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# FOREST TAXATION AND ITS IMPACT ON THE SUPPLY OF ROUNDWOOD

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# 1. Introduction

In this study we will analyse the effects on the supply of roundwood of different kinds of tax systems.

The reason for this analysis is some ideas that has been brought out in Sweden lately, about the system of taxation being a cause of the lack of raw material for the Swedish forest industry.

The supply of roundwood has for several years been less than the requirement of the industry. In spite of raises in timber prices, the supply has not increased sufficiently to clear the market, and the tendency of excess demand remains.

Some people claim that the potential resources of wood permit a higher felling quantity and the reason why we are left with a shortage of roundwood is the Swedish system of taxation.

In Sweden there are two different ways of imposing taxes upon forest firms. One is the company taxation, which is a proportional taxation of profit with a fixed tax rate. This form of taxation is applied to forest management in the form of limited companies, incorporated associations and foundations. The other is the progressive taxation of private incomes, were the tax rate is rising with the income. This form of taxation is applied to management of private owned forests.

About 47 % of the total forest area is owned and managed by private persons, and for this part of forestry, the progressive tax system is relevant. The other half of the forest is owned and managed by companies, the governmental and local authorities and the Church, and these face a fixed proportional tax rate.

One opinion is that the progressive taxation of income makes the private forest farmer insensitive to variations in gross incomes and prices, and unwilling to increase his felling or even unwilling to fell at all. If the forestry is combined with some other permanent source of income, e.g. agricultural farming or employment within industry, and income from forestry is merely additional, the marginal rates of taxation makes the additional net income very small, as the average tax rate for the total income rises. The forest farmers' argument for not increasing the felling would be that 'the whole increase in income goes off as taxes'. Combined with a high rate of inflation in relation to the nominal interest rate the real interest rate on savings will be negative, and it will be profitable to save in the form of growing trees.

Comparisons have been made with our neighbour country Finland, where the forest industries have no, or at least small problems with its raw-material supplies.

The comparison is relevant, as these two countries, as far as the forest resources are concerned, are much the same in many respects. In both countries the forestry management and the forest industry play an important role in the national economy, being one of the oldest and most extensive branches in the economy. And in both countries, the natural resource wood is fully utilized. The countries, however, differ with respect to the systems of forest taxation.

In Finland a lump-sum taxation is used, i.e. the forest owner pays a certain amount every year, which is related to the yearly growth of the biomass on his forest land. He has to pay regularly whether he has incomes from forestry or not. Hence, there is a taxation of the very possession of forest land. On the other hand, he has to pay no taxes on the proceeds from the felling. And there is no taxes on the extra incomes he receives if he through silviculture gets a higher forest growth, than what have been calculated as 'normal growth' and what he has paid taxes for. This is the main difference between the forest management in the two countries, and the one which has been indicated as an explanation to the differences in the roundwood supply.

Our task in this study is to analyse whether the systems of taxation have any impact on the supply of roundwood, and, if so, what exactly makes the difference, and which system is to be preferred from supply and efficiency considerations.

We will treat all three kinds of taxation, which are applied in Sweden and Finland, i.e. the lump-sum tax, which is a constant amount and independent on the size quantity produced, the proportional profit tax with a fixed rate of taxation, and the progressive profit tax where the rate of taxation is a variable and dependent on the income. These taxes will be compared with respect to their effects on forest management.

## 2. The supply of roundwood in a forestry firm

In this section, we will introduce a simple model of the supply of roundwood from a particular forestry firm. To begin with no explicit taxes are introduced.

We will make a couple of simplifying assumptions, which will generate the linear programming problem of maximizing the present value the income stream from felling subject to the constraints of a linear technology[1].

The first simplifying assumption that we make is to assume that the planning horizon is finite. This assumption is, however, not quite logical. At any given future point of time the age distribution of the forest will have implications for the future. If one stops the analysis after a finite number of periods, say T, it would in principle be necessary to include some 'scrap value' of the forest stand. The only logical way of doing so is to determine the maximum present value attainable in the further future, starting with any given age distribution of the forest.

On the other hand, astronomy teach us that the world, as we know it, is finite, and, moreover, if one assumes that prices are bounded it is easy to show that the loss from stopping after a finite time can be made arbitrary small by making T large enough.

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1) This formulation has been borrowed from Berck (1976). A similar formulation of the finite planning horizon, discrete time case is due to Karl G Jungenfelt, and can be found in SOU 1973:14, *Mål och medel i skogspolitiken*.

It is also appropriate to say a few words about our target function. The maximization of present value presupposes a perfect capital market. If the planning horizon is finite, and there are two different interest rates, it can be shown[2)] that an objective investment decision could be made provided that the present value calculated at all the $2^{T-1}$ different ways are positive. It is easy to imagine that - if this was the only imperfection - it would be possible to generalize the profit function by searching for a maintenance program which has the largest present value independently of which of the $2^{T-1}$ possible r-tuples of interest rates that is used. This method would, provided that such a maintenance program exists, generate an objective investment decision in this case.

However, there are other possible capital market imperfections in real life, and we will only indicate below how these could be introduced into the analysis by the more pragmatic method of adding further restrictions.

The technology of the forestry firm can be introduced by defining

$x_{ti}$ = the quantity of land occupied at time t by trees of age i

As an initial condition one has

$$\bar{x} = \bar{x}_{00} + \bar{x}_{01}, \ldots, + \bar{x}_{0n} \tag{1}$$

i.e. trees of different age classes and seeded land ($\bar{x}_{00}$) cover the initial amount of land ($\bar{M}$). Clearly it holds for all t that

$$\sum_{i=0}^{n} x_{ti} = \bar{M} \qquad \text{all } t \tag{2}$$

It is also convenient to define the harvest as a joint product consisting of timber and seeded land and to assume that there is a highest age class (n), and that the lowest age class is seeded bare land (0). Hence, bare land at the end of the period 1 is

$$x_{10} = c_{11} + c_{12} + c_{13} \ldots + c_{1n} \tag{3}$$

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2) Compare Puu (1967)

i.e. equal to the sum of the cut in all age classes during period 1. The area of land occupied by trees of age $i < n$ at time $t \leqq T$ will be equal to

$$x_{ti} = -c_{ti} + x_{t-1\ i-1} \qquad \text{all } t \tag{4}$$

i.e. equal to what was left from the previous period minus what was cut during the period. For $i = n$, we will assume, to end the recursion, that what was left from the previous period of trees belonging to age class n will remain in this age class for ever. In other words

$$x_{tn} = -c_{tn} + x_{t-1\ n-1} + x_{t-1n} \tag{5}$$

Combined with the growth function we will apply below, this can be interpreted as if the growth of the biomass in the forest ended after n periods. One knows that this is not true for the single tree that stays alive from one period to another; there is always a growth ring. On the other hand, this does not guarantee that the biomass of the forest stand grows from one period to the next, and for all practical purposes the assumption behind equation (5) is no large violation of reality. It can be viewed upon as a pragmatic method of keeping the dimensions of the problem finite.

The growth function is specified as a vector $G' = (0\ g_1, \ldots, g_n)$, and the net price vector is defined as $p = (p_1, \ldots, p_T)$, where

$p_t > 0$ is the present value of the constant unit profit in period t.

Maximization problem can now be specified as

$$\underset{c}{\text{Max}} \sum_{t=1}^{T} p(t)c'(t)G \tag{6}$$

subject to the restriction specified by equations (1)-(5) where

$$c'(t)G = c_{t1}g_1 + \ldots, + c_{tn}g_n = c_t$$

If we define $c = (c_1, \ldots, c_T)$ the 'profit function' can be written as

$$\max_{c} \pi(p) = pc \tag{6a}$$

subject to equation (1)-(5), and we will work with this variation of the model. Further restrictions could be added without any substantial change in the qualitative properties of the model.

## 2.1 The properties of the supply function

The supply of roundwood from a particular forestry firm will depend on the initial endowment $\bar{x}(0)$ and the constant net price vector (p). We will now derive some properties of the solution of the forest management problem (6a), which are relevant for the effects of taxation.

Let us assume that the felling program c* solves the maximization problem (6a), when the net price system is p*; by definition it then holds that

$$p^*c^* \geqq p^*c \qquad \text{all } c \in C \tag{7}$$

where C denotes the feasible choice set, i.e. the compact set[3] defined by the restrictions (1)-(5). Equation (7) simply says that c* gives a present value at least as large as any other feasible felling program. Now, multiply the net price system by a scalar $\lambda > 0$. Since, the restrictions are not changed by the change of the price system and since (7) holds, it must also hold that

$$\lambda p^*c^* \geqq \lambda p^*c \qquad \text{all } c \in C \tag{8}$$

and we have shown

*<u>Homogenity of degree zero (H)</u>. If $c^*$ is profit maximizing for the net price system $p^*$, then $c^*$ is profit maximizing for the net price system $\lambda p^*$, $\lambda > 0$.*

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[3] A compact set is a closed and bounded set.

It obviously also holds that we can add a constant cost ($T_x$), independent of the felling program, to the problem 6a without any change in the present value maximizing felling program c*, i.e.

$$p^*c^* - T_x \geqq p^*c - T_x \qquad \text{all } c \in C \tag{7a}$$

and we have shown

*<u>Indenpendence of addition of constant (I)</u>. If c* is present value maximizing for the price system p*, it will be present value maximizing if a constant is added to the 'profit function'.*

Let us now introduce the following definition

*Definition* *A feasible supply program c is efficient if and only if there is no other feasible supply program c' such that $c'_t \geqq c$ all t and $c'_t > c_t$ for at least one t.*

The definition says simply that a felling program is efficient if it is impossible to increase the supply of roundwood in one period without at the same time decreasing it in another period. It should, however, be pointed out that the definition presupposes a given (biological) production technology and not necessarily the best technology. It could, however, be argued that it is in the interest of the present value maximizing forest owner to use the best known 'production function', as a more efficient technology would increase net revenues from forestry.

It is easy to show that a present value maximizing program according to equation (7) is efficient provided that

$$p^* >> 0$$

If c* is present value maximizing, but not efficient, then there exists a $c^o$ such that $c^o_t \geq c^*_t$ for all t and $c^o_t > c^*_t$ for at least one t. From the assumption that $c^o$ is efficient and c* is inefficient it follows that $p^*c^o - p^*c^* > 0$. This statement contradicts the assumption that c* is the profit maximizing relative to p*, which means that $p^*c^* - p^*c^0 \geqq 0$. Thus, if $c^*$ is profit maximizing relative to $p^*$, it is also efficient.

*Efficiency (E). The present value maximizing felling program $c^*$ is efficient, provided that $p_t > 0$, all t.*

For the two period case (T=2) this could be illustrated by the following diagram. The feasible felling possibilities are given by the shaded area in figure 1. The efficient programs are on the boundary AB. All interior points, such as $c^0$, are inefficient

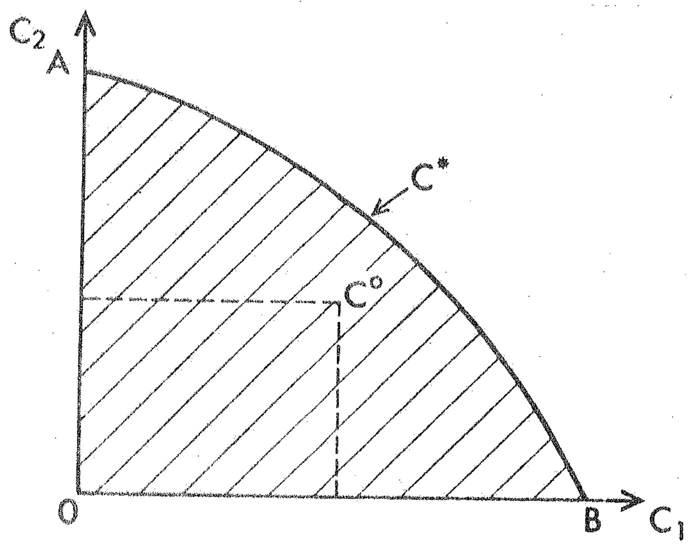


Figure 1. The feasible felling possibilities and the efficient points

## 2.2 The effects of taxes

A proportional tax on profits is equivalent to a scalar multiplication of the price system $p^*$ by the scalar $\lambda = 1-t > 0$, where $0 < t < 1$ is the proportional tax rate. Hence, the optimal felling program $c^*$ is not changed by the imposition of a proportional tax. (This follows directly from the (H) homogenity property of $c^*$.)

The Finnish tax system is equivalent to a lump-sum tax paid during each period, and the total sum that has to be paid up to last period will also be a constant. Hence, we can use the property (I) dervied above, and conclude that the optimal felling program $c^*$ will be independent of lump-sum taxes ($T_x$). Hence, if the tax system is changed from the Swedish corporate tax, which is a proportional tax to the

Finnish lump-sum tax system, this will not change the supply of roundwood within the forest firm. The principles which are involved can be illuminated by the following diagram. It appears from the figure 2 below that the lump-sum tax does not change the volume of production that gives maximum profit, it just shifts the profit function $\pi(c)$ downwards.

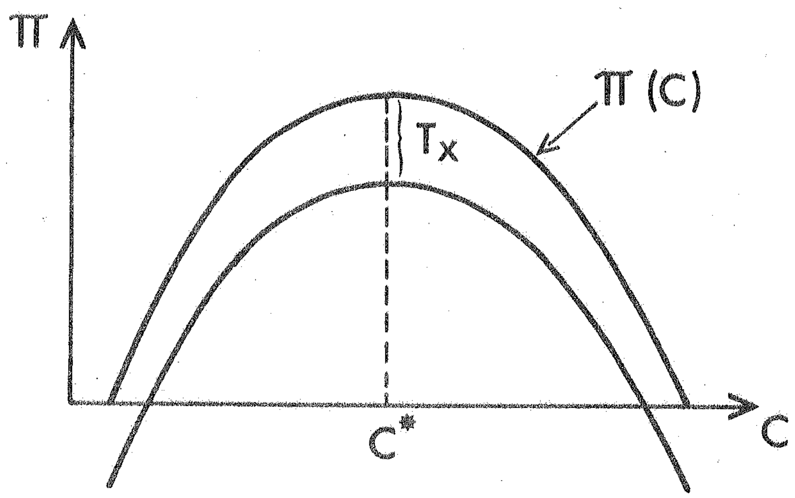


Figure 2. The effects of a lump-sum tax

The proportional profit tax does not affect the determination of the optimal felling program, since it does not influence the relationship between costs and revenues. This is displayed in figure 3 below.

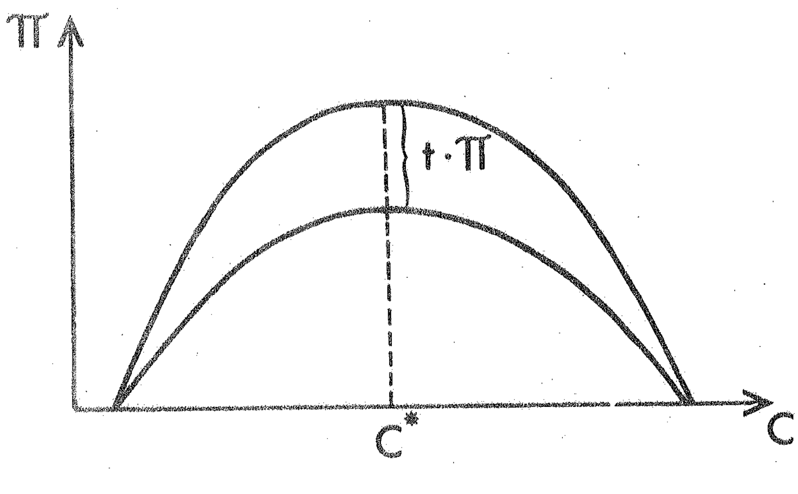


Figure 3. The effects of a proportional profit tax

Needless to say, as it coincides with the solution in the non-taxation case, the solution $c^*$ is efficient.

Although the Swedish corporate tax is proportional tax, we will discuss what happens if profits are taxed by a progressive tax rate. When the rate of taxation is variable and dependent on profits the maximization problem becomes

$$\max_{c} \pi(p) = \sum_{t=1}^{T} (1-t(p_t^* c_t))p_t^* c_t \tag{9}$$

subject to restrictions (1)-(5)
where

$$t_x(p_t^* c_t) = t_x(y_t) \qquad \frac{\partial t_x}{\partial y_t} \geqq 0$$

is the progressive tax rate. When we consider the whole planning period $t = 1,\ldots, T$, the tax rate has different values in different periods depending on the total income during each particular period. As the tax rate is variable, it is not a scalar multiplication of the net price vector this time, and the profit maximizing felling program ($c'$) will in general differ from $c^*$. If, however, $p_t > 0$ all $t$ and $0 < t(y_t) < 1$ all $y_t$, it can be proved that $c'$ is an efficient solution relative to the net price system $p' = (p_1(1-t_x(y_t')) ,\ldots, p_T(1-t_x(y_t')))$, and the solution in the two period case will be situated on the border of the feasible region OAB in figure 1.

## 2.3 Conclusions

If the forest firm is profit maximizing with respect to the present value of the total profit over a planning period longer than one year, the felling programs that solve the maximization problems, when the forest taxation consists of a lump-sum tax, a proportional profit tax, and when it's exempted from taxation, are identical. The progressive taxation of the profit income gives a felling program that in general differs from the one obtained when the other forms of taxation are used.

Translated to comparison between the Finnish and Swedish systems of forest taxation the results mean that a switch from the Swedish proportional tax to the Finnish lump-sum tax would not change the felling policy of a forest corporation of the same kind as the Swedish Domänverket. The private forest owner, on the other hand, would be affected of a change from a progressive taxation of his incomes from forestry to a lump-sum tax. His felling policy would change from one efficent solution to another.

## 3. The self-active forest farmer

In this section we will turn to an analysis of the self-active forest farmer, who has the choice between to work on his forest farm, to work on his agricultural farm, or to work in the industry.

To deal with his management problem we will assume that he possesses the utility function

$$U = U(y, \bar{L}-\ell_1 - \ell_2 - \ell_3) \tag{10}$$

which is assumed to be quasi-concave, twice continuously differentiable, and increasing in each argument ($\frac{\delta U}{\delta y} > 0$, $-\frac{\partial U}{\partial \ell_i} > 0$). Moreover,

$\bar{L}$ = the total available number of hours

$\ell_1$ = the total number of labor supplied for work in forestry

$\ell_2$ = the total number of labor supplied for work in industry

$\ell_3$ = the total number of labor supplied for work in agriculture

$y$ = the consumption of consumer goods

$\bar{L} - \ell_1 - \ell_2 - \ell_3$ = leisure time

The fact that $\bar{L} - \ell_1 - \ell_2 - \ell_3$ is included as an argument in the utility function means that the disutility of one hours work is independent of where the self-active farmer works. In other words, he has no preference for any particular working place. This might seem a bit unrealistic, but is no large violation of reality, and it simplifies the analysis considerably.

The utility function is maximized subject to a budget constraint.

$$\pi_f(\ell_1) + w\ell_2 + \pi_a(\ell_3) - py \geqq 0 \tag{11}$$

where

$\pi_f(\ell_1)$ = the imputed income[4)] from $\ell_1$ hours of work in forestry

$w$ = the wage rate ($w\ell_2$ = the income from work in industry)

$\pi_a(\ell_3)$ = the imputed income[4)] from $\ell_3$ hours of work in agriculture

$p$ = the price of consumer goods

$\pi_f(\ell_3)$ and $\pi_a(\ell_3)$ could be apprehended as the yearly interest on the capital stocks of the agricultural and forest farm respectively. We will assume that

$$\begin{aligned} &\pi_f(0) \geqq 0 \\ &\pi_a(0) \geqq 0 \end{aligned} \tag{12}$$

where

$$\frac{\partial \pi_f}{\partial \ell_1} > 0, \qquad \frac{\partial \pi_a}{\partial \ell_3} > 0, \qquad \frac{\partial^2 \pi_f}{\partial \ell_1^2} < 0 \quad \text{and} \quad \frac{\partial^2 \pi_a}{\partial \ell_3^2} < 0$$

This means that the incomes from forestry and agriculture are non-negative and that they are differentiable, and increasing functions of the input of labor $(\frac{\partial \pi_f}{\partial \ell} = \pi_f'(\ell_1) > 0$, $\frac{\partial \pi_a}{\partial \ell_3} = \pi_a'(\ell_3) > 0$. Moreover, there are dimishing returns to the inputs of labor.

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4) From the Faustmann - Ohlin theorem, we know that the land value of a forest stand (the value of the capital) is given by

$$\text{Max} = \frac{pf(T)e^{-rT}}{1-e^{-rT}} = \frac{pf(T)}{e^{rT}-1} = K$$

$T$ = time for cutting
$p$ = the price of lumber
$r$ = the interest rate
$K$ = the value of the capital

If $T^*$ solves the problem, we have from the fact that income equals interest on the capital that imputed income from forestry equals

$$\pi_f = rK^* = \frac{rpf(T^*)}{e^{rT^*}-1}$$

$$\left(\frac{\partial^2 \pi_f}{\partial \ell_1^2} = \pi_f''(\ell_1) < 0, \quad \text{and} \quad \frac{\partial^2 \pi_a}{\partial \ell_3^2} = \pi_a''(\ell_3) < 0\right)$$

A possible shape of the $\pi_f(\ell_1)$ function is depicted in figure 4 below

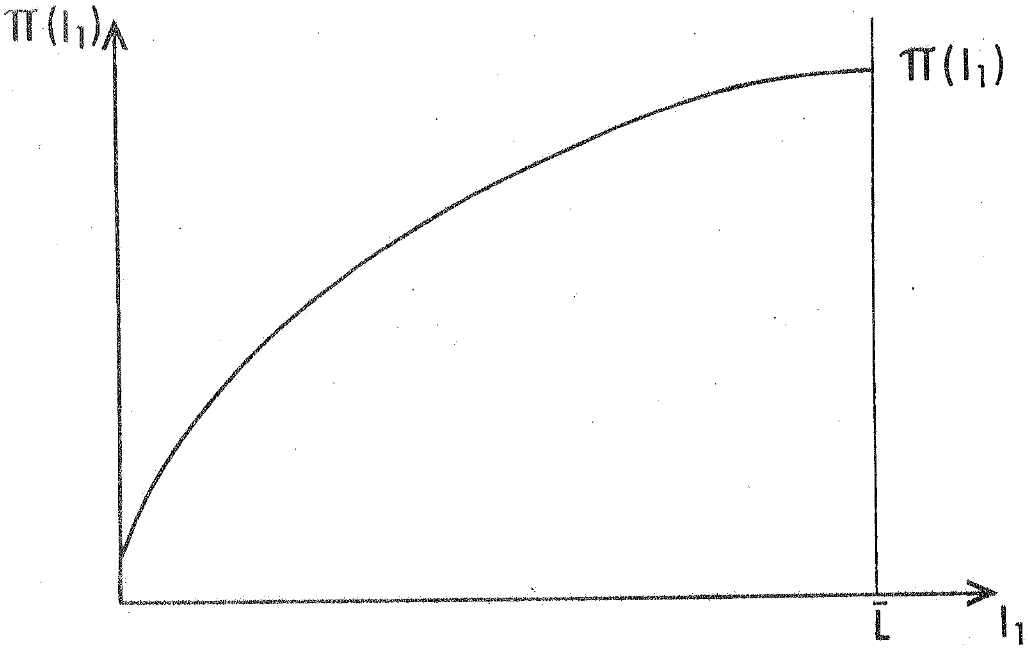


Figure 4. The capital income from forestry

The budget restriction simply says that the value of consumption cannot exceed the "income" of the self-active forest farmer.

Let us now introduce taxation: We assume that the income from forestry and agriculture are taxed by the proportional tax rate $1-\alpha$, $0 < \alpha \leqq 1$, while wage income is taxed by the proportional tax rate $1-\beta$, $0 < \beta \leqq 1$. (Nothing essential is changed if the tax rates are assumed to be progressive.) Moreover, we introduce a lump-sum tax $T_x$. This budget constraint can now be written as[5)]

$$\alpha(\pi_f(\ell_1) + \pi_a(\ell_3)) + \beta w \ell_2 - py - T_x = 0 \tag{12}$$

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5) As the utility function is increasing in each argument the budget constraint will hold with equality.

The self-active forest farmer's choice theoretical problem is now to maximize the utility function (10) subject to the budget constraint (12). The first order conditions for an interior solution can be written[6)]

$$\begin{aligned}
&\text{a)} \quad U_y - \lambda p = 0 \\
&\text{b)} \quad - U_{\bar{L}} + \lambda \alpha \pi_f'(\ell_1) = 0 \\
&\text{c)} \quad - U_{\bar{L}} + \lambda \beta w = 0 \\
&\text{d)} \quad - U_{\bar{L}} + \lambda \alpha \pi_a'(\ell_3) = 0 \\
&\text{e)} \quad \alpha(\pi_f(\ell_1) + \pi_a(\ell_3)) + \beta w \ell_2 - py - T_x = 0
\end{aligned} \tag{13}$$

where

$\frac{\partial U}{\partial y} = Uy =$ the marginal utility of consumer good

$- \frac{\partial U}{\partial \ell} = - U_{\bar{L}} =$ the marginal utility of leisure

$\pi_a'(\ell_3)$, $\pi_f'(\ell_1)$ = the increase in income of the last hour spent in agriculture and forestry respectively

$\lambda$ = a Lagrange multiplier

These are five equations in five variables ($y$, $\ell_1$, $\ell_2$, $\ell_3$ and $\lambda$), and it is particularly easy to solve for the degree of self-activity in forestry, and agriculture ($\ell_1$ and $\ell_3$). By combining equations (13b) and (13c) one obtains

$$\alpha \pi_f'(\ell_1) = \beta w \tag{14}$$

which can be interpreted as an equality between the net return from the last hours of work in industry and forestry respectively. As equation (14) only contains one unknown, one can solve to obtain

$$\ell_1 = \pi_f'^{-1} \left(\frac{\beta w}{\alpha}\right) \tag{14a}$$

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6) Form a Lagrange-function and differentiate with respect to the choice variables $y$, $\ell_1$, $\ell_2$, $\ell_3$ and $\lambda$.

Two things should be noted. The number of hours worked as a self-active forest farmer is independent of the lump-sum tax, and if income from work in forestry and income from work in industry is taxed in the same manner ($\beta=\alpha$), then the degree of self-activity will only depend on the wage rate (w).

If we put $\beta w \alpha^{-1} = \bar{z}$ we can rewrite (14) as

$$\pi_f'(\ell_1) = \bar{z} \tag{14b}$$

and the solution is found diagrammatically, where the slope of the income function coincides with the number $\bar{z}$. Compare figure 5 below

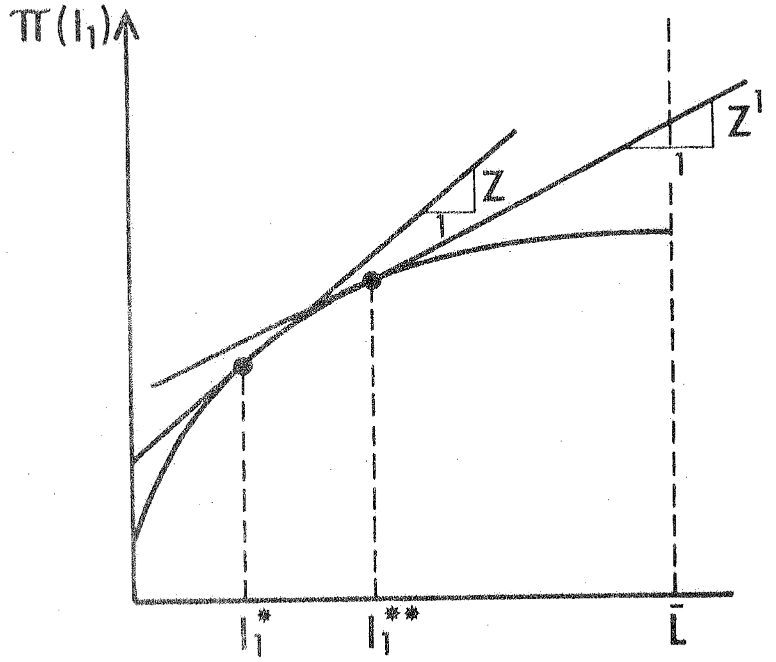


Figure 5. The optimal self-activity in forestry

The optimal self-activity in forestry given $\bar{z}$ is equal to $\ell_1^*$. If we eliminate the tax on income from forestry ($\alpha \to 1$), e.g. by introducing a lump-sum tax on the possession of the forest land, $\bar{z}$ decreases to $\bar{z}'$ and self-activity in forestry increases.

Hence, we have shown two things: in the first place we have shown that the degree of self-activity in forestry is a decreasing function of $\bar{z} = \beta w \alpha^{-1}$ i.e.

$$\ell_1 = \pi'^{-1}(\underset{-}{\bar{z}}) \tag{14a}$$

where the sign under the argument denotes the sign of the derivative with respect to $\bar{z}$. In the second place, we have shown that a switch from the Swedish system of taxing income from forestry to the Finnish system will increase self-activity, and, hence, probably also the supply of roundwood.

## 4. Income constraints and the backward bending supply curve

### 4.1 A general approach

It has been argued that many forest owners have restrictions on the net income from forestry during specific periods. Such a constraint is the following binding income target

$$p_t c_t = \bar{y}_t \tag{15}$$

where

$\bar{y}_t$ = the income target

We will now compare the effects on the supply of roundwood of a lump-sum tax, and a proportional profit tax, when there is a income target in period t. If the net price is $p_t$, the income target is $\bar{y}_t$, and there is no taxation, then supply equals

$$c_t^1 = \frac{\bar{y}_t}{p_t} \tag{16}$$

The supply decreases when the net price increases and vice versa,

Having the same price $p_t$ and income target $\bar{Y}_t$, but introducing a lump-sum tax, we get

$$p_t c_t - T_x = \bar{y}_t \tag{17}$$

and the supply becomes

$$c_t^2 = \frac{\bar{y}_t + T_x}{p_t} \tag{18}$$

As

$$\frac{\bar{y}_t + T_x}{p_t} > \frac{\bar{y}_t}{p_t}$$

when

$$T_x > 0$$

$$c_t^2 > c_t^1 \tag{19}$$

As the lump-sum tax lowers the net profit, the supply has to be increased if the income target is going to be hit. When a proportional profit tax is introduced, the income target can be written

$$(1-t_x)p_t c_t = \bar{y}_t \tag{20}$$

and the supply becomes

$$c_t^3 = \frac{\bar{y}_t}{(1-t_x)p_t}$$

As

$$0 < (1-t_x) < 1, \qquad \frac{\bar{y}_t}{(1-t_x)p_t} > \frac{\bar{y}_t}{p_t}$$

and

$$c_t^3 > c_t^1 \tag{21}$$

Also in this case the supply will be larger than the corresponding supply in the non-taxation case.

But what about $c_t^2$ and $c_t^3$? Which one of the two forms of taxation gives the larger supply, when the forest farmer has an income target?

As the income target is the same in (17) and (20) we can write

$$p_t c_t^2 - T_x = (1-t_x) p_t c_t^3 \tag{22}$$

If the amount of taxes that has to be paid, is the same in the two systems of taxation, i.e. the two forms of taxes are equal from the point of view of tax revenue, then we can write

$$p_t c_t^2 - T_x = p_t c_t^3 - t_x p_t c_t^3$$

with

$$T_x = t_x p_t c_t^3$$

if follows that

$$p_t c_t^2 = p_t c_t^3 \tag{23}$$

which implies that

$$c_t^2 = c_t^3 \tag{24}$$

This conclusion holds also for the progressive taxation of income as $t_x$ can be regarded as the average rate of taxation during period t. Thus, if the forest farmer has an income target, the kind of taxation that is applied, is of no importance for the supply of roundwood.

## 4.2 The self-active forest farmer with income constraints

The backward bending supply curve derived in the previous analysis is due to the fact that the income constraint is solely formulated in terms of income from forestry.

The income constraint is more likely to be relevant for the self-active forestry farmer, whose income from forestry farming is combined with other sources of income. In that case, a more logical way to introduce an income constraint is to allow income from all sources to add up to the income target. We will now carry out an analysis analogous to the one in chapter 3 of the self-active forest farmer, and his choice between working on his forest farm, agricultural farm or in the industry. The only difference is that we now introduce a binding income constraint. We assume that the working time in forest farming can be combined with work in industry, and that the incomes from these two sources has to add up to an income target.

Then the income constraint can be written

$$\alpha\pi_f(\ell_1) + \beta w\ell_2 - T_x = \bar{\pi} \tag{25}$$

where $\bar{\pi}$ is the income target.

When the self-active forest farmer maximizes the utility function (10) subject to the budget constraint (12) and the income constraint (25), the first order conditions for an interior solution can be written

$$\begin{aligned}
&\text{a)} \quad U_y - \lambda p = 0 \\
&\text{b)} \quad -U_{\bar{L}} + (\lambda+\mu)\alpha\pi_f' = 0 \\
&\text{c)} \quad -U_{\bar{L}} + (\lambda+\mu)\beta w = 0 \\
&\text{d)} \quad -U_{\bar{L}} + \lambda\pi_a' = 0 \\
&\text{e)} \quad \alpha(\pi_f(\ell_1) + \pi_a(\ell_3)) + \beta w\ell_2 - py - T_x = 0 \\
&\text{f)} \quad \alpha\pi_f(\ell_1) + \beta w\ell_2 - T_x = \bar{\pi}
\end{aligned} \tag{26}$$

The tax rates on incomes from forestry is $(1-\alpha)$ and on incomes from industry $(1-\beta)$, and $\mu$ is the Lagrange multiplier corresponding to the constraint introduced in equation (25).

By combining (26b) and (26c) one obtains

$$\pi_f'(\ell_1) = \beta w \tag{27}$$

which implies that

$$\ell_1 = \pi_f'^{-1}(\frac{\beta}{\alpha} w) \tag{27a}$$

The number of hours worked as a self-active forest farmer is independent of the lump-sum tax. If the income from forestry and industry is taxed in the same manner ($\alpha=\beta$), the degree of self-activity will only depend on wage rate in the industry.

The other thing to be noted is that the supply curve will no longer be backward bending. The intuitive reason is very simple. The utility maximizing self-active forest farmer will strive to satisfy the income constraint with minimum loss of leisure time. If the marginal revenue from work in forestry increases, he will under the restriction (25) increase working time in forestry and decrease working time in the industry.

## 5. Summary and concluding comments

In this paper we have formally analysed the effects of a changed forest taxation on the supply of roundwood, and on the efficiency of forestry. We have shown that the optimal felling program in a present value maximizing forest firm, like the Swedish Domänverket, will be unaffected of a switch from the present tax system to the Finnish lump-sum tax system, where the site quality classification determine the magnitude of the lump-sum.

However, when we considered the behaviour of a self-active forest farmer, we found that the number of hours spent in forestry would very likely increase, if the above mentioned change of the tax system is carried out. It would be strange if this did not result in an increased supply of roundwood.

Let us for a moment leave the world of models, and ask us a little tentatively what will happen to the felling activities among those who have small forest areas, which they manage inoptimally, due to the small losses involved, and perhaps also due to a speculation in future price increases. The magnitude of the inoptimality losses are not affected by a proportional tax or a progressive tax on the income from forestry. A lump-sum tax would, however, make it more expensive to manage a forest inoptimally, and make it most expensive on the land with the highest site quality classifications. A switch to a lump-sum tax might therefore improve on the distribution between active and inactive forest owner of two reasons. Former inactive forest owners might turn active, or choose to sell their land to active forest farmers.

Finally, it should be mentioned that we in this paper have analysed very limited aspects of the tax system. In a perfect market economy all kinds of taxes, except lump-sum taxes, can deteriorate the efficiency of the market economy. To tax income from forestry differently from income out of other sources does not necessarily improve efficiency of the Swedish economy. The non-clearing roundwood market, however, indicates that there are social benefits from measures that can increase the supply of roundwood.

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