pollution emissions reduction over time (Bostian, Färe, Grosskopf, Lundgren, & Weber, 2018; Färe et al., 2012); and work to separate the abatement technology into separate subtechnologies for pollution prevention and pollution treatment (Bostian et al., 2022). Bostian, Färe, Grosskopf, and Lundgren (2018) reformulate both the Murty et al. (2012) by-production model and the Førsund (2018a, 2018b) single good/single bad output model as network models with multiple subtechnologies.

In another related application to agriculture, Ang and Kerstens (2016) develop a network model to estimate optimal coordination of activities between crop and livestock production subtechnologies. Bostian, Färe, Grosskopf, and Lundgren (2018) survey the pollution-generating technology literature more broadly, with an emphasis on multi-stage approaches. Kao (2017) provides a more general overview of nonparametric network technology estimation.

Our network approach to track the propagation of N use, crop and soil N uptake, and N loss to the environment also connects to previous work to impose the materials balance condition for pollution-generating technologies (Coelli et al., 2007; Førsund, 2018a, 2018b; Hampf, 2014; Rødseth, 2013, 2017). Rather than imposing the materials balance condition in our production model, we use a biophysical model for crop growth to explicitly account for these N flows over the crop cycle.

2.4 | Lessons from the literature and contributions of the current study

Persistent N loss to the environment due to over-application of N fertilizer is well established in the literature. Recent literature, using a variety of modeling techniques and application settings, finds that the overall reduction of N fertilizer application rates, as well as moving the timing of the application toward later stages of crop development, is both economically and environmentally beneficial (Cai et al., 2023; De Laporte et al., 2021; Hoang & Wilson, 2017). We contribute further evidence that is consistent with these recent findings for N rate and timing, but using a production theoretical approach.

More broadly, we contribute to the three strains of literature discussed above in a number of ways. We estimate optimal fertilizer rates and timing using a production technology-based approach which models the production process as a network of the stages of crop development. This is particularly important as the effective use of inputs by crops critically depends on the stage of crop development. Our interest in the timing of N fertilizer use differs from previous related network applications (e.g., Ang & Kerstens, 2016; Wang et al., 2023), which connect different production systems (crops, livestock, and emissions), aggregated to the growing year. Moreover, we apply the network production model at the field scale, which has not yet been done for network models in the context of NUE. In addition, our analysis comprises simultaneously both productivity and environmental objectives, allowing us to assess the associated trade-offs between crop output and N losses at the field scale.

3 | THE PRODUCTION TECHNOLOGY

We assume a stylized crop production process, in which a single production input, N fertilizer, is used to produce a single final crop output. This model could be readily expanded to include multiple inputs and outputs. In addition, we consider pollution resulting from this process, in the form of N losses to the surrounding environment. We begin by presenting a traditional, single-stage, joint production representation of the technology, which we then use as our base model for comparison to the more structurally explicit network technology.

3.1 | The single-stage joint production technology

We first model the production technology as a single-stage joint production process, as in Chung et al. (1997), by letting x denote total applied N, y denote final harvested crop yields, and u denote total N loss to the environment. In addition to the standard production inputs and outputs, we also consider environmental conditions, which influence crop production and N fate. This inclusion of environmental conditions follows Ray (2004), as well as a number of more recent studies to incorporate weather and climate conditions into the production technology (Ancev et al., 2023; Barnhart et al., 2023; Chambers et al., 2020; Chambers & Pieralli, 2020; Njuki et al., 2019, 2020; O'Donnell, 2016). We let the vector $\mathbf{w} = \mathbf{w_1}, \ldots, \mathbf{w_L}$ denote the corresponding environmental conditions.

We define the single-stage joint production technology, T_{JP}, as

$$T_{JP} = \{(\mathbf{x}, \mathbf{y}, \mathbf{u}; \mathbf{w}) : \mathbf{x} \text{ can produce } \mathbf{y} \text{ and } \mathbf{u}, \text{ given } \mathbf{w}\}, \tag{1}$$

and the corresponding output set, $P_{JP}(\mathbf{x})$, as

$$P_{JP}(\mathbf{x}; \mathbf{w}) = \{(\mathbf{y}, \mathbf{u}) : \mathbf{x} \text{ can produce } \mathbf{y} \text{ and } \mathbf{u}, \text{ given } \mathbf{w}\}, \tag{2}$$

where again, $\mathbf{x}$ represents total N input, $\mathbf{y}$ represents final crop yield, and $\mathbf{u}$ represents total N loss. We use the bold notation for the single-stage model variables, which are aggregate values over all of the production stages, to distinguish them from their separate stage-specific counterparts in the multi-stage model below. We proceed by breaking apart this single-stage joint production technology into the multi-stage network production technology to better consider the underlying structural processes at different stages of crop growth.

3.2 | The multi-stage network technology

To develop the network technology, we first distinguish the successive stages of crop production, $s = 1, \ldots, S$. Each stage includes applied N fertilizer inputs, x^s, used to produce desirable crop biomass output, y^s, and also generating undesirable N loss, u^s. We let $w^s = w_1^s, \ldots, w_L^s$ denote the corresponding environmental conditions at each stage.

Crop biomass output for stage s, y^s, serves as an intermediate output–input into production in the next stage, stage $s+1$. We also track two additional intermediate output-inputs, the quantity of N in the soil and in the crop biomass, which allows us to link both crop growth and N losses from one stage to the next. We use $v^s = v_1^s, v_2^s$ to denote the additional soil and crop N intermediate output-inputs, respectively. At the end of the final stage, harvested crop biomass constitutes the desirable final output.

We note that within a given stage, there can be trade-offs between crop uptake of N and soil uptake of N, as well as between soil uptake of N and crop biomass, whereas soil and crop uptake of N from the previous stage serve as inputs to crop biomass for the given stage. We also note that total biomass may indirectly account for crop uptake of N. In our application, total biomass is measured in terms of carbon weight, which does not directly account for crop uptake of N. Hence, our inclusion of crop N separately, in order to more explicitly track the propagation of N, in conjunction with crop biomass, over the stages of crop growth.

In the first stage of crop production, we assume producers begin with an initial level of soil N, to which they apply an initial level of N fertilizer, x^1. This results in ending values of the N in soil and the crop biomass, v^1, initial crop biomass, y^1, as well as the undesirable N flux lost to the surrounding environment, u^1. We define the first stage, $P_{NW}^1(x^1; w^1)$, as

$$P_{NW}^1\left(x^1; w^1\right) = \left\{\left(y^1, u^1\right) : x^1 \text{ can produce } y^1, v^1, \text{ and } u^1, \text{ given } w^1\right\}, \tag{3}$$

where ending crop and soil N from the first stage then serve as intermediate output-inputs into the second stage, $P^2_{NW}(y^1, v^1, x^2; w^2)$. This general pattern holds over the successive stages of production, so that for $s = 2, \ldots, S$, we define $P^s_{NW}(x^s, y^{s-1}, v^{s-1}; w^s)$ as

$$P^s_{NW}\left(x^s, y^{s-1}, v^{s-1}; w^s\right) = \left\{\left(y^s, v^s, u^s\right) : x^s y^{s-1}, v^{s-1} \text{and can produce} y^s, v^s, \text{and } u^s, \text{given } w^s\right\}, \tag{4}$$

where in the final stage, y^S denotes harvested crop biomass, so that $y^S = \mathbf{y}$. Figure 1 illustrates the network production model (represented by the circles and connectors between them) and specific linkages between individual production stages within the network (represented by the arrows), nested within the single stage joint-production approach (represented by the rectangle).

3.3 | Empirical approximation of the technology

We employ nonparametric Activity Analysis (Afriat, 1972; Farrell, 1957) or Data Envelopment Analysis (DEA) methods (Charnes et al., 1978, 1985) to construct a conservative approximation for both the single and multi-stage production technologies. For a more recent overview of this approach, see Thanassoulis et al. (2008). Our use of DEA to estimate the production technology is similar to other recent work related to agricultural productivity and environmental conditions (Ancev et al., 2023; Barnhart et al., 2023; Chambers et al., 2020; Chambers & Pieralli, 2020), as well as other work related to multi-stage or network representations of pollution-generating production technologies (Bostian et al., 2016, 2022; Bostian, Färe, Grosskopf, Lundgren, & Weber, 2018; Färe et al., 2012, 2013; Hampf, 2014; Murty et al., 2012; Murty & Russell, 2018; Rødseth, 2017). This approach relies on

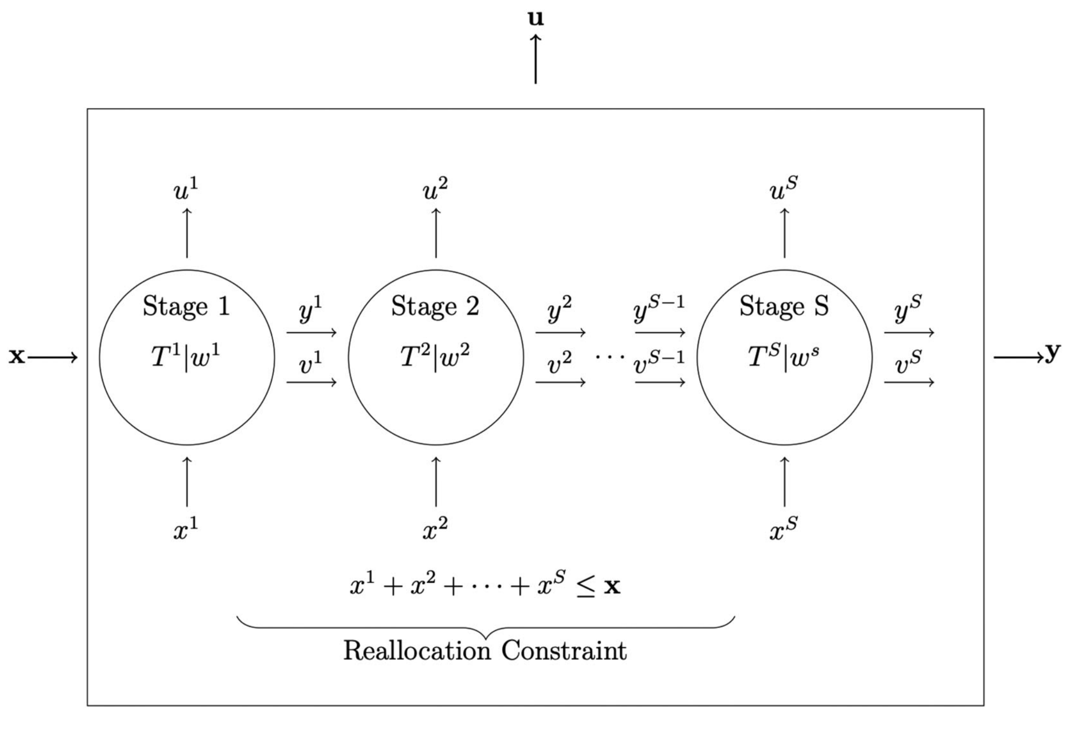

FIGURE 1 The general production network technology connecting the S stages of crop production, within the single-stage joint production technology.

linear programming methods to construct an outer hull encompassing the observed production values in the data. We provide the corresponding DEA linear programming problems and their exposition in the Online Appendix (Section A, Equations A.1–A.3).

4 | OPTIMAL N USE

We can now use the estimated network technology to consider N use efficiency, in terms of both final crop yield and total N flux losses. The network technology allows us to consider the potential to improve production outcomes, for both desirable (crop) and undesirable (N loss) outputs, by reallocating applied N fertilizer across the crop production stages, and by varying the total quantity of N fertilizer applied. Figure 1 shows this potential for reallocating fertilizer application in terms of both quantity and timing.

The dynamic optimization problem is to choose the optimal quantity of N fertilizer application at each stage, in order to jointly maximize the desirable and minimize the undesirable output. Our formulation draws on previous dynamic network production theory and applications (Bostian et al., 2016, 2022; Bostian, Färe, Grosskopf, Lundgren, & Weber, 2018; Färe et al., 2012; Färe & Grosskopf, 1996). This optimization over competing objectives is constrained by the production technology outlined in the previous section, which includes multiple inputs and multiple outputs.[1]

It is important to note that our use of the term optimal in this case refers to minimizing production inefficiency in terms of input and output quantities, including undesirable outputs, rather than maximizing profits or minimizing costs. We use this production inefficiency framework to model the physical capacity to both increase yields and decrease N losses to the environment. While this may not fully align with private producer objectives, this is also not necessarily counter to them. Producers also have an incentive to more efficiently use costly fertilizer inputs, particularly if doing so can also lead to increased crop yields. This framework also aligns with policy goals to reduce fertilizer use and loss to the environment without threatening the food supply.

4.1 | Production inefficiency

Distance functions (Shephard, 1953, 1970) can be used to represent production processes with both multiple inputs and multiple outputs. We use the directional output distance function (Chambers et al., 1996, 1998) to measure the potential to both increase crop yield and decrease pollution from N loss, subject to the production technology. These potential (but unrealized) gains constitute production inefficiency. We measure that inefficiency as the distance between potential values for N loss and crop yield at the production frontier and at the actual observed values. A directional distance value of zero implies efficient production at the frontier, while directional distance values greater than zero indicate a worsening of efficiency.

Beginning with the single-stage joint production model, similar to Chung et al. (1997), we define the directional output distance function as

$$\vec{D}_{O_{JP}}\left(\mathbf{x},\mathbf{y},\mathbf{u};\mathbf{w},g_y,g_u\right)=\max\left\{\beta:\left(\mathbf{y}+\beta g_y,\mathbf{u}-\beta g_u\right)\in P_{JP}(\mathbf{x};\mathbf{w})\right\},\tag{5}$$

[1]We note that while we use DEA solution methods, there exists a parallel literature on vector optimization, which also considers constrained multi-objective optimization problems and most commonly uses weighting-based scalarization methods for solving. Pappalardo (2008) provides an introduction to this literature. Closely related to our use of DEA, Wei et al. (2000) demonstrate the use of inverse DEA solution methods for multi-objective optimization problems.

where the directional vector, $\vec{g} = \left(g_y, g_u\right)$ specifies the simultaneous expansion of final crop output and contraction of the total N flux. We provide the corresponding DEA LP problem in the Online Appendix (Equation A.4).

The solution from (A.4) does not take into account the dynamics of N use and flux over the stages of crop growth, as it represents the single-stage joint production model. To model the optimal timing of N fertilizer use, we use the N network framework as expressed in (A.2) and (A.3), which allows for the potential to reallocate applied N fertilizer between production stages. Here, the problem is to choose the optimal applied N fertilizer at each stage to jointly maximize final crop yields, y^S, and minimize total pollution, $\sum_S u^s$. Suppressing the stage index notation for parsimony, we define the directional output distance function for the network technology as

$$\vec{D}_{O_{NW}}\left(x, y, v, u; w, g_y, g_u\right) = \max_x \left\{\beta : \left(y + \beta g_y, u - \beta g_u\right) \in P_{NW}(x; w)\right\}. \tag{6}$$

Applying the N network technology constraints from Equations (A.2) and (A.3), we solve for the optimal N use at each stage as a single problem, found in Equation (A.5), which also includes the complete stage index notation for clarity. While we solve for optimal N use as a single problem in (A.5), we treat the stages of the underlying network technology in (A.2) and (A.3) as separable subtechnologies. This separability allows the technology to differ across the separate stages of crop growth, yet still be sequentially connected via the flow of intermediate output-inputs, (y^{s-1}, v^{s-1}), from one stage to the next, and the reallocation of N fertilizer, x^s, across stages.

For the final stage, S, we add the objective for harvested crop biomass, as well as the reallocation constraint, that total chosen applied N fertilizer does not exceed original applied N fertilizer. No additional N fertilizer is applied in the final stage. We note that N use at each stage is a choice variable. The total N use across all stages is restricted to be no greater than the observed total N use. Thus, any increased N use in one stage reduces the available remaining N to other stages, and vice versa.

5 | THE DNDC MODEL AND DATA

Having described the analytical framework and the estimation strategy, we now turn to the biophysical model of crop growth, which we use to generate the data for estimating both the single-stage and network technology models.

Several models (e.g., EPIC, AGRIN, DNDC) have been used by previous studies to assess the fate of nitrogen at various stages of crop development. Among these models, the DeNitrification DeComposition (DNDC) model has been widely applied to various crops to simulate nitrogen dynamics, greenhouse gas emissions, soil organic carbon (SOC) dynamics, and crop yields (Chen et al., 2016; Cui et al., 2014; Y. Zhang & Niu, 2016). In a survey of practitioners in the field, Gilhesphy et al. (2014) found that 41% of respondents used DNDC-based models for estimating nitrogenous gas emissions. Our study employs the DNDC model to simulate wheat crop growth, nitrogen flux, nitrogen in the soil, and nitrogen uptake by plants in a wheat-based rotation cropping system.

5.1 | Model calibration

The DNDC model was originally developed for predicting carbon sequestration and trace nitrogen and greenhouse gas emissions for agricultural lands, simulating the fundamental processes controlling the interactions among various ecological drivers, soil environmental factors, and relevant biochemical or geochemical reactions (C. Li et al., 1992, 1994, 1996). The model specifies parameters for management activities (e.g., crop rotation, tillage, fertilization, manure amendment, irrigation, and weeding) and links them to the various affected biogeochemical processes (e.g., crop growth, litter production, soil water infiltration, decomposition, nitrification, denitrification etc.). The DNDC model has been used in many agricultural economics studies, including very recently by De Laporte et al. (2021). A survey of the literature by Gilhesphy et al. (2014) that documents the use of DNDC in research over a 20-year period cites numerous studies that have used DNDC in an agricultural economics context (including DNDC Europe, and EFEM-DNDC). While the DNDC model itself does not incorporate economic behavior, it is capable of quantifying the consequences of such underlying behavior in terms of soil processes, nutrient cycling, and crop growth. This capability has led to the use of DNDC in economic studies to model the environmental effects from agricultural production.

Our study employs the DNDC model to estimate nitrogen dynamics across stages of wheat production, and under a variety of climate and soil conditions. We base our model calibration on experimental data collected from trials in Narrabri, New South Wales (NSW), Australia.

5.2 | Study area

Narrabri is one of the most productive agricultural regions in NSW. Major crops in this region include cotton, wheat, and pulses. Traditionally, winter cereals such as wheat have been the dominant crop in this area, primarily in the form of wheat–canola and wheat–chickpea crop rotations.

We use results from agronomic experiments conducted at the agricultural experimental station located in Narrabri to parameterize the DNDC model. We use daily rainfall and daily maximum and minimum temperatures from 2015 to 2017 in this area, collected from the Australian Bureau of Meteorology. We use the rainfall and temperature values both to calibrate the DNDC model to the study area, and together with soil pH, as environmental condition variables, w, in our production technology models. For the production technology models, we aggregate the daily rainfall to the total rainfall for each crop stage (network models) and the growing season as a whole (single-stage model). Similarly, we aggregate daily temperature to the average temperature for each crop stage (network models) and the growing season as a whole (single-stage model).

Initial fertilizer application rates were 100 kg/ha of urea and 10 kg/ha superphosphate for both years. The initial application times were during sowing and then at tillering. The specified fertilizer application rate and timing sequences for fertilizer application are based on typical currently recommended agronomic practices in the region. Application of supplemental irrigation was 65 mm in 2015, while there was no irrigation in 2017 due to sufficient moisture in the soil from rainfall.

5.3 | Nitrogen application scenarios

The DNDC model breaks wheat crop production into seven stages: (1) Tillage to sowing (52 days); (2) Germination to seedling development (10 days); (3) Tillering to stem elongation (60 days); (4) Booting to heading (20 days); (5) Flowering to milk (30 days); (6) Dough to ripening (30 days); (7) Ripening to crop cut (31 days).

We conduct a series of simulations with the DNDC model by altering the N fertilizer application rate and the timing of application. Specifically, we consider three different scenarios under five alternative total N fertilizer application rates (50, 80, 100, 120, and 150 kg/ha of urea). The typical agronomic recommendation for the study area is to apply a total of 100 kg Urea/ha of wheat. This is the mid-point of the simulated range of urea application rates that were used in the DNDC model. The other four rates that were used in the simulation intended to represent (a) a more tactical, fine-tuning of the rate in response to changing growing conditions, given by the variation of 20% up and down from the mid-point (i.e. the 80 and 120 kg/ha); and (b) a more fundamental shift to fertilizer management with the 50 kg/ha rate representing an extensive, low-input system, while the 150 kg/

TABLE 1 The alternative N fertilizer rates (Urea kg/ha) in the DNDC scenarios for wheat production.

	Scenario 1		Scenario 2		Scenario 3	
Total N fertilizer (Urea kg/ha)	**Stage 1**	**Stage 3**	**Stage 1**	**Stage 4**	**Stage 3**	**Stage 4**
50	50	0	50	0	50	0
	25	25	25	25	25	25
	35	15	35	15	35	15
	15	35	15	35	15	35
80	80	0	80	0	80	0
	40	40	40	40	40	40
	60	20	60	20	60	20
	20	60	20	60	20	60
100	100	0	100	0	100	0
	50	50	50	50	50	50
	75	25	75	25	75	25
	25	75	25	75	25	75
120	120	0	120	0	120	0
	60	60	60	60	60	60
	80	40	80	40	80	40
	40	80	40	80	40	80
150	150	0	150	0	150	0
	75	75	75	75	75	75
	100	50	100	50	100	50
	50	100	50	100	50	100

ha rate representing a more intensive high-input system. Together, this represents a realistic range of simulation values that cover many possible approaches to fertilizer management.

We ran the DNDC under each of these scenarios to simulate results for both 2015 and 2017 climate and soil conditions, summarized in Table 1. The scenarios are as follows:

- Scenario 1. Urea application rates (50, 80, 100, 120, and 150 kg/ha) divided in various proportions between the 1st (sowing) and 3rd (tillering) stages of wheat production;
- Scenario 2. Urea application rates (50, 80, 100, 120, and 150 kg/ha) divided in various proportions between the 1st and 4th (booting) stages of wheat production;
- Scenario 3. Urea application rates (50, 80, 100, 120, and 150 kg/ha) divided in various proportions between the 3rd and 4th stages of wheat production.

Our choice of which stages of wheat development to simulate fertilizer application (Stages 1, 3, and 4) coincides with the typical practices in the study area. Other stages are either too late in the crop development cycle (Stages 5–7) for N fertilization to have an impact on yield, or are too short in duration (Stage 2) to warrant simulating fertilizer application.

In summary, we ran simulations for 2015 and 2017 climate conditions, for two separate crop rotations in each year (wheat–canola and wheat–chickpea). Each simulation includes five alternative total urea application rates (50, 80, 100, 120, and 150 kg/ha) and three urea application timing scenarios for each application rate (Stages 1 and 3, Stages 1 and 4, Stages 3 and 4). We also conduct each simulation for both 6.5 and 7.5 soil pH values. For a given year and crop rotation, we model 40 production units for each of the three scenarios. This yields 120 production units for that crop and year, of which 60 are modeled with pH of 6.5 and 60 are modeled with pH of 7.5. Two years and two crop rotations yield 4 × 120, or 480 simulated production units. Each simulated production unit represents a complete wheat production process. We note that each simulation is for a single hectare plot, so that urea application rates are equivalent to total applied urea at each stage. We use the observations of these 480 simulations as data points for our DEA estimation of both the single-stage joint production and the network production models described in the previous section.

6 | RESULTS

6.1 | Results from the DNDC simulations

Table 2 summarizes the simulation data and results by crop rotation. For each rotation, exogenous production inputs include N fertilizer applied, initial soil N, soil pH, temperature, rainfall, and irrigation. The DNDC model uses these to simulate uptake of soil N and crop N, N losses in the form of NH_3 flux, and final harvested grain (Grain C), which is measured in units of carbon/ha.

Table 3 tracks the propagation of N stocks and flows through the crop growth cycle, by production stage. This includes N applied, soil and crop uptake, and losses to NH_3 flux. As expected, we see that the N application rates and NH_3 flux losses occur primarily in the initial crop stages, before crop uptake begins. Soil N levels increase with the initial application rates, then diminish with crop uptake in the latter stages.

TABLE 2 Summary statistics for the Narrabri DNDC simulated crop production data, years 2015 and 2017, wheat–chickpea and wheat–canola crop rotations (240 obs, each rotation).

Variable	Mean	Std. dev.	Min	Max
Wheat–chickpea rotation				
N applied (kg)	100.00	34.07	50.00	150.00
Soil N (kg)	5.31	0.61	4.48	5.99
Crop N (kg)	46.42	6.80	31.25	53.86
NH_3 flux (kg)	58.38	24.69	19.28	104.89
Grain C (kg)	1451.87	272.86	839.09	1751.76
Soil pH	7.00	0.50	6.50	7.50
Temperature (°C)	15.73	3.93	10.28	23.19
Rainfall (mm)	269.00	92.03	177.00	361.00
Irrigation (mm)	32.50	32.51	0.00	65.01
Wheat–canola rotation				
N applied (kg)	100.00	34.07	50.00	150.00
Soil N (kg)	4.01	0.18	3.82	4.30
Crop N (kg)	45.25	7.07	30.69	53.71
NH_3 flux (kg)	58.85	24.57	18.59	109.95
Grain C (kg)	1404.83	283.64	816.37	1751.76
Soil pH	7.00	0.50	6.50	7.50
Temperature (°C)	15.73	3.93	10.28	23.19
Rainfall (mm)	269.00	92.03	177.00	361.00
Irrigation (mm)	32.50	32.51	0.00	65.01

Note: Each DNDC simulation observation represents a single-hectare plot.

TABLE 3 Mean N levels (kg/ha) by crop stage for the Narrabri DNDC simulated crop production data, years 2015 and 2017, wheat–chickpea, and wheat–canola crop rotations (240 obs, each rotation).

Crop rotation	Stage	N applied	Soil N	Crop N	NH_3 flux
Wheat–chickpea	1	41.67	6.10	0.00	7.83
	2	0.00	22.61	0.00	12.49
	3	33.33	16.33	12.29	23.19
	4	25.00	17.51	25.09	13.44
	5	0.00	5.83	45.28	0.84
	6	0.00	1.29	46.42	0.30
	7	0.00	1.62	46.42	0.29
Wheat–canola	1	41.67	5.00	0.00	7.78
	2	0.00	21.78	0.00	12.44
	3	33.33	15.84	12.47	30.69
	4	25.00	16.03	25.28	6.69
	5	0.00	4.82	43.86	0.68
	6	0.00	0.98	45.25	0.28
	7	0.00	1.47	45.25	0.29

6.2 | Inefficiency results

We summarize the obtained inefficiency results for the three alternative fertilization scenarios in Table 4. For scaling purposes, we weight each of the model variables by its respective sample mean. We also specify a unit direction vector, setting $\left(g_y, -g_u\right) = (1, -1)$, to facilitate aggregation and direct comparison of distance values (Färe & Grosskopf, 2003; Zelenyuk, 2002). Our use of mean-weighting, coupled with the unit direction vector, changes the interpretation of distance values from levels to percentages. All distance values represent the maximal feasible expansion or contraction of output as a percentage of the respective sample mean. Using the desirable output final crop harvested, y^k, to explain this relationship for each kth observation, relative to the sample mean, $\overline{y}$,

$$\frac{y^k}{\overline{y}} + \vec{D}_O{}^k = \frac{y^{k^*}}{\overline{y}}, \tag{7}$$

and

$$y^k + \vec{D}_O{}^k \, \overline{y} = y^{k^*}, \tag{8}$$

so that the distance value for a hypothetical observation at the mean can be interpreted as the percent increase in observed final crop harvested required to reach the corresponding frontier value y^{k^*}.

The single-stage joint production results represent potential increases to final harvested grain yields and potential decreases to total NH_3 flux, as a percent of observed values. We find, across fertilization scenarios, that mean potential increases to grain yields range from 3.6% to 9.1% for the wheat–canola rotation and from 3.6% to 9.5% for the wheat–chickpea rotation. The corresponding mean potential decreases to NH_3 flux range from 3.8% to 9.4% and 3.8% to 10.2%. For both crop rotations, mean inefficiency results for the single-stage joint production model are lowest for Scenario 2 and highest for Scenario 1. In addition, we find higher mean inefficiency values across scenarios for the wheat–chickpea rotation.

TABLE 4 Mean percent inefficiency results by fertilization scenario.

Crop rotation	Inefficiency	Scenario (1)	Scenario (2)	Scenario (3)	All
Wheat–chickpea	JP grain C	0.095	0.036	0.051	0.061
	JP NH_3 flux	0.102	0.038	0.058	0.066
	NW grain C	0.003	0.003	0.009	0.005
	NW NH_3 flux				
	1	0.045	0.149	0.000	0.065
	2	0.038	0.015	0.000	0.018
	3	0.141	0.026	0.132	0.100
	4	0.001	0.058	0.051	0.036
	5	0.001	0.004	0.015	0.007
	6	0.001	0.001	0.001	0.001
	7	0.000	0.000	0.000	0.000
	Total	0.226	0.253	0.199	0.226
Wheat–canola	JP grain C	0.091	0.036	0.044	0.057
	JP NH_3 flux	0.094	0.038	0.049	0.060
	NW grain C	0.006	0.007	0.014	0.009
	NW NH_3 flux				
	1	0.041	0.066	0.000	0.036
	2	0.013	0.009	0.000	0.007
	3	0.040	0.076	0.052	0.056
	4	0.001	0.028	0.019	0.016
	5	0.001	0.003	0.013	0.006
	6	0.001	0.001	0.001	0.001
	7	0.000	0.000	0.000	0.000
	Total	0.096	0.184	0.085	0.122

Turning to the results from the multi-stage network technology model, the inefficiency values represent potential increases to final harvested grain yield, as before, and potential decreases to NH_3 flux *at each production stage*, rather than for the overall single-stage joint production process. We note that on the biophysical side, runoff can and does occur in stages without N application. See Table 3. For instance, though no fertilizer is applied in stage 2, there is still runoff in Stage 2 resulting from fertilizer that was previously applied in Stage 1. All distance values are in percent of sample mean, so that they can be compared across stages. Because runoff levels are already so low in the final stages, the distance function values for those stages will also be very low, which we see in the network model results.

These network model results reflect the potential to reallocate N fertilizer application between crop production Stages 1, 3, and 4 to improve NUE. We find that for both crop rotations, this reallocation leads to even greater potential reductions to N losses, while maintaining existing yields. For instance, we find potential total reductions to NH_3 flux range from 8.5% to 18.4% for the wheat–canola rotation and from 20.0% to 25.3% for the wheat–chickpea rotation. The overall N flux inefficiency is the smallest for Scenario 3, indicating that applying in later stages (Stages 3 and 4) is the least inefficient approach across the three scenarios. Targeting more efficient N use over the initial stages with an aim to reduce N losses seems to also restrict potential additional crop uptake of N in the latter stages.

We summarize the results for optimal reallocation of N fertilizer application in Table 5. First, consistent with our inefficiency results above, we find greater potential decreases to total N use, in comparing observed application rates to optimal solution values, for the wheat–chickpea rotation.

TABLE 5 Optimal N reallocation results by fertilization scenario.

Stage	Scenario 1 Observed	Scenario 1 Optimal	Scenario 2 Observed	Scenario 2 Optimal	Scenario 3 Observed	Scenario 3 Optimal
Wheat–chickpea						
1	62.50	53.62	62.50	26.20	0.00	0.30
3	37.50	26.69	0.00	1.03	62.50	56.71
4	0.00	0.03	37.50	35.65	37.50	39.38
Total N App.	100.00	80.33	100.00	62.88	100.00	96.39
NH_3 Flux	61.25	49.59	56.59	42.79	57.29	46.34
Grain C	1411.07	1414.77	1483.84	1497.65	1460.70	1472.75
Stage	**Scenario 1 Observed**	**Scenario 1 Optimal**	**Scenario 2 Observed**	**Scenario 2 Optimal**	**Scenario 3 Observed**	**Scenario 3 Optimal**
Wheat–canola						
1	62.50	58.97	62.50	54.40	0.00	0.18
3	37.50	35.43	0.00	9.35	62.50	63.23
4	0.00	0.00	37.50	33.20	37.50	35.33
Total N app.	100.00	94.40	100.00	96.95	100.00	98.75
NH_3 flux	60.83	55.53	57.59	46.95	58.12	53.12
Grain C	1356.56	1364.60	1436.60	1445.52	1421.34	1426.35

Note: All N values are measured in kg urea/ha.

For instance, these values represent approximately 20% reductions to N use under Scenario 1 and 40% under Scenario 2, while at the same time maintaining and even slightly increasing crop yield. These results correspond to larger potential decreases to total NH_3 Flux with N reallocation, as well as overall N reduction, for the wheat–chickpea rotation across scenarios. However, we find similar overall increases to yield for both crop rotations. When we shift to the timing of N application, we generally find larger optimal reductions for Stage 1, versus either lower reductions, or for some scenarios, slight increases to N application for Stages 3 and 4. These shifts are more pronounced for the wheat–chickpea rotation.

It is also important that we qualify these results. First, we apply our production modeling framework to simulated data, rather than empirical observations. The DNDC simulation model was also calibrated to experimental plot data. Thus, the simulated crop production data that we use for this application may not fully reflect the heterogeneity of crop production outcomes in practice. Inefficiency represents a form of residual, as the difference between observed production values and what we approximate as best possible production values at the production frontier. Given that our approximation of the production technology incorporates the same main variables from the DNDC simulation model, we would expect to find relatively low levels of inefficiency, which we do here. What is gained by using the DNDC model is that it allows us to decompose the production technology into the separate stages of crop growth, which is not generally possible with observational data at the field level, especially outside of experimental settings.

7 | CONCLUSION

Most crops grow and develop in stages. However, the vast majority of agricultural productivity analyses have traditionally modeled crop production as a single-stage joint production process, without explicitly taking into account the separate stages of crop development. We bridge this gap by explicitly modeling these separate stages of crop development in a network production model. Our approach is novel, in that it tracks N fertilizer applied, the fate of N in soil and crop uptake of N, as well as N flux losses to the environment from one stage of crop development to the next. This is particularly useful for modeling the productivity and environmental consequences of N fertilizer use, especially for policy and management goals to reduce the use of costly, environmentally damaging inputs without compromising crop production.

Even more generally, the approach and the findings in this paper point to the possibilities for modeling production processes that can be characterized by a multi-stage or network technology, as well as by the use of environmentally sensitive inputs that can result in environmental damages. The optimal intensity and timing of the use of such inputs, which this paper directly addresses, would be relevant in many contexts in agriculture and manufacturing where there are substantial environmental considerations.

We apply the network model to determine the efficiency of N fertilizer use and to derive optimal N fertilizer application for the case of wheat production in NSW Australia. We find that reallocating fertilizer across the stages of production and overall lowering of application rates is beneficial in terms of reducing N loss into the environment, without loss to productivity. Specifically, for the case study in question, we find that reallocating some of the N fertilizer application from earlier to later stages of crop development leads to substantial potential reductions to N loss while maintaining existing crop yields.

It is important that this type of modeling is done at the field scale, as this is the scale that is most relevant for the evaluation of the environmental effects from excess N fertilizer. We conduct the modeling at the field scale by utilizing a bio-physical model (DNDC) calibrated on actual experimental data for a wheat field in Narrabri, NSW, Australia.

By modeling at the field scale, and by explicitly considering the stages of crop development, our findings reinforce findings from other recent research that lowering N fertilizer application rates, and shifting the timing to later stages of crop development are sound practices from both an environmental and economic perspective. The robustness of these findings across methods, contexts, scales, and geographies augments their credibility. It is now up to research translation professionals to develop approaches for wider adoption and implementation of these practices.

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REFERENCES

Afriat, Sydney N. 1972. "Efficiency Estimation of Production Functions." *International Economic Review* 13(3): 568–598.

Ancev, Tihomir, Moriah Bostian, and Bradley Barnhart. 2023. "Productivity-Based Indicators for Nitrogen Use Efficiency." *Journal of Agricultural and Resource Economics* 48(1): 178–201.

Ang, Frederic, and Pieter J. Kerstens. 2016. "To Mix or Specialise? A Coordination Productivity Indicator for English and Welsh Farms." *Journal of Agricultural Economics* 67(3): 779–798.

Babcock, Bruce A. 1992. "The Effects of Uncertainty on Optimal Nitrogen Applications." *Review of Agricultural Economics* 14: 271–280.

Babcock, Bruce A., and Alfred M. Blackmer. 1994. "The Ex Post Relationship Between Growing Conditions and Optimal Fertilizer Levels." *Review of Agricultural Economics* 16: 353–362.

Barnhart, Bradley, Moriah Bostian, Shawna Grosskopf, Rolf Färe, and Sophia Lochner. 2023. "A Productivity Indicator for Adaptation to Climate Change." *PLOS Climate* 2(11): e0000199.

Barnhart, Bradley, Moriah Bostian, Gerald Whittaker, Shawna Grosskopf, and Rolf Färe. 2016. "Prioritizing Conservation for the Reduction of Gulf Hypoxia Using an Environmental Performance Index." *Ecological Indicators* 66: 235–241.

Berbel, Julio, and Javier Martínez-Dalmau. 2021. "A Simple Agro-Economic Model for Optimal Farm Nitrogen Application under Yield Uncertainty." *Agronomy* 11: 1107.

Bergtold, Jason, Steven Ramsey, Lucas Maddy, and Jeffery Williams. 2019. "A Review of Economic Considerations for Cover Crops as a Conservation Practice." *Renewable Agriculture and Food Systems* 34(1): 62–76.

Bostian, Moriah, Rolf Färe, Shawna Grosskopf, and Tommy Lundgren. 2016. "Environmental Investment and Firm Performance: A Network Approach." *Energy Economics* 57: 243–255.

Bostian, Moriah, Rolf Färe, Shawna Grosskopf, and Tommy Lundgren. 2018. "Network Representations of Pollution-Generating Technologies." *International Review of Environmental and Resource Economics* 11(3): 193–231.

Bostian, Moriah, Rolf Färe, Shawna Grosskopf, and Tommy Lundgren. 2022. "Prevention or Cure? Optimal Abatement Mix." *Environmental Economics & Policy Studies* 24(4): 503–531.

Bostian, Moriah, Rolf Färe, Shawna Grosskopf, Tommy Lundgren, and William Weber. 2018. "Time Substitution for Environmental Performance: The Case of Sweden Manufacturing." *Empirical Economics* 54(1): 129–152.

Cai, Siyuan, Xu Zhao, Cameron Pittelkow, Mingsheng Fan, Xin Zhang, and Xiaoyuan Yan. 2023. "Optimal Nitrogen Rate Strategy for Sustainable Rice Production in China." *Nature* 615: 73–79.

Chambers, Robert G., Yangho Chung, and Rolf Färe. 1996. "Benefit and Distance Functions." *Journal of Economic Theory* 70: 407–419.

Chambers, Robert G., Yangho Chung, and Rolf Färe. 1998. "Profit, Directional Distance Functions, and Nerlovian Efficiency." *Journal of Optimization and Applications* 98: 52–364.

Chambers, Robert G., and Simone Pieralli. 2020. "The Sources of Measured US Agricultural Productivity Growth: Weather, Technological Change, and Adaptation." *American Journal of Agricultural Economics* 102(4): 1198–1226.

Chambers, Robert G., Simone Pieralli, and Yu Sheng. 2020. "The Millennium Droughts and Australian Agricultural Productivity Performance: A Nonparametric Analysis." *American Journal of Agricultural Economics* 102(5): 1383–1403.

Charnes, Abraham, William W. Cooper, Boaz Golany, Lawrence M. Seiford, and Joel Stutz. 1985. "Foundations of Data Envelopment Analysis for Pareto-Koopmans Efficient Empirical Production Functions." *Journal of Econometrics* 30: 91–107.

Charnes, Abraham, William W. Cooper, and E. L. Rhodes. 1978. "Measuring Efficiency of Decisionmaking Units." *European Journal of Operational Research* 2(6): 429–444.

Chen, Han, Chooqing Yu, Changsheng Li, Qinchuan Xin, Xiao Huang, Jie Zhang, Yali Yue, Guorui Huang, Xuecao Li, and Wei Wang. 2016. "Modeling the Impacts of Water and Fertilizer Management on the Ecosystem Service of Rice Rotated Cropping Systems in China." *Agriculture, Ecosystems & Environment* 219: 49–57.

Chung, Yangho, Rolf Färe, and Shawna Grosskopf. 1997. "Productivity and Undesirable Outputs." *Journal of Environmental Management* 51(3): 229–240.

Coelli, Tim, Ludwig Lauwers, and Guido Van Huylenbroeck. 2007. "Environmental Efficiency Measurement and the Materials Balance Condition." *Journal of Productivity Analysis* 28: 3–12.

Cui, F., X. Zheng, C. Liu, K. Wang, Z. Zhou, and J. Deng. 2014. "Assessing Biogeochemical Effects and Best Management Practice for a Wheat-Maize Cropping System Using the DNDC Model." *Biogeosciences* 11(1): 91–107.

Dakpo, K. Hervé, and Alfons O. Lansink. 2019. "Dynamic Pollution-Adjusted Inefficiency Under the By-Production of Bad Outputs." *European Journal of Operational Research* 276(1): 202–211.

De Laporte, Aaron, Kamaljit Banger, Alfons Weersink, Claudia Wagner-Riddle, Brian Grant, and Ward Smith. 2021. "Economic and Environmental Consequences of Nitrogen Application Rates, Timing and Methods on Corn in Ontario." *Agricultural Systems* 188: 103018.

Del Rossi, Gemma, Mohammad M. Hoque, Yangjie Ji, and Catherine L. Kling. 2023. "The Economics of Nutrient Pollution From Agriculture." *Annual Review of Resource Economics* 15: 105–130.

FAO. 2022. "The State of the World's Land and Water Resources for Food and Agriculture—Systems at Breaking Point." Main Report. Rome.

Färe, Rolf, and Shawna Grosskopf. 1996. *Intertemporal Production Frontiers: With Dynamic DEA*. Norwell, MA: Springer.

Färe, Rolf, and Shawna Grosskopf. 2003. *New Directions: Efficiency and Productivity*. Boston, MA: Kluwer Academic Publishers.

Färe, Rolf, Shawna Grosskopf, Dimitris Margaritis, and William Weber. 2012. "Technological Change and Timing Reductions in Greenhouse Gas Emissions." *Journal of Productivity Analysis* 37: 205–216.

Färe, Rolf, Shawna Grosskopf, and Carl Pasurka. 2013. "Joint Production of Good and Bad Outputs With a Network Application." In *Encyclopedia of Energy, Natural Resource, and Environmental Economics*, Vol 2, edited by Jay F. Shogren, 109–118. Amsterdam: Elsevier.

Farrell, Michael J. 1957. "The Measurement of Productive Efficiency." *Journal of the Royal Statistical Society (A, General)* 120-(Part 3): 253–290.

Fixen, Paul E. 2020. "A Brief Account of the Genesis of 4R Nutrient Stewardship." *Agronomy Journal* 112: 4511–18.

Førsund, Finn. 2018a. “Pollution Meets Efficiency: Multi-Equation Modelling of Generation of Pollution and Related Efficiency Measures.” In *Energy, Environment and Transitional Green Growth in China*, edited by R. Pang, X. Bai, and K. Lovell. Singapore: Springer.

Førsund, Finn. 2018b. “Multi-Equation Modeling of Desirable and Undesirable Outputs Satisfying the Materials Balance.” *Empirical Economics* 54: 67–99.

Frisch, Ragnar. 1965. *Theory of Production*. Dordrecht: D. Reidel.

Gilhesphy, Sarah L., Steven Anthony, Laura Cardonas, David Chadwick, Agustin Prado, Changsheng Li, Thomas Misselbrook, et al. 2014. “First 20 Years of DNDC(DeNitrification DeComposition): Model Evaluation.” *Ecological Economics* 292: 51–62.

Hampf, Benjamin. 2014. “Separating Environmental Efficiency Into Production and Abatement Efficiency–A Nonparametric Model With Application to U.S. Power Plants.” *Journal of Productivity Analysis* 41(3): 457–473.

Hoang, Viet-Ngu, and Clevo Wilson. 2017. “Accounting for Nutrient Pollution in Measuring Agricultural Total Factor Productivity: A Study of OECD Economies.” In *New Directions in Productivity Measurement and Efficiency Analysis*, edited by Tihomir Ancev, M. A. S. Azad, and Francesco Hernández-Sancho. Cheltenham, UK: Edgar Elgar Online.

Hou, Yong, Gerard L. Velthof, Jan P. Lesschen, Igor G. Staritsky, and Oene Oenema. 2017. “Nutrient Recovery and Emissions of Ammonia, Nitrous Oxide, and Methane From Animal Manure in Europe: Effects of Manure Treatment Technologies.” *Environmental Science & Technology* 51: 375–383.

Isik, Murat, and Madhu Khanna. 2002. “Variable-Rate Nitrogen Application Under Uncertainty: Implications for Profitability and Nitrogen Use.” *Journal of Agricultural and Resource Economics* 27(1): 61–76.

Kao, Chiang. 2017. “Network Data Envelopment Analysis—Foundations and Extensions.” In *International Series in Operations Research and Management Science 240*. Switzerland: Springer International Publishing.

Kazemi-Matin, Reza, and Roza Azizi. 2016. “Efficiency Evaluation of Wheat Farming: A Network Data Envelopment Analysis Approach.” *Journal of Optimization in Industrial Engineering* 9(20): 103–9.

Lamkowsky, Melina, Oene Oenema, Miranda Meuwissen, and Frederic Ang. 2021. “Closing Productivity Gaps among Dutch Dairy Farms Can Boost Profit and Reduce Nitrogen Pollution.” *Environmental Research Letters* 16(12): 124003.

Li, Changsheng, Steve Frolking, and Todd Frolking. 1992. “A Model of Nitrous Oxide Evolution from Soil Driven by Rainfall Events: 1. Model Structure and Sensitivity.” *Journal of Geophysical Research* 97: 9759–76.

Li, Changsheng, Steve Frolking, and Robert Harriss. 1994. “Modelling Carbon Biogeochemistry in Agricultural Soils.” *Global Biogeochemical Cycles* 8: 237–254.

Li, Changsheng, Vijay Narayanan, and Robert Harriss. 1996. “Model Estimates of Nitrous Oxide Emissions from Agricultural Lands in the United States.” *Global Biogeochemical Cycles* 10: 297–306.

Liu, Wen-Bin, Chin-Wei Tso, Ke-Chung Peng, Chao-Chang Chiu, and Meing-Long Shih. 2012. “Application of Network DEA in the Assessment of Operating Performances of Farmers Associations in Taiwan.” *Journal of Interdisciplinary Mathematics* 15(1): 65–81.

Murty, Sushama, and Robert R. Russell. 2018. “Modeling Emission-Generating Technologies: Reconciliation of Axiomatic and By-Production Approaches.” *Empirical Economics* 54: 7–30.

Murty, Sushama, Robert R. Russell, and Steven B. Levkoff. 2012. “On Modeling Pollution-Generating Technologies.” *Journal of Environmental Economics and Management* 64: 117–135.

Njuki, Eric, Boris E. Bravo-Ureta, and Victor E. Cabrera. 2020. “Climatic Effects and Total Factor Productivity: Econometric Evidence for Wisconsin Dairy Farms.” *European Review of Agricultural Economics* 47(3): 1276–1301.

Njuki, Eric, Boris E. Bravo-Ureta, and Christopher J. O’Donnell. 2019. “Decomposing Agricultural Productivity Growth Using a Random-Parameters Stochastic Production Frontier.” *Empirical Economics* 57(3): 839–860.

O’Donnell, Christopher J. 2016. “Using Information About Technologies, Markets and Firm Behaviour to Decompose a Proper Productivity Index.” *Journal of Econometrics* 190(2): 328–340.

Pappalardo, Massimo. 2008. “Vector Optimization.” In *Encyclopedia of Optimization*, edited by C. Floudas and P. Pardalos. Boston, MA: Springer.

Paulson, N., and B. Babcock. 2010. “Readdressing the Fertilizer Problem.” *Journal of Agricultural and Resource Economics* 35: 368–384.

Prieto, Angel M., and José L. Zofio. 2007. “Network DEA Efficiency in Input–Output Models: With an Application to OECD Countries.” *European Journal of Operational Research* 177: 292–304.

Rabotyagov, Sergey, Todd Campbell, Manoj Jha, Philip Gassman, Jeffrey Arnold, Lyubov Kurkalova, Silvia Secchi, Hongli Feng, and Catherine Kling. 2010. “Least-Cost Control of Agricultural Nutrient Contributions to the Gulf of Mexico Hypoxic Zone.” *Ecological Applications* 20(6): 1542–55.

Rabotyagov, Sergey, Todd Campbell, Michael White, Jeffrey Arnold, Jay Atwood, M. Lee Norfleet, Catherine Kling, et al. 2014. “Cost-Effective Targeting of Conservation Investments to Reduce the Northern Gulf of Mexico Hypoxic Zone.” *Proceedings of the National Academy of Sciences of the United States of America* 111(52): 18530–35.

Ray, Subhash. 2004. *Data Envelopment Analysis: Theory and Techniques for Economics and Operations Research*. Cambridge, UK: Cambridge University Press.

Reinhard, Stijn, C. A. Knox Lovell, and Geert Thijssen. 1999. “Econometric Estimation of Technical and Environmental Efficiency: An Application to Dutch Dairy Farms.” *American Journal of Agricultural Economics* 81(1): 44–60.

Reinhard, Stijn, C. A. Knox Lovell, and Geert Thijssen. 2000. "Environmental Efficiency With Multiple Environmentally Detrimental Variables; Estimated With SFA and DEA." *European Journal of Operational Research* 121(2): 287–303.

Reinhard, Stijn, and Geert Thijssen. 2000. "Nitrogen Efficiency of Dutch Dairy Farms: A Shadow Cost System Approach." *European Review of Agricultural Economics* 27(2): 167–186.

Robertson, Dale, David Saad, and Gregory Schwarz. 2014. "Spatial Variability in Nutrient Transport by HUC8, State, and Subbasin Based on Mississippi/Atchafalaya River Basin SPARROW Models." *JAWRA Journal of the American Water Resources Association* 50(4): 988–1009.

Robertson, Dale, Gregory Schwarz, David Saad, and Richard Alexander. 2009. "Incorporating Uncertainty Into the Ranking of Sparrow Model Nutrient Yields From Mississippi/Atchafalaya River Basin Watersheds." *JAWRA Journal of the American Water Resources Association* 45: 534–549.

Rødseth, Kenneth. 2013. "Capturing the Least Costly Way of Reducing Pollution: A Shadow Price Approach." *Ecological Economics* 92: 16–24.

Rødseth, Kenneth. 2017. "Axioms of a Polluting Technology: A Materials Balance Approach." *Environmental and Resource Economics* 67(1): 1–22.

Sellers, Sarah, Gary Schnitkey, and Laura Gentry. 2020. "*Do Illinois Farmers Follow University-Based Nitrogen Recommendations?*" In Proceedings of the Agricultural and Applied Economics Association Annual Meeting, Kansas City, MO, USA, July 26–28.

Shephard, Ronald W. 1953. *Cost and Production Functions*. Princeton, NJ: Princeton University Press.

Shephard, Ronald W. 1970. *Theory of Cost and Production Functions*. Princeton, NJ: Princeton University Press.

Sheriff, Glenn. 2005. "Efficient Waste? Why Farmers Over-Apply Nutrients and the Implications for Policy Design." *Review of Agricultural Economics* 27(4): 542–557.

Sinha, Eva, Anna M. Michalak, and Venkatramani Balaji. 2017. "Eutrophication Will Increase During the 21st Century as a Result of Precipitation Changes." *Science* 357: 405–8.

Tembo, Gelson, B. Wade Brorsen, Francis Epplin, and Emilio Tostao. 2008. "Crop Input Response Functions With Stochastic Plateaus." *American Journal of Agricultural Economics* 90: 424–434.

Thanassoulis, Emmanuel, Maria C. S. Portela, and Ozren Despić. 2008. "Data Envelopment Analysis: The Mathematical Programming Approach to Efficiency Analysis." In *The Measurement of Productive Efficiency and Productivity Growth*, edited by Harold Fried, C. A. Knox Lovell, and Shelton S. Schmidt, 251–420. Oxford, UK: Oxford University Press.

Tumisiime, Emmanuel, B. Wade Brorsen, Jagadeesh Mosali, Jim Johnson, James Locke, and Jon T. Biermacher. 2011. "Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models." *Journal of Agricultural and Applied Economics* 43: 541–552.

Wang, Scarlett, Frederic Ang, and Alfons O. Lansink. 2023. "Mitigating Greenhouse Gas Emissions on Dutch Dairy Farms. an Efficiency Analysis Incorporating the Circularity Principle." *Agricultural Economics* 54(6): 819–837.

Wei, Quanling, Jianzhong Zhang, and Xiangsun Zhang. 2000. "An Inverse DEA Model for Inputs/Outputs Estimate." *European Journal of Operational Research* 121(1): 151–163.

Yanni, Sandra, Predrag Rajsic, Claudia Wagner-Riddle, and Alfons Weersink. 2018. "*A Review of the Efficacy and Cost-Effectiveness of On-Farm BMP's for Mitigating Soil-Related GHG Emissions*." Institute for the Advanced Study of Food and Agricultural Policy, Department of Food, Agriculture, and Resource Economics, University of Guelph. Working Paper Series WP 18-05.

Zelenyuk, Valentin. 2002. "*Aggregation of the Nerlovian Profit Indicator*." In Essays in Efficiency and Productivity Analysis of Economic Systems. Oregon State University Doctor of Philosophy Dissertation.

Zhang, Xin, Eric A. Davidson, Denise L. Mauzerall, Timothy D. Searchinger, Patrice Dumas, and Ye Shen. 2015. "Managing Nitrogen for Sustainable Development." *Nature* 528: 51–59.

Zhang, Yajie, and Haishan Niu. 2016. "The Development of the DNDC Plant Growth Sub-Model and the Application of DNDC in Agriculture: A Review." *Agriculture, Ecosystems & Environment* 230: 271–282.

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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