The Q model, a Simple Conceptual Model for Runoff Simulation in Catchment Areas.

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Abstract

A model called the Q model for estimating daily runoff is described. The driving variables are precipitation and mean temperature and the model runs on a daily time step. In order to meet special requirements, the Q model was constructed using elements from different existing models. It was shown that a simpler structure with fewer parameters to calibrate gave equal model performance. The Q model has got an automatic calibration routine which considerably improves both handling and calibration procedures.

Introduction

In connection with the running programme on integrated monitoring in Sweden there is a need for supplementary surface runoff data in periods when measurements for some reason are missing. To meet this, a model was designed based on components of already existing models. The demands were twofold, the model should be able to simulate runoff properly with a minimum of driving variables (temperature and precipitation), and it should be easy to calibrate and use. Further, it should be able to operate in small catchments. The models examined were the HBV model (Bergström 1992), the SOIL model (Jansson 1991), the Birkenes model (Lundquist 1976), and a modified version of the Birkenes model (Grip 1982), here named the GRIP model. None of these models could fully meet the demands above and therefore a ”new” model was created with elements from both the HBV and GRIP models. This model, named the Q model, has been used since 1989 with subsequent modifications on several occasions. The model has functioned properly in many different catchments, ranging from small forested areas (down to 0.2 km^2) to large agricultural catchment areas (up to 2000 km^2). So far about thirty areas in Sweden have been modelled.

The model and some applications will first be described, followed by a discussion of calibration procedures, before finally proposing some model modifications.

Model Description

The Q model is a conceptual, simply structured, model for simulation of runoff in different types of catchments. The driving variables are daily precipitation and mean temperature and the model runs on a daily time step. In order to function properly, the model has to be calibrated against observations of runoff for a limited time period. If no calibration is performed, the quality achieved will not be fully satisfactory. The model has only four state variables; two for treatment of snow and two state variables that describe water content in the unsaturated zone.

Snow and Rain

The equations treating snow and rain are very similar to those in the HBV and GRIP models. Snow cover is handled by two state variables; one for frozen (S) and one for liquid water (W). The amount of snow and its content of water is controlled by precipitation and air temperature, together with model parameters. A threshold value on air temperature (T_{snow}) decides if precipitation is snow or rain. The precipitation is corrected with different correction

factors for snow and rain, respectively. Another threshold value (T_{melt}) decides if snow is freezing or melting.

A melt parameter (C_{melt}) controls snowmelt, which is proportional to the difference between air temperature and T_{melt}. A refreezing constant regulates the transition of water from liquid to frozen form if air temperature is below T_{melt}. Infiltration from snow starts when the water content of snow exceeds its water-holding capacity.

The equations for snow and its content of liquid water are:

$$\frac{dS}{dt} = snow - melt + freeze \qquad 1$$

$$\frac{dW}{dt} = rain + melt - freeze - infil \qquad 2$$

where:

S	=amount of frozen water in snow	mm
W	=amount of liquid water in snow	mm
Snow	=corrected snow precipitation	mm/day
Rain	=corrected rain precipitation	mm/day
Melt	=melting intensity	mm/day
Freeze	=freezing intensity	mm/day
Infil	=amount of water which infiltrates to soil	mm/day

The Unsaturated Zone

The unsaturated zone is divided into two compartments. The equations for water content in the unsaturated zone are mainly identical to those in the GRIP model (Figure1). When modifications have been made they are reported in text.

$$\frac{dA}{dt} = infil - E_A - Q_A \qquad 3$$

$$\frac{dB}{dt} = A_{div} * Q_A - E_B - Q_B \qquad 4$$

where:

A	=water content of upper soil layer	mm
E_A	=evaporation from upper soil layer	mm/day
Q_A	=runoff from upper soil layer	mm/day
A_{div}	=f(B) (see below)	-
B	=water content of lower soil layer	mm
E_B	=evaporation from lower soil layer	mm/day
Q_B	=runoff from lower soil layer	mm/day

The upper soil layer (A), which has been compared with the humus layer, receives the infiltrated water. Water in layer A evaporates at the same time as it percolates to the lower layer (B). Under certain circumstances, regulated by the parameter A_{div}, a direct runoff to the river occurs. The lower layer (B) receives percolated water ($A_{div}*Q_A$) and loses water through evaporation and runoff. A_{div} is a factor that regulates how much of the runoff from A is surface runoff, and depends on the amount of water in the B-layer according to the following equations:

$$A_{div} = 1 \qquad \text{for } B \le B_{min} \qquad 5$$

$$A_{div} = 1 - R_{div} * \frac{B - B_{min}}{B_{max} - B_{min}} \qquad \text{for } B > B_{min} \qquad 6$$

where

R_{div}	= a non dimensional percolation constant	-
B_{min}	= a threshold above which A_{div} becomes 1.	mm
B_{max}	= maximum volume of B-layer	mm
A_{min}	= a threshold value (in equations below)	mm

Runoff from A and B is defined as follows:

$$Q_A = A_k * (A - A_{min}) \qquad 7$$

$$Q_B = B_k * (B - B_{min}) \qquad 8$$

where A_k and B_k are recession constants for the A- and B-layers.
Below layer B the ground is assumed to be impervious to water and the formation of runoff water is formulated as:

$$Q = (1 - A_{div}) * Q_A + Q_B \qquad 9$$

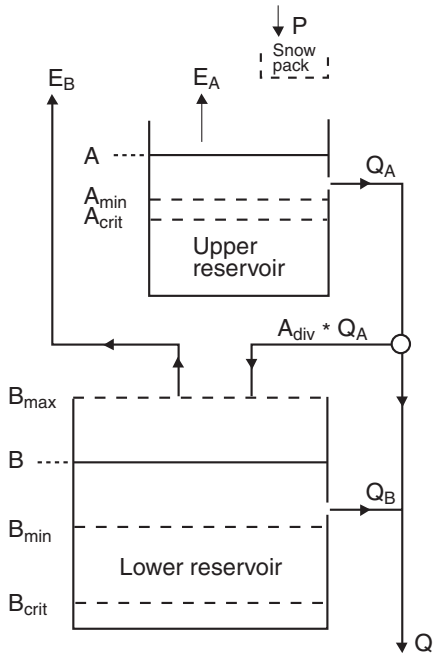

Figure 1*. Model structure (from Grip 1982).*

Evaporation

Evaporation is considered potential if the water content in both layers is high enough. In the model, evaporation occurs from both layers. In periods of dry weather the evaporation is reduced in proportion to water content of each layer.

Potential evaporation (PE) is calculated by a simplified version (Linacre, 1977) of the Pennman equation (Pennman, 1948):

$$PE = \frac{500 * \frac{T + 0{,}006*H}{100 - Lat} + 15 * \Delta T}{80 - T} \qquad 10$$

where

T	= daily mean temperature	°C
H	= Altitude	m a.s.l.
Lat	= Latitude	°
DT	= dew point depression T-Td	°C

The factors MR_A and MR_B are expressions for water availability in each respective layer. They are defined as:

$$MR_A = \frac{A - A_{crit}}{A_{min} - A_{crit}} \qquad 11$$

$$MR_B = \frac{B - B_{crit}}{B_{ncrit} - B_{crit}} \qquad 12$$

where

A_{min}	= a threshold above which MR_A is equal to 1	mm
A_{crit}	= a threshold below which MR_A is equal to 0	mm
B_{ncrit}	= a threshold above which MR_B is equal to 1	mm
B_{crit}	= a threshold below which MR_B is equal to 0	mm

Evaporation from A- and B-layer are :

$$E_A = PE * \frac{\max(MR_A , MR_B)}{MR_A + MR_B} *(MR_A)^a \qquad 13$$

$$E_B = PE * \frac{\max(MR_A , MR_B)}{MR_A + MR_B} *(MR_B)^a \qquad 14$$

In this present version of the model, the value of parameter a is set equal to 1. If a is allowed to vary it can be shown that this will lead to apparently incorrect values of the evaporation. Another modification of the GRIP model is that the parameter B_{min} has been replaced by a new parameter B_{ncrit} in equation 12. By introducing this threshold value for evaporation, both watercourses that dries completely and those that never dry out can easily be calibrated. This is achieved by allowing B_{ncrit} to vary on both sides of B_{min}.

Calibration routine

The model has to be calibrated against observations of runoff. In some cases also snow observations are used for tuning of parameters belonging to the snow routine. A total of fourteen parameters have to be calibrated, and these are summarised in Table 1.

When the model is calibrated, all parameters are varied both step-wise and in clusters until no further improvements are archived. As a measure of fitness, least squares, in the text named N^2-norm according to equation 15, are minimised:

$$N^2 = \frac{\Sigma(q_{mod} - q_{obs})^2}{n} \qquad 15$$

The model has a routine for automatic calibration. This routine is an algorithm for least square determination, named the DUD method or method of false position (Ralston and Jenirich 1978). When this method was introduced, the model performance improved considerably in comparison with the manual method used earlier. This is exemplified in Table 2. where the automatic method is compared with the manual. A calibration of a single catchment will normally be done within less than one hour and with a minimum of time request by the operator.

Table 1. The physical parameters of the modified Q model which have to be derived by calibration.

Process	Parameter	Equation	Name	Variation	Unit
Rain and snow	Tsnow		Precipitation threshold	-0.5-+2.0	°C
	Rcorr		Rain correction factor	0.9-1.9	-
	Scorr		Snow correction factor	1.0-2.0	-
	Tmelt		Melting threshold	-0.5-+2.0	° C
	Cmelt		Melting constant	3.0-6.0	° C/day
Evaporation	Acrit	11	Evaporation threshold	0-100	mm
	Bcrit	12	Evaporation threshold	0-200	mm
	Bncrit	12	Evaporation threshold	100-300	mm
Upper layer	Amin	8.12	Runoff threshold	0-30	mm
	Ak	8	Recession constant	0.10-0.4	mm/day
	Rdiv	6	Percolation constant	0.2-.06	-
Lower layer	Bmax	6	Maximum volume	150-250	mm
	Bmin	6.9	Runoff threshold	0-130	mm
	Bk	9	Recession constant	0.10-0.4	mm/day

Table 2. Comparison between manual and automatic calibration in some integrated monitoring areas expressed as N2-norm.

Area name	Station	Period	Manuel N^2-norm	Automatic N^2-norm
Tresticklan	501	871001-890531	1.53	1.51
Svartedalen	502	890401-911231	1.17	0.62
Berg	504	900401-911231	0.54	0.52
Tiveden	506	881001-901231	0.50	0.36
Tiveden	508	871101-890431	0.62	0.46
Reivo	511	900301-910931	0.32	0.29
Stormyran	519	900101-910931	0.62	0.50
Söderåsen	522	891001-901231	0.22	0.18

Model applications

The model has been applied to about 30 different areas in Sweden (Figure 2). Most areas are small, down to 0.2 km2 forested areas. But also large low slope agricultural areas have been modelled, eg River Fyris which has a total drainage area of 2000 km2. Data from Svartedalen Berg, Tiveden, Söderåsen, Stormyran, Reivo (Figure 3a,b) and the River Fyrisån (Figure 4) are presented as examples of some simulations. Model results are plotted as time diagrams. The upper curve is the accumulated difference between observed and calculated runoff. The lower curves are observed (thin line), and calculated (thicker line) runoff is expressed as mm/day. The figures illustrate that the ability to simulate runoff also during independent periods is usually good and does not significantly differ from the calibration period (dependent period). It can also be seen that N^2 norm differs considerably between different sites, an obvious explanation being the big variation in precipitation. Problems of this kind are common in small drainage areas, especially if the distance to the meteorological reference station is long. The errors are most frequent during summer, when convective precipitation dominates. This is the case for both Tiveden and Söderåsen, two areas situated in the border between hilly and plain landscapes. Tiveden is situated very close to Lake Vättern and Söderåsen is on the northeast slope of a large horst formation in a rain shadow.

In larger areas, eg River Fyris, these problems are less frequent because there are more meteorological stations in the area and also because the runoff fluctuations are smoothed out. The model has functioned well in catchments with high percentages of lakes eg Svartedalen (18%) and Tiveden (4%). Both areas have lakes close to the outlet. Attempts to introduce different types of smoothing as a result of the lake did not improve the results at all.
In the River Fyris catchment, which also includes River Sävja, the Q model is divided into eleven sub-models. Runoff stations at Vattholma, Ulva Kvarn and Kuggebro are used for calibration. In this case, the model results are used for monthly determination of nutrient transport in connection with a recipient control programme.

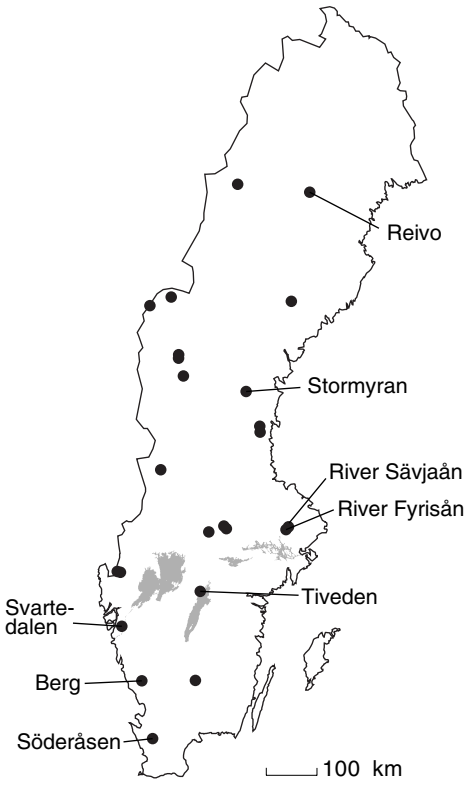

Figure 2. *Locations of studied areas (dots) in Sweden where the Q model has been used for runoff simulation.*

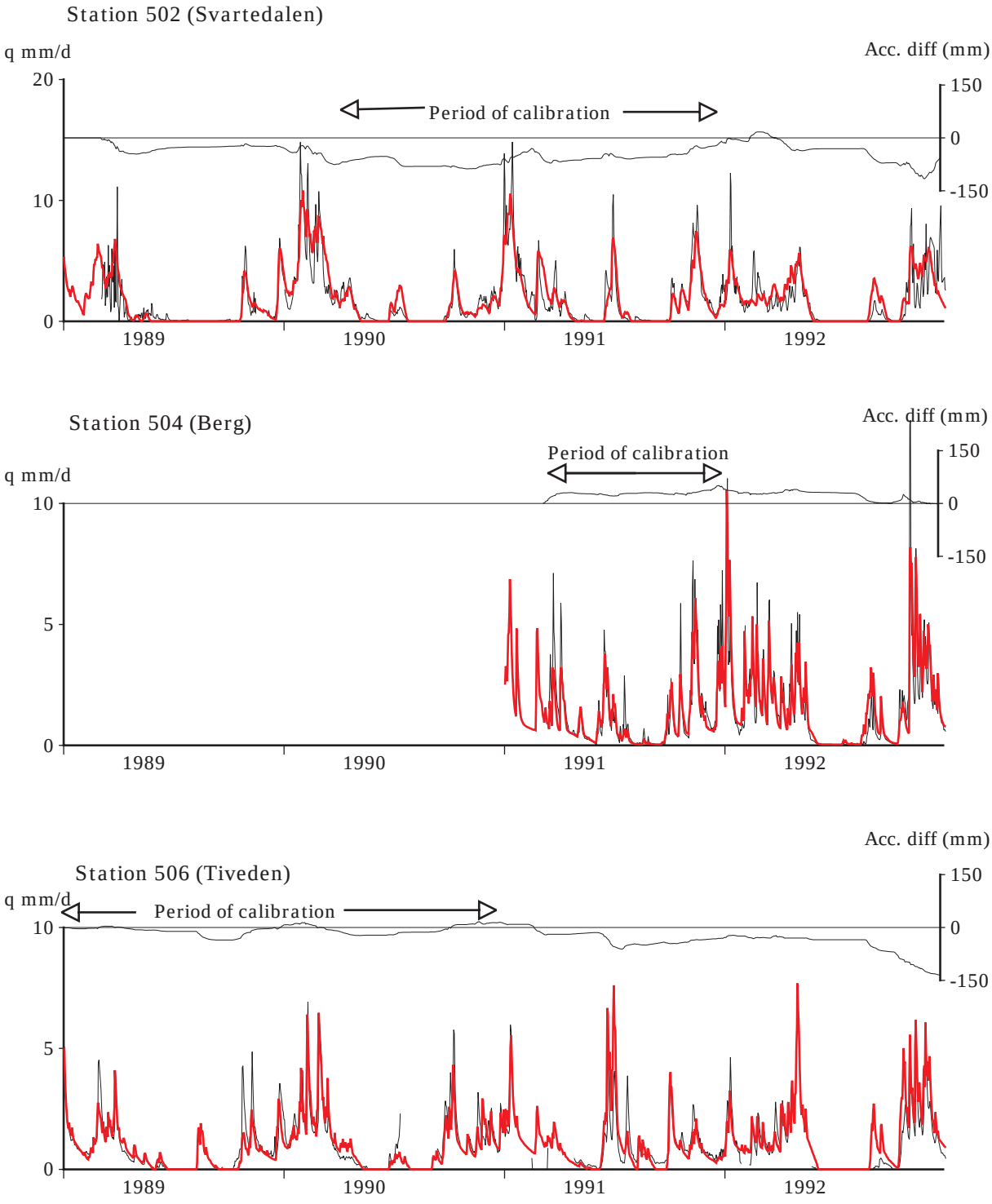

Figure 3a. *Simulated and observed runoff at some stations in the programme on integrated monitoring in Sweden. The upper curve is the accumulated difference between observed and calculated runoff. The lower curves are observed (thin line) and calculated (thicker line) runoff expressed as mm/day.*

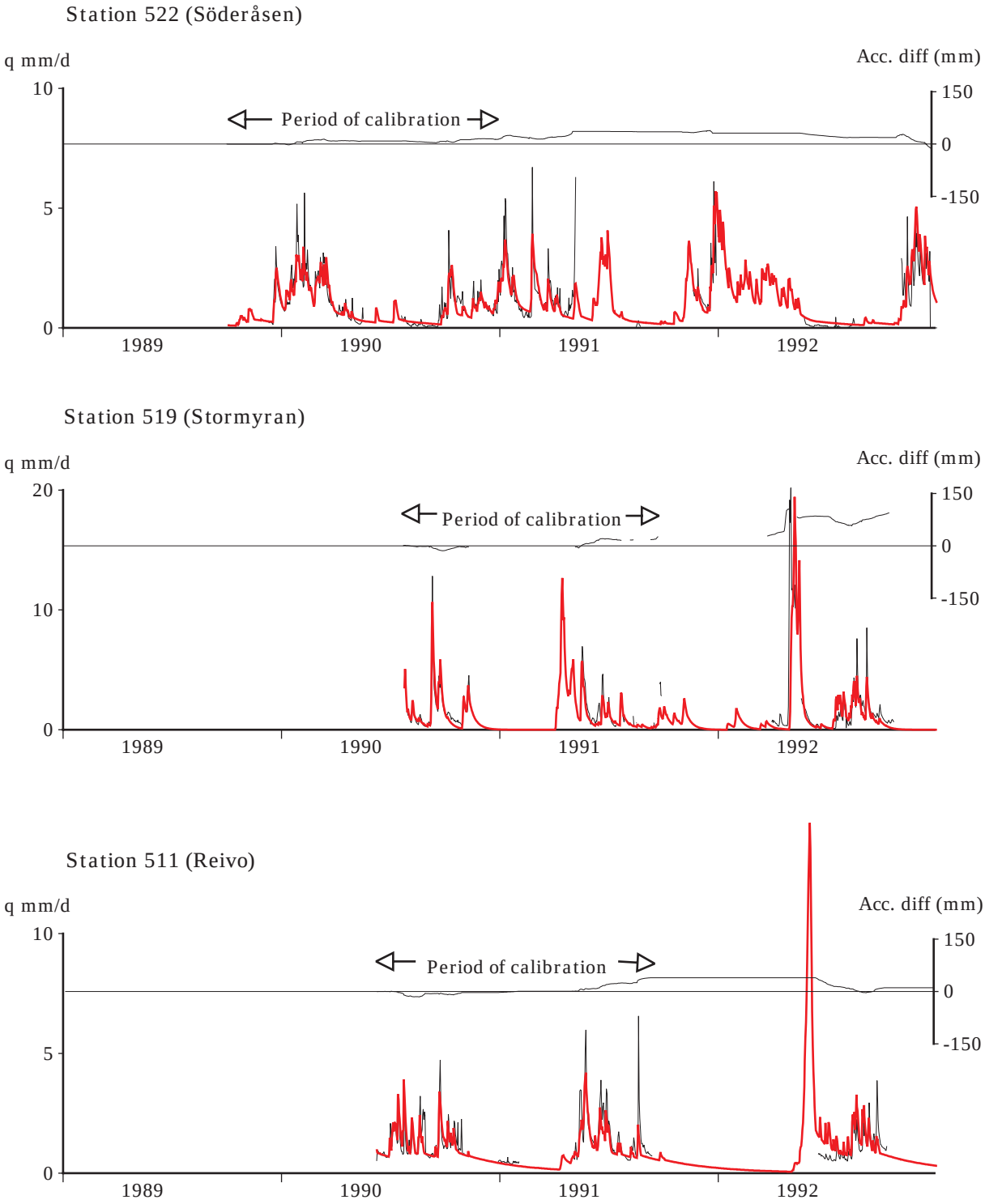

Figure 3b. *Simulated and observed runoff at some stations in the programme on integrated monitoring in Sweden. The upper curve is the accumulated difference between observed and calculated runoff. The lower curves are observed (thin line) and calculated (thicker line) runoff expressed as mm/day.*

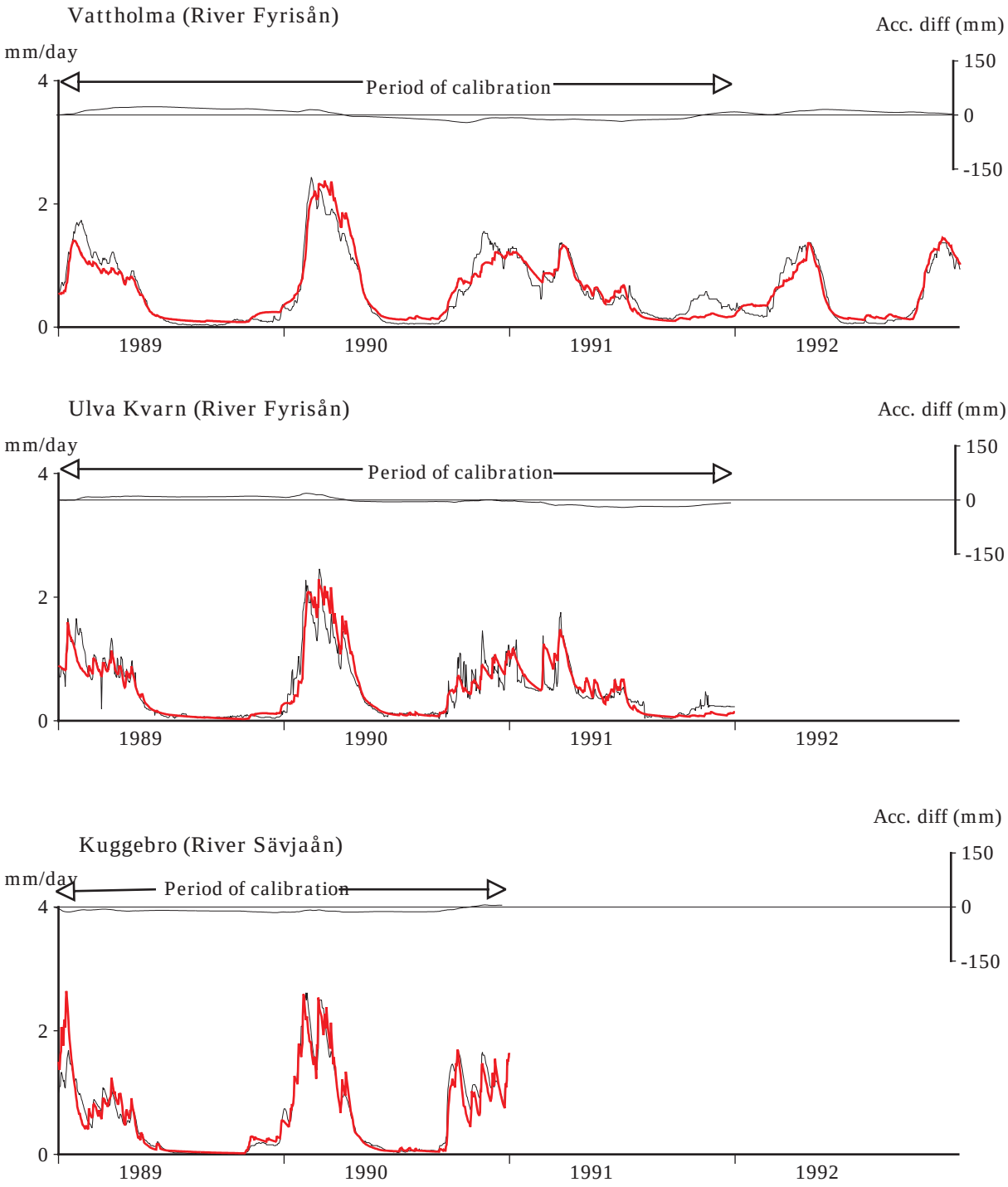

Figure 4. *Simulated and observed runoff at some stations in the River Fyris catchment, Sweden. The upper curve is the accumulated difference between observed and calculated.*

Parameter optimisation

Parameter reduction

If the number of parameters to calibrate is limited the risk of overparameterisation is reduced. At the same, time calibration becomes less complicated.

There are reason to believe that a model with only two driving variables and which is designed barely to predict runoff should not be too complicated with regards to the number of equations and parameters. An exercise to find out whether any of the model parameters could be reduced was made.

During this experiment, recalibrations were made after some of the parameters successively were locked at certain preset values. Altogether six smaller areas were examined (Table 3). DUD technique was used and least squares according to equation 15 characterised fitness between observed and calculated values. The start position was a complete calibration where all parameters varied. After that, the order of the locked parameters was A_{min}, B_{max}, B_{min}, R_{div}, B_{ncrit} and B_{crit}.

Table 3. Area characteristics of some integrated monitoring areas.

PMK-area	Station	Latitude	Longitude	Area	Altitude	Soil depth	Lake	Outflow area	Exposed bedrock	Till	Mire
				km^2	m a.s.l.	meters	%	%	%	%	%
Svartedalen	502	580124	120194	1.95	130	0.5	18.0	39.0	64.2	7.2	9.9
Berg	504	570399	124754	0.93	120	1.8	4.0	30.4	0.5	64.1	31.4
Tiveden	506	584202	143856	1.04	175	0.6	4.0	24.0	84.6	3.7	7.7
Reivo	511	654722	190550	10.9	520	2.7	0.7	30.3	0.3	84.0	13.9
Stormyran	519	621562	161628	3.21	430	2.1	3.7	55.0	0.4	64.0	31.9
Söderåsen	522	560261	131383	1.67	80	6.2	0.0	20.4	0.1	99.6	0.3

Table 4. Calibration results expressed in terms of N2-norm after looking of parameters Amin. Bmax. Bmin. Rdiv. Bncrit and Bcrit.

Station	All parameters	Amin=100	Amin=100 Bmax=200	Amin=100 Bmax=200 Rdiv=0.4	Amin=100 Bmax=200 Rdiv=0.4 Bncrit=Bmin	Amin=100 Bmax=200 Rdiv=0.4 Bncrit=Bmin Bmin=150
502	1.026	1.026	1.026	1.032	-	-
504	0.391	0.391	0.391	0.391	0.399	0.438
506	0.299	0.299	0.299	0.299	-	-
511	0.278	0.278	0.278	0.278	-	-
519	0.503	0.503	0.503	0.514	-	-
522	0.175	0.175	0.175	0.175	0.181	-

The results are presented in Table 4. It is demonstrated that model performance presented as N^2-norm does not significantly alter if the locked parameters are introduced.

Unchanged results show up when parameters A_{min} and B_{max} are set at locked values (100 respectively 200 mm) in all examined stations. If also R_{div} is locked (R_{div} = 0.4) only a small increase in the N^2 -norm occurs in two of the areas. If further parameters are locked (columns 6 and 7 of Table 3) the N^2-norm is increased unacceptably. This was found for stations 504 and 522. On the basis of these results, there is strong evidence that the model in question was overparameterised. The following changes are therefore proposed:

1. The parameter a in equations 13 and 14 is set to 1.0.
2. In equation 12, B_{min} has been replaced by a new parameter By.
3. A_{min} =100 (equation 8,11)
4. B_{max}=200 (equation 6)
5. R_{div}=0,4 (equation 6)

By means of these modifications the number of parameters needing calibration is reduced from 14 to 11. If the parameters of the snow routine are excluded the, free parameters are reduced from 9 to 6.

Conclusions

With regard to model structure, this model does not differ very much when compared with other models with similar purposes. Our confidence in this model as a tool for operative work is similar to that for comparable models.

The model structure has moved into a less complex direction by means of parameter reduction. This is important with regard to model behaviour during the ”independent period”. Introduction of DUD-optimisation leads to a more objective way of calibration. At the same time, this technique eases and speeds up modelling work.

References

Bergström, S. (1992) The HBV model - its structure and applications, SMHI RH No. 4, Norrköping.

Grip, H. (1982). Water chemistry and runoff in forest streams at Kloten. UNGI Report 58, Uppsala University, Sweden.

Ralston, M. , Jennrich R. , 1978. Dud, A Derivative-Free Algorithm for Nonlinear Least Squares. TECHNOMETRICS, vol. 20, no 1.