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Magnus Ekström
Sara Sjöstedt-deLuna

Arbetsrapport 90 2001
Working Paper 90 2001

SWEDISH UNIVERSITY OF
AGRICULTURAL SCIENCES (SLU)
Department of Forest Resource
Management and Geomatics
S-901 83 UMEÅ
Phone: 090-786 58 25 Fax: 090-14 19 15, 77 81 16

ISSN 1401-1204
ISRN SLU-SRG-AR--90 --SE

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Magnus Ekström
Swedish University of Agricultural Sciences
S-901 83 Umeå, Sweden
Magnus.Ekstrom@resgeom.slu.se

Sara Sjöstedt-de Luna
Umeå University
S-901 87 Umeå, Sweden
sara@matstat.umu.se

Abstract

Subsampling and block resampling methods have been suggested in the literature to nonparametrically estimate the variance of some statistic computed from spatial data. Usually stationary data are required. However, in empirical applications, the assumption of stationarity can often be rejected. This paper proposes nonparametric methods to estimate the variance of sample means based on nonstationary spatial data using subsampling. It is assumed that data is observed on a rectangular lattice in some subregion of $\mathbf{R}^2$. The kind of data we consider is of the following type: The information in the different picture elements (pixels) of the lattice are allowed to come from different distributions, with smoothly varying expected values, or with expected values decomposed additively into directional components. Furthermore, pixels are assumed to be locally dependent, and the dependence structure is allowed to differ over the lattice. Consistent variance estimators for sample means, and convergence rates in mean square, are provided under these assumptions. An example with applications to forestry, using satellite data, is discussed.

Key words: bootstrap, nonidentically distributed variables, nonindependent variables, resampling, subsampling.

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*Magnus Ekström is Assistant Professor, Department of Forest Resource Management and Geomatics, Swedish University of Agricultural Sciences, S-901 83 Umeå, Sweden (E-mail: Magnus.Ekstrom@resgeom.slu.se); and Sara Sjöstedt-de Luna is Assistant Professor, Department of Mathematical Statistics, Umeå University, S-901 87 Umeå, Sweden (E-mail: sara@matstat.umu.se). This work is a part of the Swedish research programme Remote Sensing for the Environment, RESE (home page http://rese.satellus.se), financed by the Swedish Foundation for Strategic Environmental Research, MISTRA. The authors thank Mats Nilsson for valuable help in the preparation of the forestry example.

1 INTRODUCTION

Suppose we have observed data on a spatial lattice over some (possibly irregularly shaped) region A. Suppose further that a statistic $s(A)$ is computed based on this data, which estimates some unknown parameter. In order to do inference it is desirable to estimate the variance of $s(A)$. In many cases this can be done without the need to know the underlying dependence structure, and the marginal and joint distribution of the data. For instance, Hall (1985, 1988), Possolo (1991), Politis and Romano (1993), Sherman and Carlstein (1994), and Sherman (1996) propose such nonparametric variance estimators, based on different block resampling and subsampling methods, under the assumption of stationarity. However, in many empirical applications, e.g. in forestry, agricultural experiments and fishery, the hypothesis of stationarity can easily be rejected:

Example In forestry, it is important to know different characteristics about the forest, such as wood volume, age, changes, etc.. Traditionally, this information has been obtained from small samples of field data. In the last decades, people have looked at the possibilities of using remotely sensed data, like satellite images, for this purpose as well (see e.g. Kilkki and Päivinen 1987; Fazakas, Nilsson, and Olsson 1999; Nilsson and Sandström 2001). These spatially dense data give information on pixel level (e.g. the Landsat TM has pixel size 25×25 m). By integrating field data and satellite data, prediction at the pixel level for different forest variables can be made (by using regression techniques, kriging, the k Nearest Neighbor (kNN) method, etc.). This opens up the possibilities for receiving good information on smaller regions than was previously possible. Suppose we are interested in the total wood volume over some region A. Let $\mu_{\boldsymbol{i}}$ denote the true wood volume in pixel $\boldsymbol{i}= (i_1, i_2)$, and let $X_{\boldsymbol{i}}$ denote an unbiased predictor of $\mu_{\boldsymbol{i}}$. Then we have $X_{\boldsymbol{i}} = \mu_{\boldsymbol{i}}+\varepsilon_{\boldsymbol{i}}$, where $\varepsilon_{\boldsymbol{i}}$ is the prediction error. To estimate the total wood volume $\mu_A = \sum \mu_{\boldsymbol{i}}$ over A, we use the (observable) total of the predicted values, $X_A = \sum X_{\boldsymbol{i}}$, over the same region as a point estimate. Due to variation on the ground, we may run into difficulties when trying to construct a valid variance estimator of X_A:

- The prediction errors $\{\varepsilon_{\boldsymbol{i}}\}$ may be differently distributed.
- The $\{\varepsilon_{\boldsymbol{i}}\}$ may be dependent with different types of dependence in different parts of the region.
- The true values $\{\mu_{\boldsymbol{i}}\}$ may vary, but usually tend to be similar for pixels close to each other.

Moreover, the marginal and joint distributions and the dependence structure are usually unknown. Hence, we want a variance estimator that can handle these types of nonstationarity without assuming too much about the distribution and the dependence structure.

Recently, some attention has been drawn to *nonstationary* spatial lattice data. Ekström (2001) showed, when the statistic is the sample mean, that the resampling

methods derived for dependent but stationary spatial observations (Politis and Romano 1993; Sherman 1996) can still be employed if the assumption of stationarity is violated. The observations in Ekström (2001) are allowed to be differently distributed and locally dependent, but with the same expected value, or with expected values that can be decomposed additively into directional components. Sjöstedt-de Luna (2001) focused on another type of nonstationarity, where the spatial data are independent but differently distributed with smoothly varying expected values.

In the next section, we propose a subsampling variance estimator for sample means that can handle differently distributed and locally dependent observations with smoothly varying expected values. Section 3 summarizes the main results, such as consistency and convergence rates in mean square. A simulation study is presented in Section 4 to evaluate the performance of the proposed variance estimator. Section 5 provides an illustration and application to forestry with data from northern Sweden. A summary and concluding remarks are given in Section 6. Proofs are found in the Appendix.

We use the following notation: $a_n \sim b_n$ means that for two sequences $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ there exist two constants, $0 < c_1 < c_2 < \infty$, such that $c_1 < a_n/b_n < c_2$, $n = 1, 2, ...$; $\mathbf{a} = (a_1, a_2) \to \infty$ means that *both* a_1 and a_2 tend to ∞.

2 ESTIMATION OF THE VARIANCE

In this section, we discuss nonparametric ways of estimating the variance of sample means based on nonstationary lattice data from a region $A \subset \mathbf{R}^2$. For simplicity, we assume that A is of rectangular shape, consisting of $N = n_1 n_2$ pixels, although other shapes are possible. Hence, we have observed spatially indexed data $\{X_{\boldsymbol{i}} : \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}\}$, where $\mathcal{I}_{\boldsymbol{n}} = \{\boldsymbol{i} = (i_1, i_2) : i_1 = 1, ..., n_1, \text{ and } i_2 = 1, ..., n_2\}$, $\boldsymbol{n} = (n_1, n_2)$. If we had independent replicates of $\bar{X}_A = X_A/N$, we would estimate the variance of $\bar{X}_A$ by calculating the sample variance over the replicates. Usually, $\bar{X}_A$ is observed only once, and therefore we would like to construct pseudoreplicates of $\bar{X}_A$ from the data available in A, and use them instead.

Let $B_{\boldsymbol{i}}$ denote a subshape of A positioned at pixel $\boldsymbol{i}$, i.e. a rectangular block consisting of $K = k_1 k_2 < N$ pixels, with the lower left corner in pixel $\boldsymbol{i}$. If the region A is not rectangular, subshapes can be constructed as described by e.g. Sherman (1996). We can construct $N' = n_1' n_2'$ different (overlapping) blocks that lie within A, where $n_j' = n_j - k_j + 1$, $j = 1, 2$, $\boldsymbol{n}' = (n_1', n_2')$. Let $\bar{X}_{\boldsymbol{i}}$ denote the mean value of the predicted values in block $B_{\boldsymbol{i}}$. In the case when $\{X_{\boldsymbol{i}}\}$ is stationary, one can regard the subshape means $\bar{X}_{\boldsymbol{i}}$ as pseudoreplicates of $\bar{X}_A$. Therefore, a reasonable estimator of the variance $\gamma_{\boldsymbol{n}} = NVar[\bar{X}_A]$, would be the sample variance over the subshape means, normalized with the subshape size K, i.e.

$$\tilde{\gamma}_{\boldsymbol{n}} = \frac{K}{N'} \sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}'}} (\bar{X}_{\boldsymbol{i}} - \bar{X}')^2,$$

where $\bar{X}' = \sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}'}} \bar{X}_{\boldsymbol{i}}/N'$. Under mixing conditions, consistency is achieved if the

size of the subshapes (K) increases with N, see e.g. Sherman (1996). The subsampling estimator $\tilde{\gamma}_{\boldsymbol{n}}$ also works for locally dependent and possibly *differently* distributed spatial data if $\mu_{\boldsymbol{i}} \equiv \mu$ (see Lemma 2). Locally dependent in this context means that the predicted values $\{X_{\boldsymbol{i}}\}$ are *spatially* $\boldsymbol{m}$*-dependent,* $\boldsymbol{m} = (m_1, m_2)$, in the sense that

$$X_{\boldsymbol{i}} \text{ and } X_{\boldsymbol{j}} \text{ are independent whenever } |i_1 - j_1| > m_1 \text{ or } |i_2 - j_2| > m_2. \qquad (1)$$

However, if the $\mu_{\boldsymbol{i}}$'s vary smoothly over region A, this estimator will not be consistent due to variation coming from the expected values. To reduce this variation we propose a modified subsampling method based on the crosswise block differences, $Z_{\boldsymbol{i}} = \bar{X}_{\boldsymbol{i}} - \bar{X}_{i_1+d_1+k_1,i_2} + \bar{X}_{i_1+d_1+k_1,i_2+d_2+k_2} - \bar{X}_{i_1,i_2+d_2+k_2}, i_j = 1..., n''_j$, where $n''_j = n'_j - k_j - d_j$, $j = 1, 2$, $\boldsymbol{n}'' = (n''_1, n''_2)$, see Figure 1. Here $\boldsymbol{d} = (d_1, d_2)$ measures the horizontal and vertical distance between blocks.

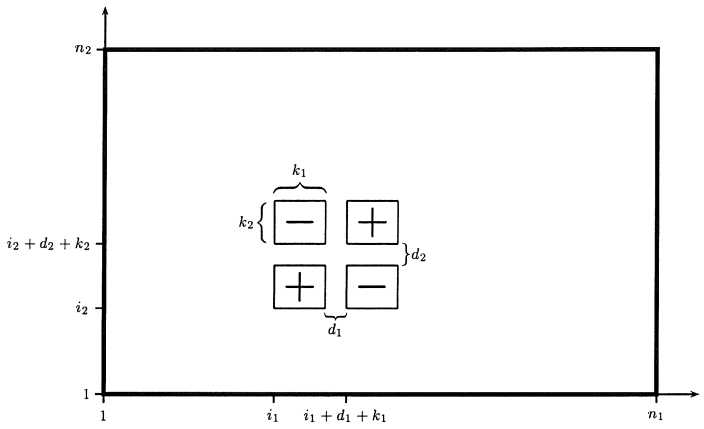

Figure 1. Placement of the four blocks defining a crosswise block difference at position $\boldsymbol{i}$.

The idea is that the $\mu_{\boldsymbol{i}}$'s in the blocks with plus signs in Figure 1 will tend to be similar to those in blocks with minus signs. Therefore, the crosswise block differences $\{Z_{\boldsymbol{i}}\}$ will have more or less constant expected values, and thus reduce the variation coming from the $\mu_{\boldsymbol{i}}$'s. An estimator, $\hat{\gamma}_{\boldsymbol{n}}$, of $\gamma_{\boldsymbol{n}}$ can be formed as the sample variance over the crosswise block differences, normalized by $K/4$,

$$\hat{\gamma}_{\boldsymbol{n}} = \frac{K}{4N''} \sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}''}} (Z_{\boldsymbol{i}} - \bar{Z})^2,$$

where $N'' = n''_1 n''_2$, and $\bar{Z} = \sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}''}} Z_{\boldsymbol{i}}/N''$.

In order for the estimators $\tilde{\gamma}_{\boldsymbol{n}}$ and $\hat{\gamma}_{\boldsymbol{n}}$ to be consistent, we have to let the block size (but not necessarily $\boldsymbol{d}$) grow with the number of observations. A discussion on choices of $\boldsymbol{d}$ and $\boldsymbol{k} = (k_1, k_2)$ are found in Sections 4 and 5.

Note that even if the predictors $\{X_{\boldsymbol{i}}\}$ in the example in the previous section are biased, $\hat{\gamma}_{\boldsymbol{n}}$ still provides a consistent estimator of the variance $\gamma_{\boldsymbol{n}}$, if the expected values, $E[X_{\boldsymbol{i}}]$, tend to be similar for neighboring pixels.

3 MAIN RESULTS

Assume that for each $\boldsymbol{n}$, the data $\{X_{\boldsymbol{i}} : \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}\}$ are spatially $\boldsymbol{m}$-dependent with uniformly bounded moments of order $2+\delta$, $\delta > 0$, such that

$$E[|X_{\boldsymbol{i}}|^{2+\delta}] < \tau_\delta < \infty, \quad \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}, \text{ for all } \boldsymbol{n}. \tag{2}$$

Furthermore, assume that the expected values, $E[X_{\boldsymbol{i}}] = \mu_{\boldsymbol{i}}$, are smoothly varying in the sense that they satisfy a *Lipschitz condition of order* α. More specifically, it is assumed that for all $\boldsymbol{n}$, $\mu_{\boldsymbol{i}} = f(i_1/n_1, i_2/n_2)$, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}$, where

$$|f(\mathbf{x}) - f(\mathbf{y})| \leq c\,\|\mathbf{x}-\mathbf{y}\|^\alpha, \text{ for some } 0 < \alpha \leq 1. \tag{3}$$

Here $\|\cdot\|$ is the Euclidean norm, and c is some positive constant. Alternatively, the expected values may be decomposed additively into directional components so that for all $\boldsymbol{n}$,

$$E[X_{\boldsymbol{i}}] = \mu + c_{i_1} + r_{i_2}, \text{ where } \sum_{i_1=1}^{n_1} c_{i_1} = 0 \text{ and } \sum_{i_2=1}^{n_2} r_{i_2} = 0, \ \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}. \tag{4}$$

A combination of the two assumptions for the expected values yield, for all $\boldsymbol{n}$,

$$X_{\boldsymbol{i}} = Y_{\boldsymbol{i}} + c_{i_1} + r_{i_2}, \text{ where } \sum_{i_1=1}^{n_1} c_{i_1} = 0 \text{ and } \sum_{i_2=1}^{n_2} r_{i_2} = 0, \ \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}, \tag{5}$$

and where $E[Y_{\boldsymbol{i}}] = f(i_1/n_1, i_2/n_2)$, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}$, satisfy (3). The consistency results of $\hat{\gamma}_{\boldsymbol{n}}$ are summarized in the two theorems below. The assumptions A1 and A2 that control $\boldsymbol{k}$ and $\boldsymbol{d}$ are presented in the beginning of the Appendix.

Theorem 1 *Assume that (1), (2), (5), A1, and A2 hold. Then* $\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ *as* $\boldsymbol{k}, \boldsymbol{n} \to \infty$. *If* $\delta \geq 2$, $n_1 \sim n_2$, *and* $k_1 \sim k_2$, *we also have* $E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + 1/K + K^2(K/N)^{2\alpha})$.

Theorem 2 *Assume that (1), (2), (4), and A1 hold. Then* $\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ *as* $\boldsymbol{k}, \boldsymbol{n} \to \infty$. *If* $\delta \geq 2$, $n_1 \sim n_2$, *and* $k_1 \sim k_2$, *we also have* $E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + 1/K)$.

In the two theorems it is not necessary to assume that $\gamma_{\boldsymbol{n}}$ converges to some limit value γ. The distances $\boldsymbol{d}$ between the blocks in the crosswise differences should be chosen as $d_i = O(k_i)$ (see A1), but note that this does not exclude $\boldsymbol{d}$ from being fixed. If $\alpha = 1$ in Theorem 1, which corresponds to f being continuously differentiable, then the optimal convergence rate in mean square is $O(N^{-2/5})$, achieved if we choose $k_i = O(n_i^{2/5})$. It is also interesting to observe that in Theorem 2 the optimal convergence rate in mean square is $O(N^{-1/2})$, which we get if we choose $k_i = O(n_i^{1/2})$. This is the same rate of convergence as for stationary sequences (e.g. Sherman 1996) where $\tilde{\gamma}_{\boldsymbol{n}}$ is used.

Remark 1 The results in this paper hold for more general data than indicated, i.e. the results are valid for triangular arrays $\mathbf{X}_{\boldsymbol{n}} = \{X_{\boldsymbol{i},\boldsymbol{n}} : \boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}\}$, $n_1 = 1, 2, ..., n_2 = 1, 2, ...$, of collections of random variables, such that for each $\boldsymbol{n}$, the random variables in $\mathbf{X}_{\boldsymbol{n}}$ are $\boldsymbol{m}$-dependent, and where $f(\cdot)$, μ, r_{i_2}, and c_{i_1}, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}$, may all depend on $\boldsymbol{n}$.

Remark 2 For the convergence rates in their most general form, where $n_1 \sim n_2$ and $k_1 \sim k_2$ are not required, see (18) for Theorem 1 and (16) for Theorem 2.

Remark 3 If the expected values of $X_{\boldsymbol{i}}$ are smoothly varying such that they satisfy (3), then we do not need crosswise differences. We could instead use the differences between two blocks like e.g. $Z_{\boldsymbol{i}} = \bar{X}_{\boldsymbol{i}} - \bar{X}_{i_1+d_1+k_1,i_2}$ or $Z_{\boldsymbol{i}} = \bar{X}_{\boldsymbol{i}} - \bar{X}_{i_1,i_2+d_2+k_2}$. This can be seen from the proof of Theorem 1. However, by using crosswise differences we have a more robust variance estimator that can handle additive row and column effects as well.

4 A SIMULATION STUDY

In this Monte Carlo study, we investigate and compare the performance of the two variance estimators, $\tilde{\gamma}_{\boldsymbol{n}}$ and $\hat{\gamma}_{\boldsymbol{n}}$.

First, we generated *stationary*, spatially $\boldsymbol{m}$-dependent, and lognormally distributed data $X_{\boldsymbol{i}}$. Here, each $X_{\boldsymbol{i}} = \prod_{j_1=i_1-l_1}^{i_1+l_1} \prod_{j_2=i_2-l_2}^{i_2+l_2} Y_{\boldsymbol{j}}$, for some integers l_1 and l_2, where the $Y_{\boldsymbol{j}}$'s are independent and lognormally distributed with $E[\log Y_{\boldsymbol{j}}] = 0$ and $Var[\log Y_{\boldsymbol{j}}] = \sigma^2 = 0.02^2$ for all $\boldsymbol{j}$ and $\boldsymbol{n}$. Thus, the $X_{\boldsymbol{i}}$'s are spatially $\boldsymbol{m}$-dependent with $m_i = 2l_i$, $i = 1, 2$, and lognormally distributed with $E[\log X_{\boldsymbol{i}}] = 0$ and $Var[\log X_{\boldsymbol{i}}] = (m_1 + 1)(m_2 + 1)\sigma^2$ for all $\boldsymbol{i}$ and $\boldsymbol{n}$. In Figures 2 and 3 we see, for different values of $\boldsymbol{m}$ and $\boldsymbol{n}$, that $\tilde{\gamma}_{\boldsymbol{n}}$ performs better than $\hat{\gamma}_{\boldsymbol{n}}$ in terms of root mean square error (RMSE), as was to be expected. However, the difference is small if we are able to choose $\boldsymbol{k}$ and $\boldsymbol{d}$ in a good way. Each value of RMSE in this section is based on 1000 replicates of $\tilde{\gamma}_{\boldsymbol{n}}$ or $\hat{\gamma}_{\boldsymbol{n}}$.

As seen in Figures 2 and 3, there is no advantage in choosing the d_i's larger than

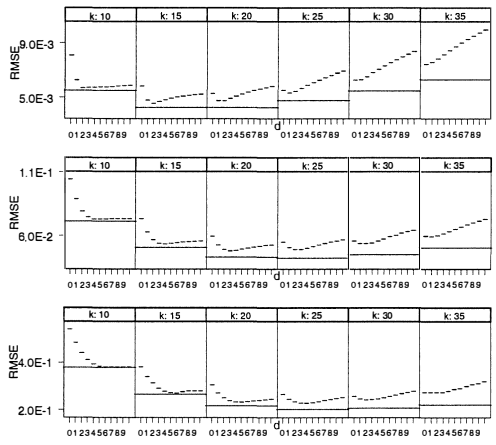

Figure 2. The RMSE for $\tilde{\gamma}_{\boldsymbol{n}}$ (long lines) and $\hat{\gamma}_{\boldsymbol{n}}$ (short lines) using different block sizes $k \times k$ and distances $\boldsymbol{d} = (d, d)$. The top picture corresponds to $\boldsymbol{m} = \mathbf{2} = (2, 2)$ and $\gamma_{\boldsymbol{n}} \approx 0.032$, the middle picture to $\boldsymbol{m} = \mathbf{4}$ and $\gamma_{\boldsymbol{n}} \approx 0.249$, and the bottom picture to $\boldsymbol{m} = \mathbf{6}$ and $\gamma_{\boldsymbol{n}} \approx 0.961$. Here $\boldsymbol{n} = (200, 200)$.

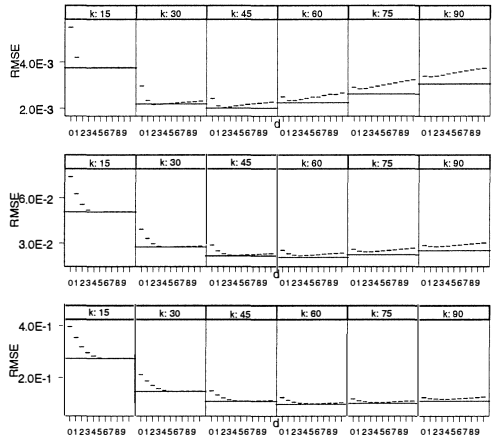

Figure 3. The RMSE for $\tilde{\gamma}_{\boldsymbol{n}}$ (long lines) and $\hat{\gamma}_{\boldsymbol{n}}$ (short lines) using different block sizes $k \times k$ and distances $\boldsymbol{d}=(d,d)$. The top picture corresponds to $\boldsymbol{m}=\boldsymbol{2}=(2,2)$ and $\gamma_{\boldsymbol{n}} \approx 0.033$, the middle picture to $\boldsymbol{m}=\boldsymbol{4}$ and $\gamma_{\boldsymbol{n}} \approx 0.252$, and the bottom picture to $\boldsymbol{m}=\boldsymbol{6}$ and $\gamma_{\boldsymbol{n}} \approx 0.979$. Here $\boldsymbol{n}=(1000,1000)$.

the m_i's, but if the chosen block size is rather large, then there is an advantage making d_i smaller than m_i, $i = 1, 2$. In the choice of block size, it should be noted that Theorem 2 gives only the correct asymptotic order. In practice we need to choose $k_i = c_i n_i^{1/2}$, for some constants $c_i > 0$, $i = 1, 2$. Clearly, the stronger the strength of dependence is, the larger the values of c_1 and c_2 that need to be chosen (as indicated in Figures 2-3). The choice of block size in practice is indeed an important and difficult task, but only a few guidelines exist; see e.g., Politis, Romano, and Wolf (Chapter 9, 1999), who discuss this topic in the case of stationary sequences. If little is known about the dependence structure, a safe policy is to use relatively large blocks (Sherman 1996).

When considering the same data, but with added smoothly varying expected values, i.e. $\tilde{X}_{\boldsymbol{i}} = X_{\boldsymbol{i}} + \nu_{\boldsymbol{i}}$, where $\nu_{\boldsymbol{i}} = \sin(\pi i_1/n_1 + \pi \sin(\pi i_2/n_2))$, for all $\boldsymbol{i}$ and $\boldsymbol{n}$, the estimator $\hat{\gamma}_{\boldsymbol{n}}$ is clearly the winner, and it is obvious that $\tilde{\gamma}_{\boldsymbol{n}}$ fails to provide valid estimates in this case (Figure 4). Note that we have taken the logarithm of the RMSEs in Figure 4.

In Figure 4, we see that the d_i's should be large if the blocks are small, and small if the blocks are large. A plausible reason could be the following: For small blocks

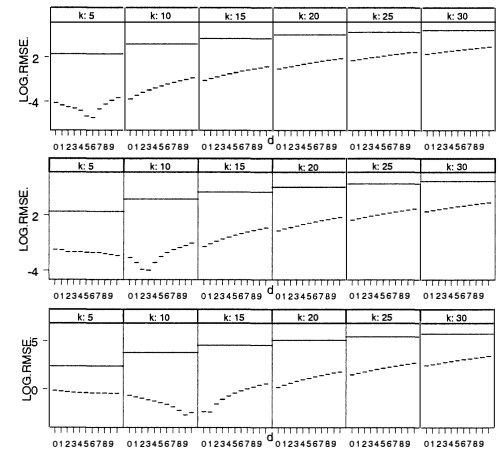

Figure 4. The logarithm of RMSE for $\tilde{\gamma}_{\boldsymbol{n}}$ (long lines) and $\hat{\gamma}_{\boldsymbol{n}}$ (short lines), using different block sizes $k \times k$ and distances $\boldsymbol{d}=(d,d)$, when generating data with varying expected values. The top picture corresponds to $\boldsymbol{m}=\boldsymbol{2}=(2,2)$ and $\gamma_{\boldsymbol{n}} \approx 0.032$, the middle picture to $\boldsymbol{m}=\boldsymbol{4}$ and $\gamma_{\boldsymbol{n}} \approx 0.249$, and the bottom picture to $\boldsymbol{m}=\boldsymbol{6}$ and $\gamma_{\boldsymbol{n}} \approx 0.961$. Here $\boldsymbol{n}=(200,200)$.

and positively correlated data, the estimator $\hat{\gamma}_n$ has a relatively large negative bias. In this case, the bias can be reduced by including some of the variation coming from the varying expected values.

When comparing Figure 4 with Figure 2, it is clear that the optimal block size in the case of varying expected values is smaller than when having constant expected values, and this corresponds well with the results on convergence rates obtained in Theorems 1 and 2.

5 ACCURACY OF ESTIMATES OF FOREST PARAMETERS

General forest parameters can be estimated by combining satellite data with sparsely distributed field data. In Nilsson and Sandström (2001), estimates of wood volume are derived from a Landsat TM scene, together with Swedish National Forest Inventory (NFI) data, by applying the kNN method (Kilkki and Päivinen 1987; Fazakas et al. 1999). The NFI is conducted exclusively as a field survey, and has low spatial resolution. By using information collected in the NFI, together with satellite data, it is believed that reliable estimates of forest parameters can be obtained for smaller areas than what is otherwise possible. In the kNN method, values of forest parameters are calculated for pixels as weighted averages of the k spectrally most similar plots. A map of kNN estimates, from the study of Nilsson and Sandström, is given in Figure 5 for the Brattåker area, northern Sweden. The value of k used in the kNN method is equal to 5. We wish to estimate the accuracy of the estimate of total volume of wood in the two rectangular areas in Figure 5.

Since an estimate of wood volume per hectare over some area is obtained by aggregating kNN estimates given for 25×25 m pixels in the Landsat TM scene, estimates of accuracy can be obtained by using the method derived in the current paper. Care needs to be taken though, since it is clear that discontinuities exist in the mean structure; e.g. when we go from a forested area to a clear cut area.

To see if our method gives reasonable results, we compare the subsampling estimates $\hat{\gamma}_n^{1/2}$ of standard deviation with estimates of RMSE from Nilsson and Sandström, obtained from a validation data set consisting of 2400 field plots available for the Brattåker area (Figure 6). In order to compute RMSE's, they split the region containing the field plots into s disjoint and equally-sized squares. Only squares in which 75% of the area belonged to the land cover classes "forest" and "forested wetlands" were used (the average proportion of the square area belonging to these two classes of forest is given in the second column of Table 1). For each of these squares, the difference between the kNN estimate of wood volume per hectare and the corresponding estimate from the field inventory was calculated. The estimated RMSE's were simply calculated as the square root of the average of the s squared differences, and were obtained at different aggregation levels, of which three are presented in Table 1. It should be noted that Nilsson and Sandström estimated wood volume and RMSE's only on the forest and forested wetlands land cover classes, while we

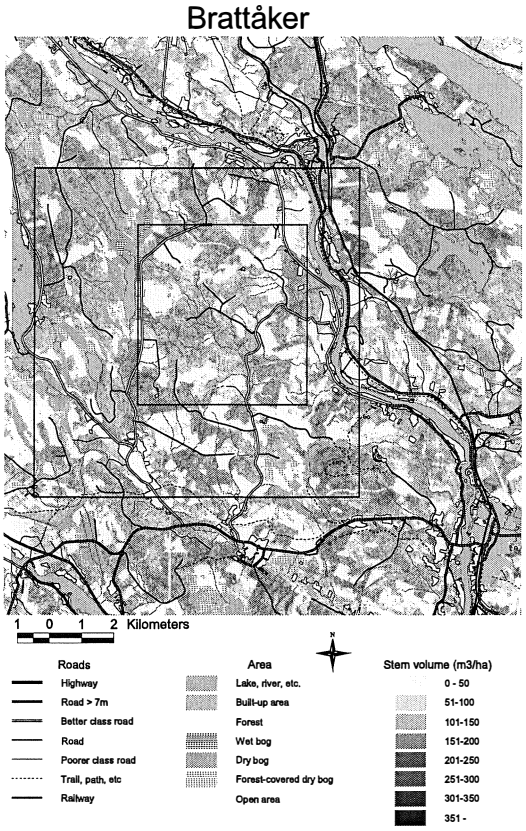

Figure 5. Map of kNN estimates. The two rectangular areas are 2940 ha and 10,302 ha, respectively. The larger area consists of 408×404 *pixels, while the smaller one consists of* 224×210 *pixels.*

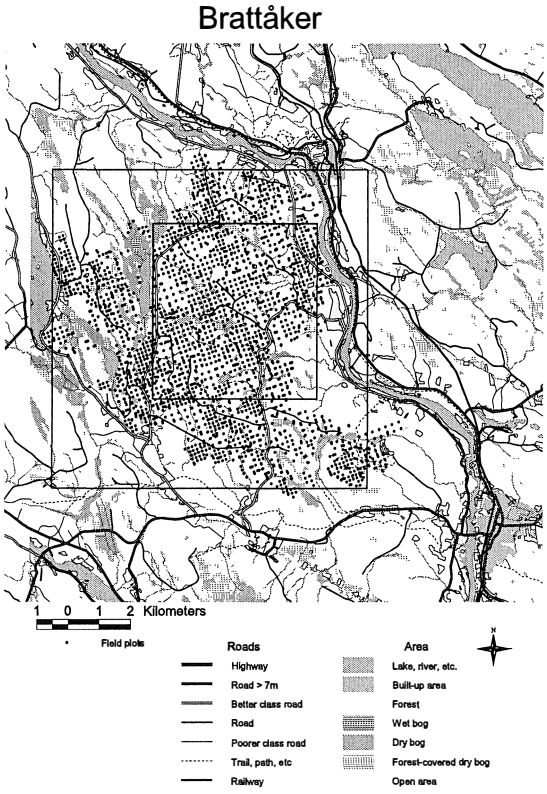

Figure 6. Locations of the 2400 field plots available from the Brattåker area. The plots are circular with a 10 m radius.

calculated estimates of standard deviation of wood volume in the two different areas indicated in Figure 5 by giving a zero volume of wood to each pixel not belonging to these two classes.

Square size (ha)	Proportion of forest	s	RMSE
100	89.1%	43	886
225	87.2%	18	1052
400	90.6%	6	867

Table 1. Estimates of RMSE at different aggregation levels, derived from the field inventory.

In general, a good choice of block size depends on the strength of dependence (the stronger dependence, the larger the blocks) and on the variability of the expected values (the more they vary, the smaller the blocks). Thus, a compromise between these two objectives is needed. In Table 2 we used the block size 20×20 as well as 30×30 for the larger rectangular area in Figure 5. The value of $d = d_1 = d_2$ in this case was chosen to be 10. Since smaller block sizes require larger values of d, and larger blocks smaller values of d (recall Figure 4), we also considered $d = 20$ and $d = 0$ for the smaller and larger block size, respectively. The choices of $k_1 \times k_2$ and d for the smaller rectangular area in Figure 5 were chosen in a similar manner.

$k_1 \times k_2$	d	Area (ha)	$\hat{\gamma}_{\boldsymbol{n}}^{1/2}$
15×15	10	2940	554
15×15	20	2940	613
25×25	0	2940	624
25×25	10	2940	759
20×20	10	10302	729
20×20	20	10302	805
30×30	0	10302	786
30×30	10	10302	939

Table 2. Subsampling estimates of standard deviation assuming varying mean values.

When comparing our estimates $\hat{\gamma}_{\boldsymbol{n}}^{1/2}$ of standard deviation with the estimates of RMSE (Tables 2 and 1, respectively), it appears that our modified subsampling method yields reasonable estimates. The rather low values of $\hat{\gamma}_{\boldsymbol{n}}^{1/2}$ when considering the 2940 ha area and 15×15 block size, is likely due to the fact that the pixels of kNN estimates are spatially dependent up to approximately 500 m, suggesting that this block size is too small to be used.

6 SUMMARY AND CONCLUDING REMARKS

We have proposed a subsampling variance estimator for sample means that can handle differently distributed and locally dependent observations with smoothly varying expected values. The nonstationary spatial lattice data are collected over

some (possibly irregularly shaped) region, and the variance estimator, $\hat{\gamma}_n$, is computed from crosswise subshape differences. In particular, if the region is rectangular, $\hat{\gamma}_n$ is calculated from crosswise block differences, see Figure 1. The idea is that the crosswise subshape differences will reduce the variation coming from the varying expected values. The simulation study shows that the variance estimator performs well and that we need to use crosswise subshape differences instead of ordinary subshapes ($\tilde{\gamma}_n$) when we have varying expected values. On the other hand, we do not seem to loose much by using $\hat{\gamma}_n$ instead of $\tilde{\gamma}_n$ when we have data with constant expected values. Thus, if we do not have good knowledge of whether the data have constant expected values, a safe policy is to choose $\hat{\gamma}_n$ instead of $\tilde{\gamma}_n$.

As in all block resampling and subsampling methods, the choice of block or subshape size is not obvious. For $\hat{\gamma}_n$, it is a tradeoff between the strength of dependence (the stronger the dependence, the larger the subshapes) and the variability of the expected values (the more they vary, the smaller the subshapes). Although the optimal asymptotic order is given in the two theorems in terms of minimizing the MSE, it only gives a rough guideline of how the subshape size should be chosen for a particular situation. Moreover, special care needs to be taken if there are discontinuities in the smoothly varying expected values, which may arise in real applications. The choice of subshape size in practice is a difficult and challenging topic for future research.

The currently proposed variance estimator can be generalized to vector valued sample means collected over regions in $\mathbf{R}^d$, where d is some finite integer. We also believe that it is possible to come up with similar variance estimators for functions of sample means under the assumed types of nonstationarity.

APPENDIX: PROOFS

The conditions for the block (subshape) sizes that are needed to give consistent variance estimators are summarized in the following two assumptions:

A1: If $\delta \geq 2$, then $k_i = o(n_i)$, $i = 1, 2$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$. If $0 < \delta < 2$, then $(k_1/k_2)((K/N)\log k_2)^{\delta}$, $(k_2/k_1)((K/N)\log k_1)^{\delta}$, and $(k_i/n_i)\log k_i$, $i = 1, 2$, all tend to zero as $\boldsymbol{k}, \boldsymbol{n} \to \infty$. Also, $d_i = O(k_i)$, $i = 1, 2$.

A2: We have $K(k_1/n_1)^{2\alpha} \to 0$ or $K(k_2/n_2)^{2\alpha} \to 0$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$.

Remark 4 If $n_1 \sim n_2$, $k_1 \sim k_2$, and $k_1 n_1^{-1} \log k_1 \to 0$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$, then A1 holds for any $\delta > 0$.

The proofs of the two theorems in Section 2 are given below, where the following inequality and three lemmata are used: For any real numbers $a_1, ..., a_r$ and $\lambda \geq 1$ we have, from Jensen's inequality, that

$$(a_1 + \cdots + a_r)^{\lambda} \leq r^{\lambda - 1}(|a_1|^{\lambda} + \cdots + |a_r|^{\lambda}). \tag{6}$$

Lemma 1 *(Ekström 2001) Assume that* $|\beta_{\boldsymbol{i}}| \leq 1$, $i_j = 1, ..., b_j \geq m_j$, $j = 1, 2$. *If (1), (2), and A1 hold, and* $E[X_{\boldsymbol{i}}] = 0$, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}$, *then* $E[|\sum_{i_1=1}^{b_1}\sum_{i_2=1}^{b_2}\beta_{\boldsymbol{i}}X_{\boldsymbol{i}}|^{2+\delta}] = O\left((b_1 b_2)^{1+\delta/2}\right)$.

Lemma 2 *Assume that (1), (2), and A1 hold, and that* $E[X_{\boldsymbol{i}}] = \mu$, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}$. *Then* $\tilde{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ *as* $\boldsymbol{k}, \boldsymbol{n} \to \infty$. *If* $\delta \geq 2$, *we also have* $E[(\tilde{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2})$.

Proof of Lemma 2 Without loss of generality we assume that $\mu = 0$. From straightforward calculations we have

$$\tilde{\gamma}_{\boldsymbol{n}} + K(\bar{X}' - \bar{X})^2 = K(N')^{-1}\sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}'}}(\bar{X}_{\boldsymbol{i}} - \bar{X})^2 = N(N')^{-1}\tilde{\gamma}^*_{\boldsymbol{n}} - A_1 - A_2, \tag{7}$$

where $\bar{X} = \sum_{\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}}} X_{\boldsymbol{i}}/N$,

$$\tilde{\gamma}^*_{\boldsymbol{n}} = \frac{1}{KN}\sum_{i_1=2-k_1}^{n_1}\sum_{i_2=2-k_2}^{n_2} D_{\boldsymbol{i}}, \quad D_{\boldsymbol{i}} = \left(\sum_{j_1=i_1}^{i_1+k_1-1}\sum_{j_2=i_2}^{i_2+k_2-1}(X_{\boldsymbol{j}} - \bar{X})I_{\boldsymbol{j}}\right)^2,$$

and

$$A_1 = \frac{1}{KN'}\sum_{i_1=2-k_1}^{0}\sum_{i_2=1}^{n_2'}(D_{\boldsymbol{i}} + D_{i_1+n_1, i_2}), \quad A_2 = \frac{1}{KN'}\sum_{i_1=2-k_1}^{n_1}\sum_{i_2=2-k_2}^{0}(D_{\boldsymbol{i}} + D_{i_1, i_2+n_2}).$$

Here $I_{\boldsymbol{j}}$ equals one if $\boldsymbol{j} \in \mathcal{I}_{\boldsymbol{n}}$, and zero otherwise. Note that from Lemma 1

$$E[|\bar{X}'|^{2+\delta}] \leq E|\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}}} \beta_{\boldsymbol{i}} X_{\boldsymbol{i}}|^{2+\delta}/(N')^{2+\delta} = O(1/N^{1+\delta/2}), \tag{8}$$

where $|\beta_{\boldsymbol{i}}| \leq 1$. Likewise, $E[|\bar{X}|^{2+\delta}] = O(1/N^{1+\delta/2})$, and by (6), $E[|K(\bar{X}' - \bar{X})^2|^{1+\delta/2}] = O((K/N)^{1+\delta/2})$. From (6),

$$E[|A_1|^{1+\delta/2}] \leq (KN')^{-1-\delta/2}\,(2k_1 n_2')^{\delta/2} \sum_{i_1=2-k_1}^{0} \sum_{i_2=1}^{n_2'} (E[|D_{\boldsymbol{i}}|^{1+\delta/2}] + E[|D_{i_1+n_1,i_2}|^{1+\delta/2}]).$$

By Lemma 1, the right hand side is of order $O((k_1/n_1)^{1+\delta/2})$, and similarly, $E[|A_2|^{1+\delta/2}] = O((k_2/n_2)^{1+\delta/2})$. Therefore, from A1 and Chebyshev's inequality (CI),

$$\tilde{\gamma}_{\boldsymbol{n}} - N(N')^{-1}\tilde{\gamma}^*_{\boldsymbol{n}} \xrightarrow{P} 0 \text{ as } \boldsymbol{k}, \boldsymbol{n} \to \infty. \tag{9}$$

From Theorem 1 in Ekström (2001) it follows that $N(N')^{-1}\tilde{\gamma}^*_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$, which, by (9), implies that $\tilde{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$. If $\delta \geq 2$, Corollary 1 in Ekström (2001), and (6) applied to (7), together with the above yield

$$E[(\tilde{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] \leq 5E[K^2(\bar{X}' - \bar{X})^4] + 5N^2(N')^{-2}E[(\tilde{\gamma}^*_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2]$$

$$+5(N/N' - 1)^2\gamma^2_{\boldsymbol{n}} + 5E[A_1^2] + 5E[A_2^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2}).$$

■

Lemma 3 *Under the assumptions of Lemma 2 we have* $\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ *as* $\boldsymbol{k}, \boldsymbol{n} \to \infty$. *If* $\delta \geq 2$, *we also have* $E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2})$.

Proof of Lemma 3 Without loss of generality we assume that $\mu = 0$. Let $s_i = d_i + k_i$, $i = 1, 2$. It follows that

$$\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} (Z_{\boldsymbol{i}} - \bar{Z})^2 = 2\sum_{h=1}^{7} Q_h - N''\bar{Z}^2, \tag{10}$$

where

$$Q_1 = \sum (\bar{X}^2_{\boldsymbol{i}} + \bar{X}^2_{i_1+s_1,i_2+s_2} + \bar{X}^2_{i_1,i_2+s_2} + \bar{X}^2_{i_1+s_1,i_2})/2,$$

$$Q_2 = -\sum \bar{X}_{\boldsymbol{i}}\bar{X}_{i_1,i_2+s_2}, \quad Q_3 = \sum \bar{X}_{\boldsymbol{i}}\bar{X}_{i_1+s_1,i_2+s_2},$$

$$Q_4 = -\sum \bar{X}_{\boldsymbol{i}}\bar{X}_{i_1+s_1,i_2}, \quad Q_5 = -\sum \bar{X}_{i_1+s_1,i_2+s_2}\bar{X}_{i_1,i_2+s_2},$$

$$Q_6 = -\sum \bar{X}_{i_1+s_1,i_2+s_2}\bar{X}_{i_1+s_1,i_2}, \quad Q_7 = \sum \bar{X}_{i_1+s_1,i_2}\bar{X}_{i_1,i_2+s_2},$$

and all sums are taken over all $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}''}$. We can rewrite Q_1 as

$$\frac{K}{2N''}Q_1 = \frac{N'}{N''}\tilde{\gamma}_{\boldsymbol{n}} + \frac{KN'}{N''}\bar{X}'^2 - \frac{K}{2N''}\sum_{i_1=1}^{n_1'}\sum_{i_2=1}^{s_2}(\bar{X}_{\boldsymbol{i}}^2 + \bar{X}_{i_1,i_2+n_2''}^2)$$

$$-\frac{K}{4N''}\sum_{i_1=1}^{s_1}\sum_{i_2=1}^{n_2''}(\bar{X}_{\boldsymbol{i}}^2 + \bar{X}_{i_1,i_2+s_2}^2) - \frac{K}{4N''}\sum_{i_1=n_1''+1}^{n_1'}\sum_{i_2=1}^{n_2''}(\bar{X}_{\boldsymbol{i}}^2 + \bar{X}_{i_1,i_2+s_2}^2). \tag{11}$$

From Lemma 2, it follows that $N'(N'')^{-1}\tilde{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ as $\boldsymbol{k},\boldsymbol{n} \to \infty$. Furthermore, (8) and CI imply that $KN'\bar{X}'^2/N'' \xrightarrow{P} 0$ as $\boldsymbol{k},\boldsymbol{n} \to \infty$. Now, by (6) and Lemma 1,

$$E[|\frac{K}{N''}\sum_{i_1=1}^{n_1'}\sum_{i_2=1}^{s_2}(\bar{X}_{\boldsymbol{i}}^2 + \bar{X}_{i_1,i_2+n_2''}^2)|^{1+\delta/2}]$$

$$\leq \left(\frac{K}{N''}\right)^{1+\delta/2}(2n_1's_2)^{\delta/2}\sum_{i_1=1}^{n_1'}\sum_{i_2=1}^{s_2}(E[|\bar{X}_{\boldsymbol{i}}|^{2+\delta}] + E[|\bar{X}_{i_1,i_2+n_2''}|^{2+\delta}]) = O((\frac{s_2}{n_2''})^{1+\delta/2}).$$

By a similar reasoning we can show that the expectation of the absolute value of each of the two last terms of (11), to the power of $1+\delta/2$, is of order $O((s_1/n_1'')^{1+\delta/2})$. An application of CI implies that $K(2N'')^{-1}Q_1 - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ as $\boldsymbol{k},\boldsymbol{n} \to \infty$. Let us consider Q_2. Define $(b)_+ = \max(0,b)$, and

$$A_{\boldsymbol{i}} = \frac{1}{K}\sum_{j_1=i_1}^{i_1+k_1-1}\sum_{j_2=i_2}^{i_2+k_2-1-(m_2-d_2)_+} X_{\boldsymbol{j}}, \quad C_{\boldsymbol{i}} = \frac{1}{K}\sum_{j_1=i_1}^{i_1+k_1-1}\sum_{j_2=i_2+k_2-(m_2-d_2)_+}^{i_2+k_2-1} X_{\boldsymbol{j}}.$$

Then, $\bar{X}_{\boldsymbol{i}} = A_{\boldsymbol{i}} + C_{\boldsymbol{i}}$, and $\bar{X}_{i_1,i_2+s_2} = A_{i_1,i_2+s_2+(m_2-d_2)_+} + C_{i_1,i_2+d_2+(m_2-d_2)_+}$, and thus

$$-Q_2 = \sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} S_{\boldsymbol{i}} + \sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} U_{\boldsymbol{i}},$$

where $S_{\boldsymbol{i}} = A_{\boldsymbol{i}}\bar{X}_{i_1,i_2+s_2} + C_{\boldsymbol{i}}A_{i_1,i_2+s_2+(m_2-d_2)_+}$, and $U_{\boldsymbol{i}} = C_{\boldsymbol{i}}C_{i_1,i_2+d_2+(m_2-d_2)_+}$, with $E[S_{\boldsymbol{i}}] = 0$, $\boldsymbol{i} \in \mathcal{I}_{\boldsymbol{n}''}$. By (6) and Lemma 1 we have

$$E[|S_{\boldsymbol{i}}|^{2+\delta}] \leq 2^{1+\delta}(E[|A_{\boldsymbol{i}}|^{2+\delta}]E[|\bar{X}_{i_1,i_2+s_2}|^{2+\delta}]$$

$$+E[|C_{\boldsymbol{i}}|^{2+\delta}]E[|A_{i_1,i_2+s_2+(m_2-d_2)_+}|^{2+\delta}]) = O(1/K^{2+\delta}). \tag{12}$$

Furthermore,

$$\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} S_{\boldsymbol{i}} = \sum_{i=1}^{k_1+m_1}\sum_{j=1}^{k_2+s_2+m_2}\sum_{h_1=1}^{p_i}\sum_{h_2=1}^{q_j} S_{i+(k_1+m_1)(h_1-1),j+(k_2+s_2+m_2)(h_2-1)}, \tag{13}$$

where $p_i = \lfloor (n_1''-i)/(k_1+m_1)\rfloor+1$, $q_j = \lfloor (n_2''-j)/(k_2+s_2+m_2)\rfloor+1$, and $\lfloor x\rfloor$ denotes the integer part of x. Note that for a given (i,j) in the first two sums of (13), the items summing over h_1, h_2 are independent. Therefore, from (6) and Petrov (1995 pp. 82-83), respectively, we have

$$E[|\frac{K}{N''}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} S_{\boldsymbol{i}}|^{2+\delta}] \le [(k_1+m_1)(k_2+s_2+m_2)]^{1+\delta}\sum_{i=1}^{k_1+m_1}\sum_{j=1}^{k_2+s_2+m_2}\eta(p_iq_j)^{\delta/2}$$

$$\times (K/N'')^{2+\delta}\sum_{h_1=1}^{p_i}\sum_{h_2=1}^{q_j} E[|S_{i+(k_1+m_1)(h_1-1),j+(k_2+s_2+m_2)(h_2-1)}|^{2+\delta}] = O\left((K/N)^{1+\delta/2}\right), \tag{14}$$

where the equality follows from (12). By (6) and Cauchy-Schwartz's inequality

$$E[|K(N'')^{-1}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} U_{\boldsymbol{i}}|^{1+\delta/2}] \le K^{1+\delta/2}(N'')^{-1}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}}\left(E[|C_{\boldsymbol{i}}|^{2+\delta}]E[|C_{i_1,i_2+d_2+(m_2-d_2)_+}|^{2+\delta}]\right)^{1/2},$$

which is of order $O(k_2^{-1-\delta/2})$ by Lemma 1. Hence, by (6) we have

$$E[|KQ_2/N''|^{1+\delta/2}] \le E[|K(N'')^{-1}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} S_{\boldsymbol{i}}|^{2+\delta}]^{1/2} + E[|K(N'')^{-1}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}} U_{\boldsymbol{i}}|^{1+\delta/2}]$$

$$= O((K/N)^{(1+\delta/2)/2}) + O(k_2^{-1-\delta/2}).$$

Reasoning in a similar way shows that $E[|KQ_i/N''|^{1+\delta/2}] = O((K/N)^{(1+\delta/2)/2} + k_1^{-1-\delta/2} + k_2^{-1-\delta/2})$, and thus, by CI, $KQ_i/N'' \xrightarrow{P} 0$ as $\boldsymbol{k},\boldsymbol{n}\to\infty$, $i = 2,...,7$. Moreover, by Lemma 1

$$E[|K\bar{Z}^2|^{1+\delta/2}] = \frac{4^{2+\delta}K^{1+\delta/2}}{(N'')^{2+\delta}}E[|\frac{1}{4}\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}}(\bar{X}_{\boldsymbol{i}} + \bar{X}_{i_1+s_1,i_2+s_2} - \bar{X}_{i_1,i_2+s_2} - \bar{X}_{i_1+s_1,i_2})|^{2+\delta}],$$

$$\le \frac{4^{2+\delta}K^{1+\delta/2}}{(N'')^{2+\delta}}E[|\sum_{\boldsymbol{i}\in\mathcal{I}_{\boldsymbol{n}''}}\beta_{\boldsymbol{i}}X_{\boldsymbol{i}}|^{2+\delta}] = O((K/N)^{1+\delta/2}), \tag{15}$$

where $|\beta_{\boldsymbol{i}}| \leq 1$. CI implies that $K\bar{Z}^2 \xrightarrow{P} 0$, and thus we have shown that $\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$. If $\delta \geq 2$, then from (6) applied to (10) we have

$$E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] \leq 8E[|\frac{K}{2N''}Q_1 - \gamma_{\boldsymbol{n}}|^2] + 8\left(\frac{K}{2N''}\right)^2 \sum_{i=2}^{7} E[Q_i^2] + 8\left(\frac{K}{4}\right)^2 E[\bar{Z}^4].$$

Note that by (6) applied to (11), Lemma 2, and the above inequalities, we have

$$E[|KQ_1/(2N'') - \gamma_{\boldsymbol{n}}|^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2}).$$

Putting this together with the knowledge that $E[(KQ_i/N'')^2] = O(K/N + k_1^{-2} + k_2^{-2})$, $i = 2, ..., 7$, and with (15), we conclude that

$$E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2}). \tag{16}$$

Hence, for $n_1 \sim n_2$ and $k_1 \sim k_2$ we have $E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + 1/K)$. ■

Proof of Theorem 1 Recall that $X_{\boldsymbol{i}} = Y_{\boldsymbol{i}} + r_{i_2} + c_{i_1}$, where $E[Y_{\boldsymbol{i}}]$ satisfies (3). Let $s_i = k_i + d_i$, $i = 1, 2$. In the crosswise differences $Z_{\boldsymbol{i}}$, the row and column effects cancel out since

$$Z_{\boldsymbol{i}} = \bar{X}_{\boldsymbol{i}} + \bar{X}_{i_1+s_1, i_2+s_2} - \bar{X}_{i_1+s_1, i_2} - \bar{X}_{i_1, i_2+s_2} = \bar{Y}_{\boldsymbol{i}} + \bar{Y}_{i_1+s_1, i_2+s_2} - \bar{Y}_{i_1+s_1, i_2} - \bar{Y}_{i_1, i_2+s_2}.$$

Hence, it is enough to prove the theorem under the assumption that the expected values $\mu_{\boldsymbol{i}}$ satisfy (3). In this proof all sums are over the index set $\mathcal{I}_{\boldsymbol{n}''}$. Let $W_{\boldsymbol{i}} = Z_{\boldsymbol{i}} - \bar{Z}$. Then

$$\sum W_{\boldsymbol{i}}^2 = \sum (W_{\boldsymbol{i}} - E[W_{\boldsymbol{i}}])^2 - \sum (E[W_{\boldsymbol{i}}])^2 + 2\sum (W_{\boldsymbol{i}} - E[W_{\boldsymbol{i}}])E[W_{\boldsymbol{i}}]. \tag{17}$$

From Lemma 3 we know that

$$K\,(4N'')^{-1} \sum (W_{\boldsymbol{i}} - E[W_{\boldsymbol{i}}])^2 - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0 \quad \text{as } \boldsymbol{k}, \boldsymbol{n} \to \infty.$$

Moreover, by (6) and (3), respectively,

$$K\,(4N'')^{-1} \sum (E[W_{\boldsymbol{i}}])^2 \leq K\,(4N'')^{-1} \sum (E[Z_{\boldsymbol{i}}])^2$$

$$\leq K(2N'')^{-1} \sum ((\mu_{\boldsymbol{i}} - \mu_{i_1+s_1, i_2})^2 + (\mu_{i_1+s_1, i_2+s_2} - \mu_{i_1, i_2+s_2})^2) = O(K\,(s_1/n_1)^{2\alpha}),$$

or alternatively,

$$\leq K(2N'')^{-1} \sum ((\mu_{\boldsymbol{i}} - \mu_{i_1, i_2+s_2})^2 + (\mu_{i_1+s_1, i_2+s_2} - \mu_{i_1+s_1, i_2})^2) = O(K\,(s_2/n_2)^{2\alpha}).$$

Hence, by A2, $K\sum (E[W_{\boldsymbol{i}}])^2\ /(4N'') \to 0$ as $\boldsymbol{k}$,$\boldsymbol{n} \to \infty$. Finally, by Cauchy-Schwartz's inequality and CI, it follows that

$$K\,(4N'')^{-1} \sum (W_{\boldsymbol{i}} - E[W_{\boldsymbol{i}}])E[W_{\boldsymbol{i}}] \xrightarrow{P} 0 \text{ as } \boldsymbol{k}, \boldsymbol{n} \to \infty,$$

which verifies that $\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}} \xrightarrow{P} 0$ as $\boldsymbol{k}, \boldsymbol{n} \to \infty$. If $\delta \geq 2$, we can apply (6) to (17) to get

$$E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] \leq 3E\left[\left(K\left(4N''\right)^{-1}\sum(W_i - E[W_i])^2 - \gamma_{\boldsymbol{n}}\right)^2\right]$$

$$+3\left(K\left(4N''\right)^{-1}\sum(E[W_i])^2\right)^2 + 3E\left[\left(K\left(2N''\right)^{-1}\sum(W_i - E[W_i])E[W_i]\right)^2\right].$$

The first term on the right hand side is of order $O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2})$ by Lemma 3. The last two terms are both of order $O(K^2(k_i/n_i)^{4\alpha})$, $i = 1$ or 2, (here we have applied Cauchy-Schwartz's inequality to the last term). Therefore it follows that

$$E[(\hat{\gamma}_{\boldsymbol{n}} - \gamma_{\boldsymbol{n}})^2] = O(K/N + k_1^{-2} + k_2^{-2} + k_1^2 n_1^{-2} + k_2^2 n_2^{-2} + K^2(k_i/n_i)^{4\alpha}), \quad i = 1 \text{ or } 2. \tag{18}$$

Thus, if $n_1 \sim n_2$ and $k_1 \sim k_2$, we obtain the desired convergence rate. ■

Proof of Theorem 2. Since the row and column effects cancel out in the crosswise differences Z_i, the proof follows from Lemma 3. ■

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Serien Arbetsrapporter utges i första hand för institutionens eget behov av viss dokumentation. Rapporterna är indelade i följande grupper: Riksskogstaxeringen, Planering och inventering, Biometri, Fjärranalys, Kompendier och undervisningsmaterial, Examensarbeten samt internationellt. Författarna svarar själva för rapporternas vetenskapliga innehåll.

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1995 1 Kempe, G. Hjälpmedel för bestämning av slutenhet i plant- och ungskog. ISRN SLU-SRG-AR--1--SE

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7 Henriksson, L. The thinning quotient - a relevant description of a thinning? Gallringskvot - en tillförlitlig beskrivning av en gallring? Examensarbete i ämnet skogsuppskattning och skogsindelning. ISRN SLU-SRG-AR--7--SE.

8 Ranvald, C. Sortimentsinriktad avverkning. Examensarbete i ämnet skogsuppskattning och skogsindelning. ISRN SLU-SRG-AR--8--SE.

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