Taper and Volume Equations for Poplar Trees Growing on Farmland in Sweden

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Abstract

Effective management of poplar plantations for high yield production would be facilitated by the availability of improved equations for predicting the taper and volume of poplar stems. Therefore, this thesis is based upon a polynomial stem taper equation and two volume equations constructed for individual poplar trees growing on farmland. Data for fitting and evaluate the taper and volume equations were collected from 51 trees growing in 27 stands in central and southern Sweden (lat. 55-60° N). The mean age of the stands was 22 years, mean density 970 stems ha^{-1}, and mean diameter at breast height 24 cm. Validation data were collected from 17 trees growing in ten stands, not used for fitting the equations.

The outputs of the polynomial taper equation were compared with five published equations. The statistical evaluation indicated that the variable exponent taper equation presented by Kozak (1988) performed best and can be recommended. Because this equation´s complex construction, alternative recommendations were made. The constructed taper equation and the segmented equation presented by Max & Burkhart (1976) were second and third ranked.

The first constructed stem volume equation is a function of diameter at breast height (DBH) and total height (H) as independent variables. In addition to these variables the other is also a function of an upper diameter. The outputs of these two equations were analyzed and compared to those of five published equations developed for, or applied on, poplar or aspen species. Of the stem volume equations examined the best performance was provided by the constructed equation with an additional upper diameter and recommended when precise and accurate volume estimations are required. However, because of difficulties to measure diameters high above ground, this equation is less practical in traditional surveys. For this purpose, the first constructed equation or the equation developed by Fowler & Hussain 1987 can be recommended.

The taper and volume equations recommended in the study are likely to be useful in optimizing the efficiency and profitability of poplar plantation management.

Keywords: poplar, taper equation, segmented taper equation, variable exponent taper equation, stem volume equation,

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Dedication

To Joel and Jonathan

Contents

List of Publications

This thesis is based on the work contained in the following papers, referred to by Roman numerals in the text:

I Hjelm B. (2011). Stem Taper Equations for Poplars Growing on Farmland in Sweden. Submitted to *Journal of Forestry Research*

II Hjelm B., Johansson T. (2011). Stem Volume Equations for Poplars Growing on Farmland in Sweden. Submitted to *Scandinavian Journal of Forestry Research*

The contribution of Birger Hjelm to the papers included in this thesis was as follows:

I Hjelm: 80 % input to the data collection, analysis and evaluation, manuscript writing. Supervisor: 20 % input to discussion and conclusion

II Hjelm: 80 % input to the data collection, analysis and evaluation, manuscript writing. Johansson: 20 % input to discussion and conclusion

1 Introduction

1.1Poplars characteristics and distribution

Poplar belongs to the genus *Populus* of the Salicaceae family, which also includes the genus *Salix*. European aspen (*Populus tremula* L.) is the only domestic species of *Populus* in Sweden. All poplar species and clones are introduced in Sweden. The natural distribution of *Populus* extends from the tropics to the latitudinal and altitudinal limits of tree growth in the Northern hemisphere (Dickman & Kuzovkina, 2008).

Members of the genus *Populus* are deciduous or (rarely) semi-evergreen and divided into six sections: *Abaso* (Mexican poplar), *Aigeiros* (Cottonwoods and black poplar), *Leucoides* (swamp poplars), *Populus* (white poplars and aspens), *Tacamahaca* (balsam poplars), and *Turanga* (arid and tropical poplars). Poplars usually have rapid growth rates, which enable some to grow to large sizes, notably some cottonwood species of North America (*P. deltoides,* Batra ex. Marsh and *P. trichocarpa,* Torrey & Grey) and some Asian balsam poplars (*P. maximowiczii,* Henry and *P. suaveolens,* Fisch). They can become enormous trees, with diameters at breast height (DBH) of 3 m and total heights (H) exceeding 40 m. Members of the genus have proved to be amenable and attractive targets for genetic mapping and cloning of desired characteristics, for example growth rate or pathogen resistance. *Populus* species are dioecious (i.e. individual trees are either male or female), and can be regenerated by coppicing and from cuttings.

Various species of the genus have been widely planted around the globe, both within and outside its natural distribution (including various sites in the southern hemisphere) and have a wide range of uses, inter alia for:

- saw timber — the wood is soft, with a low density and light colour. It is used for furniture frames and other indoor uses including framework and roof trusses.
- veneer — traditionally used for making fruit crates and boxes, since it does not taint the fruit, and a number of other products, ranging from plywood to matches.
- reconstituted wood — the development of oriented strand board (OSB) has opened new markets for poplar wood, especially in North America.
- pulp wood
- fodder
- protection of stream banks
- shelterbelts and windbreaks
- fuelwood
- phytoremediation
- ornamental and landscape uses, including screening

Until recently, planting of poplars in Sweden was confined to small plantations, established between 1980 and 1990 on set-aside farmland to assess their productivity. Plantations older than ten years in Sweden have less than 500 ha areas and are stocked (inter alia) with the well-known clone OP 42 (*P. maximowiczii* x *P. trichocarpa*). However, rising demand for biofuel has increased interest in poplar, among other species that are suitable for short rotations, in Sweden. Consequently, poplars have been planted recently, commonly at ca. 120 ha sites on forest land where previous stands were damaged by wind during the storm "Gudrun" in 2005 (Rytter et al., 2011).

The advantages of growing poplar as an exotic species in short rotation forestry have been discussed in several recent publications from a production perspective (Jonsson, 2008; Christersson, 2010). Notable findings include the following. In an early experiment Persson (1973) found the productivity of 42-year-old "Robusta" (*P. deltoides* x *P. nigra*,L.) poplar hybrids was quite high (ca. 12 m^3 ha^{-1} $year^{-1}$). However, Johansson (2010) recently showed that the productivity of hybrid poplars on former farmland is substantially higher, averaging ca. 19 m^3 ha^{-1} $year^{-1}$ (excluding branches, twigs and leafs). This is also substantially higher than the productivity of hybrid aspen plantations (ca. 13 m^3 ha^{-1} $year^{-1}$), and much higher than productivities of plantations of various domestic species, e.g. birch (*Betula* spp.), alder (*Alnus* spp.), Norway spruce (*Picea abies* (L.) Karst.), wild cherry (*Prunus avium* L.) and hybrid larch (*Larix decidua* Mill. x *Larix kaempferi* (Lamb.) Carr.), which have reported mean annual increments (MAI) ranging from 3 to 7 m^3 ha^{-1} $year^{-1}$ (Johansson 2010). Several authors have also considered ecological and environmental aspects of poplar plantations (Karacic, 2005; Christersson & Verwijst, 2006), but these will not be considered further here since the focus is on the stem volume and taper of the trees.

1.2 Taper and stem volume equations

1.2.1 Taper equations

The terms 'form' and 'taper' are often used synonymously, but as noted by (Gray, 1956) 'form' strictly describes the shape or structure of a tree's stem, e.g. a cone or paraboloid, whereas 'taper' is defined as 'the rate of narrowing in diameter in relation to increases in height of a given shape or form'. The expressions 'form factor' (for a tree, the ratio of its volume to the volume of a cylinder, usually of equal diameter to the breast height diameter of the tree) and 'slenderness' (DBH/H) provide general indication of a tree's form or shape, but do not provide any details about how the diameter narrows as the stem height increases. This detail can be provided using a taper equation. The major advantage of taper equations is their ability to predict the diameter of a stem at a given height or, following re-arrangement, to predict the height of a stem with a given diameter at a given height.

Numerous taper equations have been developed, and evaluated, for various tree species. They are generally based on trees' diameter at breast height (DBH), total height and the height above ground (to the point where the diameter will be predicted) as independent variables, providing estimates of: stem diameter at any given stem height, total stem volume, merchantable volume and merchantable height to any top diameter and from any stump height, and the volume of individual tree-logs of any length at any height from the ground (Kozak, 2004). Analysis of relationships between these above variables is important for two reasons (cf. Newnham 1988). Firstly, no single theory has been able to explain satisfactorily all the variability in tree stem shape. Secondly, taper equations provide flexible tools for estimating total and merchantable tree volumes, which can be used to adjust management objectives as market demands and product specifications change. From a practical perspective, the latter reason is the most important (Muhairwe, 1999).

Stem taper is a complex trait (Assmann, 1970) that varies substantially depending on genetic factors (within- and among-species), environmental factors (*inter alia* soil type, hydrology, altitude and climate), forest management practices (Steven & Benee, 1988; Karlsson, 2005) and interactions between all of these factors. The range of factors involved (natural and anthropogenic) complicates the development of a universal model for tree stem taper.

As noted by Sterba (1980), many forms and types of stem taper equation have been published and evaluated. Models have been constructed to describe the taper of diverse species in various regions globally, based on equations of the following three types (Diéguez-Aranda et al., 2006; Sakici et al. 2008):
1. Simple taper equations (Demaerschalk, 1972; Demaerschalk, 1973; Ormerod, 1973; Sharma & Odervald, 2001)
2. Segmented taper equations (Max & Burkhart, 1976; Clark et al. 1991)
3. Variable exponent taper equations (Kozak, 1988; Newnham, 1992)

Until the mid-1970s all of the published equations were of the simple type (Figueiredo-Filho & Schaaf, 1999), and did not account for variations in the form of different tree sections (e.g. root/base, main stem & top) and hence did not adequately describe the taper of the stem either close to the base or at the top. Therefore, alternative procedures were examined to solve these problems. Max and Burkhart (1976) developed the first segmented equation, for which the tree stem was divided into three sections (neiloid, paraboloid and cone-shaped), represented by separate sub-functions.

Variable exponent taper equations utilise an exponent that changes along the stem, reflecting differences between the neiloid, paraboloid and cone-shaped sections (Kozak, 1988; Newnham, 1992). Assumptions for these approaches are that the form of a tree’s stem varies continuously along its height (Lee et al., 2003). Variable exponent taper equations have been found to be superior to segmented and simple models for estimating stem diameters and volumes (Kozak, 1988; Newnham, 1992; Muhairwe, 1999). However, variable exponent taper equations cannot be integrated analytically to calculate total stem or log volumes (Diéguez-Arunda et al., 2006), which must be estimated instead from calculated diameters and lengths by numerical integration (Kozak, 1988).

A number of variants of both segmented and variable exponent taper models have been developed and applied, and the latter have been shown to exhibit less bias and have better predictive abilities than other models in several studies (Sakici et al., 2008; Li et al., 2010). However, despite the advantages of these two model types they have major drawbacks: statistical complexity and difficulties in estimating parameters and re-arrangement to calculate heights for given diameters. The variable exponent taper equations provide the lowest degree of local bias and the most precise predictions (Kozak, 1988; Muirhairwe, 1999), but there is a need for simple equations in practical forest management. Simple polynomial taper equations have been frequently used in forest inventories in southern Brazil, notably the simple equation developed by Kozak et al. (1969), which has been assessed in several Brazilian studies (Figueiredo-Filho et al.,, 1996).

1.2.1 Stem volume equations

Several volume equations have been developed for various species. The volume of an individual tree depends on its height, diameter and stem form. The height and diameter are easy to measure and estimate, but the stem form is a complex trait that is not straightforward to estimate (Assmann, 1970). Equations for the stem volume and commercial volume for specified commercial diameters (e.g. one equation for each specification) are the most commonly used in Scandinavian forest management. Compared to the compatible taper and volume equations they are “stiff”, equations with only one possible prediction value per tree and usually these equations are based on two independent variables: H and DBH (second entry equations). There are also equations based on the single independent variable DBH (Case & Hall, 2008; Gautam & Thapa, 2009) and equations with a third independent variable (third entry equations) or more, such as diameter at a specified upper height (Burk et al, 1989; Brandel, 1990), height at crown base, bark thickness, and/or site indicator variables such as, altitude, latitude, soil type and vegetation type. Some of the most important and well-known volume equations and stem volume models applied in Sweden are briefly described in the following text.

A study published by Jonson (1928) presented a model dealing with stem curves and form classes, and a new method was introduced in which the stem is divided into two sections, the lower (2 m long) section is directly measured and the taper of the upper section is estimated from an upper diameter and a form class assigned to the section.

Two major contributions were made by Näslund (1940; 1947), in which he presented two kinds of stem volume equations for Scots pine, Norway spruce and birch trees in northern, southern and all of Sweden: “simpler” equations using the independent variables DBH and H; and “advanced” equations using the additional variables crown height and bark thickness at breast height. The equations were constructed using data from >4000 sampled trees, and have been frequently used.

Volume equations and tables for silver fir (*Abies alba* Mill.) were constructed by Eggli (1960), and Carbonnier (1954) presented volume equations for three larch species.

Volume equations for ash *(Fraxinus excelsior* L.*)*,European aspen *(Populus tremula* L.*)*, common alder *(Alnus glutinosa* (L.) Gaertn.) and lodgepole pine *(Pinus contorta*, Douglas) were developed by Eriksson (1973). While Näslund's equations were additive polynomials Eriksson also developed multiplicative equations. Beside the variables DBH and H, Eriksson used crown height above ground and crown length in percent of tree height in some of his equations.

Hagberg and Matern (1975) developed volume equations for oak *(Quercus robur* L.*)* and beech (*Fagus sylvatica* L.), using the approach applied to construct Näslund's equations (1940; 1947).

A major study of volume equations for Scots pine, Norway spruce and birch in Sweden was published by Brandel (1990), in which a multiplicative base equation with DBH and H as independent variables was presented. Further variables (upper height diameter, crown height above ground and bark thickness at breast height) were then added to the base equation, either solely or in combination. Brandel also tested the potential for improving the volume estimations using the indicator variables altitude, latitude and forest type.

Volume equations of multiplicative, additive and logarithmic variable exponent types for common alder and grey alder (*Alnus incana* (L.) Moench) in Sweden have also been developed and assessed recently (Johansson, 2005).

2 Objectives

No taper or volume equations for individual poplar trees growing in Swedish conditions have been developed, and the applicability of published equations to poplar stands in Sweden has not been previously assessed. Therefore, the main aims of this thesis were to develop and evaluate new equations, and to evaluate the suitability of previously published equations, for estimating the taper and volume of poplar trees growing on farmland in Sweden. Based on the results of the evaluation, further aims were: to select and recommend taper and volume equations that can be conveniently applied to poplar trees in the field; and to select (a) volume equation(s) that provide(s) high levels of precision and accuracy, and can be recommended for use in evaluations of research trials and/or felled trees in routine cutting and management operations.

The main specific objectives were:

1. To develop and evaluate a simple polynomial equation for estimating the taper of poplars growing on farmland and to evaluate the performance of five published taper equations.

2. To develop and evaluate two volume equations (two and three independent variables respectively) for poplars growing on farmland and to evaluate the performance of five published volume equations.

3 Material and methods

This thesis is based on the work presented in Papers I and II. Paper I focuses on taper equations, describing the development and evaluation of a simple polynomial taper equation and evaluation of five previously published taper equations. Paper II describes the development and evaluation of two stem volume equations and evaluation of five previously published stem volume equations. The developed equations include one equation with DBH and H as independent variables (second entry equation) and another with these variables and an upper height diameter as an additional independent variable (third entry equation).

3.1 Data

The sites were located on former farmland, and most of the stands have been planted between 1988 and 1992. The stands were established as research-sites, for commercial use with focus on production, or as demonstration sites. The sites cover a variety of site- and stand characteristics, Table 1. The water table was 0.3 – 1 m deep, and apart from four fitting stands and one validation stand with till soils all other soils were clay sediments with textures ranging from light to medium clay. Data for constructing the taper and volume equations were collected from 51 poplars growing at 27 stands in central and southern Sweden between latitudes 55-60° N. The ages of the stands at the sites ranged between 14 and 43 years. The management of the stands varied; some had not been thinned at all and thinning regimes ranging from moderate to heavy had been applied in the others. The number of stems varied from 287 to 3493 per hectare, which cover most of existing stand densities. In some stands the initial spacing and number of plants was known, but for most of the stands, these figures are unknown.

The models' future prediction quality was tested on independent data for a validating process (Kozak & Kozak, 2003). This data were collected from 17 trees growing at ten stands, located within the same geographical area as the fitting data (Figure 1, Table 1). The mean age of the validation stands was 21 years (16-41) and the mean number of stems ha^{-1}, 1038 (198-2900).

The mean number of stems per hectare was calculated based on the number of stems counted in either whole stands or plots. The area of the studied plantations varied between 0.5 and 3 ha. In > 1 ha stands, a 1 ha plot in the central part of the stand was chosen, at least 5 m from the edges (to avoid edge effects caused by factors such as wind, open areas, ditches and shading by adjacent stands). The DBH was measured by cross callipering and the measurements recorded were rounded to the nearest mm. The arithmetical mean diameter was calculated for each stand.

At each site, one to five sample trees were subjectively selected for measurements (due to restrictions set by the forest owner regarding future management of the stand) that were: healthy, undamaged, with fairly straight, single stems, and neither border trees nor suppressed trees. In total, 51 trees were sampled to develop the stem taper and volume equations and 17 trees were sampled for validation of the equations. Generally, the DBH of the selected trees was within the third DBH distribution quartile of respective stand. For each tree the total height and crown height (height above ground to the base of the green crown) were measured and recorded to nearest 0.1m. The total age was defined by counting annual rings from a stem disc at stump height (0.2 m).

DBH on bark and the diameter at 1 m intervals along the stem were measured by cross-callipering. According to routine methods applied in yield studies at the Department of Energy and Technology, SLU, Uppsala, diameters were also measured with cross-calipers at six relative heights of the trees (1, 10, 30, 50, 70, and 90 % of total height). The diameter measurements recorded were rounded to the nearest mm. The recordings of the diameter at the relative heights were used for the evaluations of stem taper presented in Paper I.

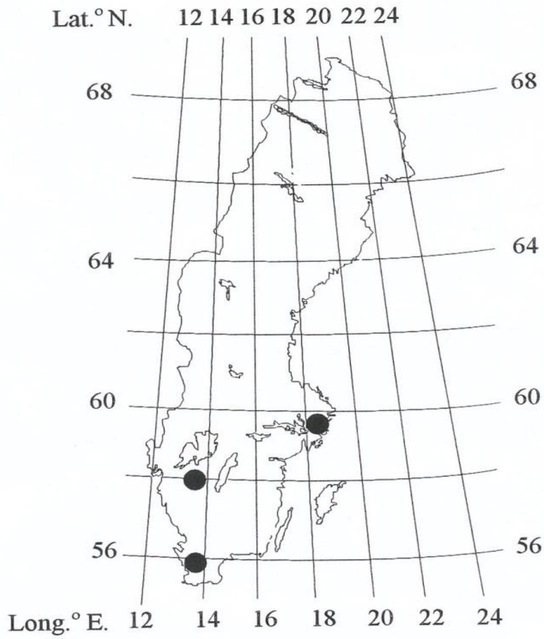

Figure 1. Map of Sweden showing the locations of the three sampling areas in this study

Table 1. *Main characteristic on hybrid poplar stands for the fitting and validation data*

Plot no	Age, yrs	Dom. Height, m	DBH, cm Mean±SD	No. of stems ha^{-1}	Basal area m^2 ha^{-1}	Soil type
Fitting data						
1	18	24.0	24.8±4.9	875	42.3	Light clay
2	41	27.0	33.4±14.5	973	87.1	Light clay
3	43	24.7	26.8±14.8	1906	107.5	Light clay
4	17	20.2	17.6±6.5	550	11.1	Medium clay
5	16	19.2	18.7±3.9	1111	30.5	Medium clay
6	21	29.2	33.0±7.6	361	30.9	Light clay
7	20	24.5	27.7±4.7	549	33.1	Light clay
8	23	22.8	19.6±6.5	632	19.1	Light clay
9	34	25.7	30.6±7.5	840	61.8	Light clay
10	15	24.0	23.4±4.4	287	12.3	Light clay
11	16	20.2	12.8±5.2	3279	42.2	Light clay
12	19	28.5	24.6±3.9	1250	59.4	Medium clay
13	19	18.5	24.0±4.1	295	13.3	Medium clay
14	34	27.2	29.1±11.9	398	26.5	Medium clay
15	24	25.9	29.3±3.9	457	30.8	Light clay
16	19	14.5	19.3±3.3	1111	32.5	Medium clay
17	20	14.6	18.2±3.3	1111	28.9	Medium clay
18	20	20.1	17.4±4.0	800	19.0	Medium clay
19	23	22.0	25.6±5.8	1005	51.7	Medium clay
20	20	22.5	23.6±4.0	1015	44.4	Medium clay
21	21	24.6	18.6±7.1	1200	32.6	Light clay
22	19	21.5	23.2±4.7	650	27.5	Light clay till
23	14	17.8	12.1±5.4	3493	40.2	Light clay
24	17	21.2	22.6±5.8	378	11.2	Sandy-Silty tills
25	21	29.1	28.3±3.7	506	31.8	Light clay tills
26	19	27.6	28.0±9.4	440	27.1	Light clay tills
27	20	29.5	25.1±3.4	707	35.0	Medium clay
Mean	22	23.2	23.8	970	37.2	
±SD	±7.3	±4.2	±5.2	±764	±20.8	
Range	14-43	14.5-29.5	12.1-33.4	287-3493	11.2–107.5	
Validation data						
1	17	24.8	29.8±4.4	520	32.4	Light clay
2	21	28.0	31.0±2.2	800	40.4	Light clay
3	20	27.0	27.2±3.6	800	32.4	Light clay
4	41	22.0	27.9±10.3	1281	77.9	Medium clay
5	18	26.0	27.3±4.6	909	45.7	Medium clay
6	16	21.2	18.6±5.4	966	25.5	Light clay
7	20	20.5	20.4±4.1	1461	47.8	Medium clay
8	20	24.5	40.4±5.7	198	23.2	Sandy-Silty tills
9	19	23.0	17.8±8.1	2900	79.7	Light clay
10	20	27.0	34.9±6.4	549	52.1	Light clay
Mean	21	24.4	27.5	1038	45.7	
±SD	±7.0	±2.5	±7.3	±7.6	±21.8	
Range	16-41	20.5-28.0	17.8-40.4	198-2900	23.2–79.7	

The two subsets of sample tree data are summarized in Table 2.

Table 2. *Summary statistics of the data set for equation construction and validation*

	No. of sites	No. of trees	No. of data points*	DBH (cm)		Height (m)	
				Mean±SD	Range	Mean±SD	Range
Parameter estimate	27	51	1285	27.6±9.1	12.4-49.5	22.7±3.4	15.0-30.0
Validate equation	10	17	431	30.2±7.4	21.4-46.5	23.5±3.3	19.4-27.6

*(used for taper equations, paper I)

The relative diameter and height points used for the fitting and validation data sets applied in Paper I are shown in Figure 2, while paired DBH and volume data points applied in Paper II are shown in Figure 3.

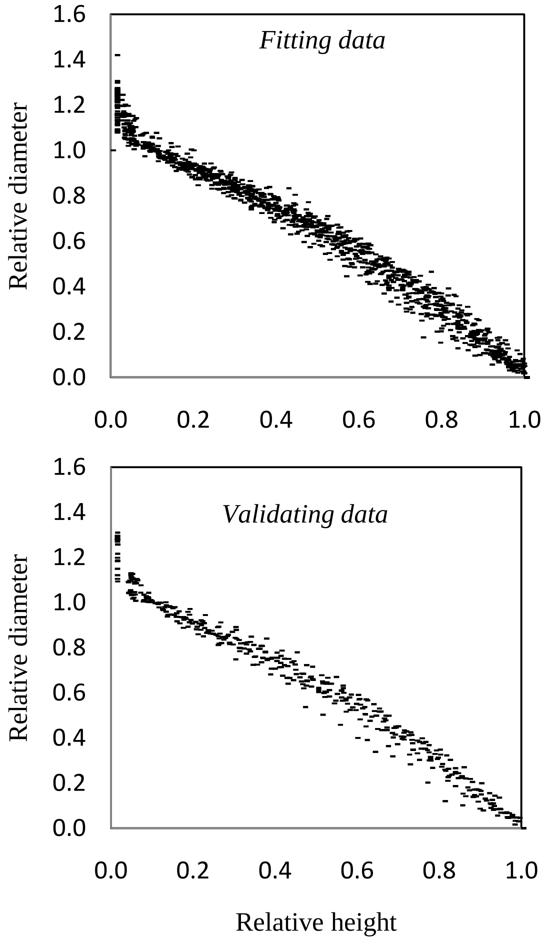

Figure 2. Paired data points of relative heights and relative diameters for the fitting and validating data sets

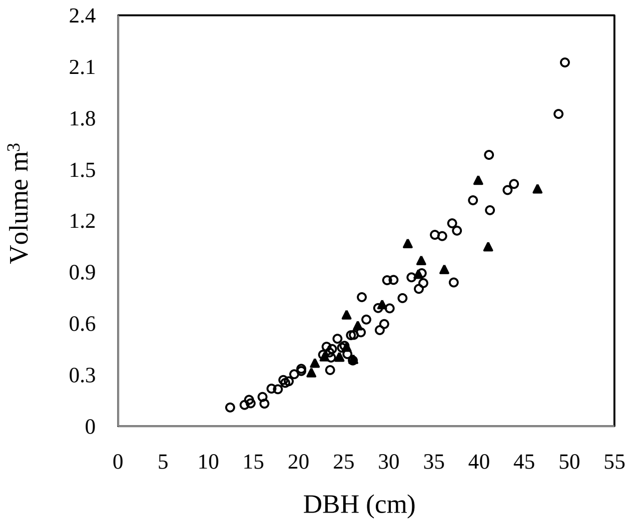

Figure 3. Paired data points of diameter at breast height (DBH) and observed volume (paper II) for the sampled trees for fitting ○ and validation data ▲.

3.2 The models

Paper I

Taper equation (I:1) was constructed and compared with five published stem taper equations (I:2-6) of which three were simple, one segmented and one variable exponent equation. The simple taper equation (I:2) was developed by Kozak et al. (1969) fitted for 19 species growing in British Columbia, Canada. This equation is not specific for poplars and has been used extensively for other species. Taper equation (I:3) published by Ormerod (1973) was developed for geometric simulation of tree component interaction during thinning extraction. Computational economy required development of a simple function. Taper equation (I:4), developed by Benbrahim & Gavaland (2003), was fitted for short rotation poplar plantations in France. The segmented taper equation (I:5) was developed by Max & Burkhart (1976) using sample tree data from plantations and natural stands of loblolly pine (*Pinus taeda* L.) in Maryland, North Carolina and Virginia, USA. The variable exponent taper equation (I:6) was developed and published by Kozak (1988) using data for several species, including cottonwood (*Populus trichocarpa*), Table 3.

Paper II

Two volume equations were constructed, one with DBH and H as independent variables (second entry equation), equation (II:1), and one with an additional variable, diameter at an upper height (third entry equation), equation (II:7). A number of published volume equations were also initially tested and five equations, (II:2-6) were chosen for further analysis and evaluation. The published second entry equations have been developed for poplar or aspen, solely or as one of a selection of species, Table 3.

Table 3. *Stem Taper- and Stem volume equations used in paper I and II*

Model	Expression	
	Stem taper equations considered in Paper I:	
Simple equations		
Hjelm (constructed)	$d=(b_1q^2+b_2q+b_3((H-h)/h)+b_4)x(D/(1-k/H))^{b5}$	(I:1)
Kozak (1969)	$(d/D)^2= b_1 + b_2\, q + b_3(h^2/H^2)$	(I:2)
Ormerod (1973)	$d=D((H-h)/(H-k))^{b1}$	(I:3)
Benbrahim &Gavaland (2003)	$d= Db - Db((ln\,(1\text{-}h/ b_1H)/\text{-}\, b_2))^{1/b3}$	(I:4)
Segmented equation:		
Max & Burkhart (1976)	$d^2=D^2(b_1(q-1)+b_2(q^2-1)+b_3(a_1-q)^2I_1+b_4(a_2-q)^2I_2)$	(I:5)
	I_1=1, if $q< a_1$; 0 otherwise	
	I_2=1, if $q< a_2$; 0 otherwise	
Variable exponent taper equation:		
Kozak (1988)	$d=b_1D^{b2}b_3^{D}((1-q^{0.5})/(1\text{-}p^{0.5}))^A$	(I:6)
	$A=(b_4q^2+b_5\ln(q+0.001)+b_6q^{0.5}+b_7e^q+b_8(D/H))$	
	Stem volume equations considered in Paper II:	
Second entry equations with independent variables D and H:		
Hjelm I (constructed)	$V = b_1\, D^{b2} + b_3\, H^{\,2} + b_4DH^2$	(II:1)
Eriksson II (1973)	$V=b_1D^2+b_2D^2H\text{-}\, b_3D^2H^2\, \text{-}b_4DH +b_5DH^2$	(II:2)
Anon. (1976)	$V= e^{\,(b1\ +b2lnD\ +b3lnH)}$	(II:3)
Fowler & Hussain (1987)	$V= b_1\, +b_2D^{\,b3}H^{b4}$	(II:4)
Opdahl (1992)	$V= b_1 + b_2D - b_3D^{\,2} + b_4D^2H$	(II:5)
Wang (2007)	$V= b_1 - b_2D^2 - b_3H^2 + b_4DH$	(II:6)
Third entry equation with the additional upper diameter variable:		
Hjelm II (constructed)	$V= b_1\, H^2 + b_2DH^2 + b_3(D_5\text{-}1)^{b4} + b_5$	(II:7)

Where:
D = diameter at breast height, cm
Db = diameter at stump height, cm
d = stem diameter, cm, at height h

H = total height, m
h = height, m, from ground to top diameter (d)
$a_{i,}$, b_i = regression coefficients estimated from sample data
$q = h/H$, relative height
HI = height, m, of the inflection point from ground
$p = HI/H$
k = 1.3 m (breast height)
V = total stem volume, in dm^3, over bark from the stump to the tree tip
D_5 = diameter at 5 m above ground

3.3 Statistical procedures

All regression analysis were carried out using the SAS statistical package (SAS, 2006). The NLIN procedure was used in both Papers I and II for fitting and developing the constructed models, estimating parameters and evaluating the previously published models considered. The following statistics were used to assess the goodness of fit for the models addressed in Papers I and II:

R^2 =	$1-\sum(d_i-\hat{d}_i)^2/(d_i-\bar{d}_i)^2$	Paper I
R^2 =	$1-\sum(v_i-\hat{v}_i)^2/(v_i-\bar{v}_i)^2$	Paper II
B =	$\sum(\mathrm{Diff})/\mathrm{n}$	Papers I & II
AB =	$\sum\lvert\mathrm{Diff}\rvert/\mathrm{n}$	Papers I & II
AB % =	$100 \times \sum\lvert\mathrm{Diff}\rvert/\sum_{i=1}^{n} d_i$	Paper I
AB % =	$100 \times \sum\lvert\mathrm{Diff}\rvert/\sum_{i=1}^{n} v_i$	Paper II
RMSE =	$\sqrt{Diff_i^2/(n-p)}$	Papers I & II
SSRR =	$\sum(Diff/d_i)^2$	Paper I

Where:
R^2 = Coefficient of determination
B = Bias
AB =Absolute Bias
AB % = Relative Absolute Bias
RMSE = Root Mean Square Error
SSRR = Sum of Squared Relative Residuals
d= stem diameter at the selected height along the stem
v= stem volume
Diff = difference between observed and predicted values

The sum of squared relative residuals (SSRR) are an important statistic in analyses of differences between outcomes of taper equations (Figueiredo-Filho et al., 1996), and according to Parresol et al. (1987) absolute bias (AB) and SSRR provide clear indications of the relative ability to model datasets. Relative absolute bias (AB %) was used by Li and Weiskittel (2010) complementary to AB when comparing and ranking the performance of different taper equations.

To test the validity of the equations in paper I and II, a paired student T-tests were applied (using the TTEST procedure in the SAS package) of the significance of differences between measured values and the values they predicted.

Multicollinearity can pose problems when constructing taper and volume equations, especially models including complex polynomial and cross-product terms. When severe multicollinearity is present in a dataset, the problems that may occur are that:
(1) minor variations in the data may substantially affect parameter estimates,
(2) the regression coefficients may have high standard errors
(3) the regression coefficients may have the wrong sign.

The level of multicollinearity of the equations tested in Papers I and II was determined by calculating condition indices, CI (the square root of the largest eigenvalue divided by the smallest eigenvalue of the correlation ratios) using the PROC REG procedure. CI values >30 are indicative of serious multicollinearity (Kozak, 1997).

Ordinary least square method relies on the assumption that residual errors are independent and identically distributed. However, in contrary to development of volume equations with one estimate per tree, stem taper models are developed by hierarchical collected diameter data at several height points on the same individual tree. Then the data between the different points on the tree are closely dependent on each other. This autocorrelation violates on the above assumption of independence.

According to Kozak (1997) autocorrelated error terms in a model can result in following consequences:
(1) the estimators of the regression coefficients are unbiased and consistent but no longer have the minimum variance property
(2) the calculated mean squared error (MSE) may underestimate the real variance of the error terms, while the standard errors of the regression coefficients may underestimate the true standard deviation
(3) statistical tests using t or F distributions and confidence intervals are no longer reliable

AIC (Akaike Information Criteria) and BIC (Bayesian Information Criteria) are common goodness of fit criteria when comparing model with dataset affected by autocorrelation. According to Li and Weiskittel (2010) these criteria are not appropriate for selecting and comparing taper equations when the response variables between the equations are not the same. The response variable for equation (1:2) and (1:5) differ from the other equations. In order to determine how well the models fit the data, instead an analyze of residual plots (figure 3 in paper I) and the above ”fit” statistics for paper I were used for the models and for data on different stem levels.

4 Results

4.1 Paper I

4.1.1 Fitting data

All six taper equations considered in Paper I yielded high correlation coefficients between predicted and measured diameters (R^2 = 0.99). The RMSE and AB values of the equations were lowest for equation (I:6), 0.89 and 0.62 respectively, indicating that it has good ability to predict stem taper, while equations (I:2) and (I:4) had the highest RMSE values of 1.48 for both and AB values of 1.13 and 1.16, respectively.
The absolute bias expressed in relative values (AB %) were between 5.6 and 7.1 % for equations (I: 1-5) and notable lower, 3.8 %, for equation (I:6).
Bias (B) was close to zero for equation (I:6) while the other equations had positive B values, ranging from 0.25 to 0.33, indicating that they slightly underestimate them. The detailed parameter estimates and results of evaluation statistics are summarized in Table 4 in Paper I.

The AB values for the diameters at the relative heights in the fitting data show that equation (I:6) had the lowest absolute bias for diameters at relative heights ranging from 10 to 90 %. There were minor differences in SSRR values between the equations at 10 to 50 % relative heights, but equation I:6 had notably lower SSRR values for the upper part of the bole (70 and 90 % relative height). Equations (I:4), (I:5) and (I:6) performed substantially better in predicting diameters at 1% relative height than the other equations. All equations yielded high SSRR values at 90% relative height, but were lowest (3.49) for equation (I:6), and highest (7.43) for equation (I:3) (Table 5 in Paper I).

Equations (I:1) – (I:4) showed low levels of multicollinearity, with CI values of 10.2, 7.6, 2.8 and 2.5, respectively, while the values for equations (I:5) and (I:6) were 47 and 585, respectively, indicative of severe multicollinearity (Kozak, 1997).

4.1.2 Validation of the equations

The statistics describing the fit of the models to diameters at the relative heights in the validation dataset applied in Paper I show similar trends to the corresponding statistics in the data used to construct the stem taper equations in Paper I. Equation (I:6) had lower AB and SSRR values than the other equations, and the differences were most pronounced for the upper part of the stem. However, equations (I:5) and (I:6) had notably higher AB values for diameter at 1% relative height (1.49 and 1.73, respectively) than their values for the fitting data (Table 5 in Paper I). Equations (I:1) and (I:4) did not met the zero criterion for predicted diameter at the top of the tree (h=H), deviating by 0.5 and 0.3 cm, respectively, for both the fitting and validation data.

The condition index, for detecting multicollinearity, showed the same trend for the validation data as for the fitting data, with CI <30 equations (I:1) – (I:4), 67 for equation (I:5) and >500 for equation (I:6).

The residual plots for the validation data show a similar pattern to those of the data used to construct the stem taper equations (Figure 3, Paper I). Residuals of equation (I:6) have a smaller distribution than those of the other equations, indicating that it has the best ability to predict stem taper. In the plot of residuals versus relative heights the residuals are well balanced and distributed in an even manner for equation (6). The other equations are unbalanced to greater degrees, equations (I:2) to (I:4) being slightly more unbalanced than equations (I:1) and (I:5), as shown in Figure 3 (Paper I).

A paired Student's t-test applied on the measured and predicted diameters showed that equations (1) to (5) have p-values > 0.05 indicating that the difference between observed and predicted diameters is not significant. The results show only minor differences in the t-test statistics for equations (1) to (5). Equation (6) has lower values of SE and a smaller range (min to max) than the other equations indicating high precision, yet less god accuracy resulting in significant difference p <0.05 between predicted and observed diameters, (Table 6 in paper I).

4.2 Paper II

4.2.1 Fitting data

All volume equations addressed in Paper II had high correlation coefficients, $R^2 > 0.989$. The RMSE and AB values of the evaluated equations were lowest for the third entry equation (II:7). The RMSE value for this equation was 26.14 and its AB value was 15.48 indicating good ability to predict stem volume, while the second entry equation (II:6) had the highest RMSE and AB values; 48.75 and 35.59, respectively. The values of relative absolute bias (AB%) shows that equations (II:1-5) have minor differences within a range of 3.7 to 4.1 %. Equation (II: 6) had the largest relative absolute bias of 5.1 % and equation (II: 7) the lowest absolute bias of 2.3 percent. Bias (B) was zero or close to zero (<0.02) for equations (II:4) and (II:6), indicating that they generally neither under- nor over-estimate the volume of poplar trees. Equation (II:3) had B value of 2.23, indicating underestimation while the other equations have a bias ± 1 indicating slight over- or under estimation. The parameter estimates and evaluation statistics for the studied equations are summarized in Table 4 in Paper II.

The crown height variable and its potential to improve volume predictions were tested by step-wise regression, but it was found to make very little or no contribution.

Equation (II:2) showed CI values >100, indicative of severe multicollinearity (Kozak, 1997).

4.2.2 Validation of the equations

Bias (B) and absolute bias (AB) values were higher for the validation data set than for the fitting data. The simple bias (B) values for the validation data ranged from -8.36 to -3.64 for the second entry equations (II:1 - 6) and -1.82 for the third entry equation (II:7). For the validation data the AB values for equations (II:1- 6) ranged from 38.6 to 43.3 and for equation (II:7) the AB value was 26.1. The results for the validation data show a similar trend to the results obtained for the data set used to construct the volume equations. The third entry equation (II:7) had notable lower B and AB values than the other equations. Among the second entry equations the constructed equation (II:1) had the lowest AB value, 38.6, but together with equation (II:2) the largest B value, -8.36 and -8.21 respectively, indicating higher tendency for overestimation. Equation (II:6) showed the highest AB value, 43.27

A paired Student’s t-test applied on the measured and predicted volumes showed that all equations in the present study had p-values > 0.05, indicating that the difference between the observed and predicted diameters was non-significant. There were minor differences in the t-test statistics for equations (1)-(6). Equation (7) has a mean more close to zero and a lower standard error (SE) and a smaller range (min to max) than the other equations, indicating both high accuracy and precision (Table 5 in paper II).

Equation (II:2) showed high levels of multicollinearity for both the fitting and validation data.

Low levels of multicollinearity and absolute bias combined with demand of variable easy to measure in field ranks equations (II:1), (II:3), (II:4) and (II:5) to be more suitable for routine forest surveys and inventories than the other equations.

4.2.3 Five volume equations results on six validation trees

The four most suitable equations (II:1, 3, 4 and 5) for routine surveys and the constructed third entry equation (II:7) were applied on six representative sample trees, two each collected from the three diameter classes: DBH < 25cm, 30cm < DBH < 35cm and DBH ≥ 40 cm. These sample trees were obtained from five stands in the validation data set.

The small and medium sized trees (no. 1-4) were collected as follows: two trees (designated nos. 1 and 4; DBH 22.8 and 33.3 cm respectively) were selected from a 18-year-old stand with 910 stems per hectare; tree nr 2 with DBH 21.4 cm from a 20-year-old stand with 1460 stems per hectare and tree nr 3 from with DBH 32.1 cm from a 21-year-old stand with 800 stems per hectare. The two larger trees (designated nos. 5 and 6, DBH 40.0 and 46.4 cm respectively) where chosen from two 20-year-old stands with 200 and 550 stems per hectare respectively.

The diameters of the larger trees (nos. 4 & 5) slightly exceeded the basal area-weighted DBH of their respective stands, while the diameters of the other sampled trees were close to the arithmetic mean diameter of their stands. The differences in predictions were greatest between the three second entry equations (II:1), (II:3), (II:4) and (II:5) and the third entry equation (II:7). While the scale of differences between observed diameters and those predicted by the second entry equation notably varied between the small, intermediate and large trees the variations in this respect for the third entry equation were minor (Table 4).

Table 4. *The performance of five equations applied on six validation trees*

Tree no.	1	2	3	4	5	6
DBH (cm)	22.8	21.4	32.1	33.3	46.4	40.0
Height (m)	23.4	19.6	27.6	25.0	23.5	27.0
Trees ha^{-1}	910	1460	800	910	200	550
Volume m^3	0.406	0.315	1.069	0.890	1.389	1.438

Equation	m3 (deviance %) [1]					
(II:1)	0.437 (107.6)	0.319 (101.3)	0.962 (90.0)	0.906 (101.8)	1.457 (104.9)	1.334 (92.8)
(II:3)	0.442 (108.9)	0.325 (103.2)	0.943 (88.3)	0.897 (100.8)	1.458 (105.0)	1.330 (92.5)
(II:4)	0.441 (108.6)	0.318 (101.0)	0.951 (89.0)	0.905 (101.7)	1.459 (105.0)	1.334 (92.8)
(II:5)	0.439 (108.1)	0.330 (104.8)	0.942 (88.1)	0.901 (101.2)	1.436 (103.4)	1.330 (92.5)
(II:7)	0.418 (103.0)	0.308 (97.8)	1.034 (96.7)	0.885 (99.4)	1.445 (104.0)	1.369 (95.2)

1) deviance % =100 x (pred vol/obs vol),
(predict value equal to observed value results in deviance % =100)

5 Discussion

In the present study the volume of the sample trees was calculated by applying Smalian's formula, Vol=($Area_1$ + $Area_2$)/2) x length, on data for the 1-m sections and summing the results for each tree, which were also compared with calculated volumes based on 2-m and 3-m sections.

Compared to the "true" volume calculated by the 1-m section the deviance was almost negligible of the volume calculations for the 2-m section lengths (equivalent to <1% of the volume for 69 % and <2% of the volume for 94 % of the sampled trees). The deviance between volume calculations based on the 1-m sections and the 3-m section lengths were larger: < 1% of the volume for 46 % and < 2% of the volume for 76 % of the sampled trees. These findings indicate that for practical purposes recording diameters at 1-m intervals may be unnecessary, since 2-m sections provide sufficient data to generate accurate taper and volume equations for poplar trees. To calculate volume on 3-m sections, or longer sections, indicate insufficient precision in the volume calculations since more than half of the sample trees has a deviance > 1% and 12 % of the sample trees show a deviance > 3% in the volume calculations compare to calculations based on the 1-m sections. It is also possible to measure sections with different lengths at fixed heights and/or relative heights along the stem, as shown in previous sampling design studies that have examined the effects of varying the numbers and lengths of sections on the performance of taper equations (Newton & Sharma, 2008).

Forest management parameters (initial spacing, cleaning intensity and thinning regimes), can affect the form and taper of individual trees (Steven & Benee, 1988; Karlsson 2005). Analysis of the slenderness (diameter/height, cm/m) of poplar trees examined in Paper I revealed that slenderness values are highest when the number of trees per hectare is less than 1500. This indicates that the stem form/slenderness, and thus the stem taper, is correlated to some degree with the stocking and closure of the stand. In the dataset used for Paper I there are only a few measurements of trees in stands with 1500 to 3000 stems per hectare, thus general conclusions about the effect of stocking densities on stem slenderness are likely to be speculative. However, future planting and management strategies for poplar plantations intended to produce timber with specific diameters should take account of the effect of stocking density on average stem diameter. Thus, further research into plantation management strategies should include detailed analyses of correlations between stocking and tree form, using tools such as the stem taper equations developed in this study. This should help to increase yields of desired assortments.

The validation dataset included measurements of trees at sites other than those used to develop the newly-constructed equations. The data are within the ranges of the fitting data set in terms of age, height and DBH, Table 2.

However, unlike the fitting data, the validation set lacks data for trees at sites with dominant heights <20.5 m and mean diameters <17.8 cm (Table 1). Few young stands were available and priority was given to using data from these stands for fitting the equations.

The systematic selection of sample trees used in this study should generally be avoided, but was necessitated by restrictions described in the Material and Methods section. According to Kozak (1997), systematic selection of sample trees could cause obtained regression coefficients to be biased and lead to greater under-estimation of true variations than a random selection strategy. These potential problems should be especially considered if the trees have been grown under various conditions within a site and there is a wide range of tree sizes. In the studies this thesis is based upon, however, this problem was minor since all stands considered were located on former farmland, conditions within the stands were nearly homogenous and the range of tree sizes was small.

When using the studied stem taper and volume equations it is important to apply them on poplar trees with heights and DBH within or close to the range limits of the fitting data. Application on trees far out from the range might cause the predictive values to be unrealistic and for small trees the predictions can have negative algebraic values.

The level of multicollinearity of the equations in both paper I and II was tested with respect to both the fitting and validation data to ensure that no potential problem of multicollinearity was present.

In Paper I equations (I:5) and (I:6) did suffer of multicollinearity while equations (I:1 – I:4) exhibit values below limit for multicollinearity.

In paper II the most suitable second entry equations (II:1), (II:4) and (II:5) and third entry equation (II:7) the CI values were lower than the commonly used limit for multicollinearity (CI<30). However, the evaluated equation (II:2) showed a high multicollinearity level (CI >100).

The results and trends are consistent for both the fitting and validation data. According to Kozak (1997) the presence of multicollinearity in a model does, however, not seriously affect its predictive capability. However, when selecting an equation, statistical models should be used to identify and give priority to equations with low multicollinearity.

Potential problems with autocorrelation are obvious when constructing taper equations. The models are developed with dependent diameter data from several heights from the same individual tree which among other can cause problem with the regression coefficients. On the other hand, autocorrelation does not seriously affect the prediction capabilities according to Kozak (1997).
The volume equations in paper II with only one independent volume data for each individual tree are not affected by autocorrelation as estimations on an individual tree is single based and not multiple hierarchical data.

5.1 Taper equations

Taper equations with a variable exponent that accounts for changes in shape along a stem (e.g. a neiloid root section, paraboloid mid-section and cone-shaped top section) provide better predictions of the diameter from ground to the top of a tree stem than simple and segmented taper equations (Kozak. 2004). Variable exponent taper equations generally have lower bias than other types of taper equations (Sakici et al. 2008) and the analysis of the taper equations in Paper I, based on the evaluation statistics presented in Tables 4, 5 and 6 in Paper I, confirms these findings. The relative absolute bias (AB %) for Kozaks variable exponent taper equation (I:6) was 3.8 %, which was lower than for the other taper equations in the study.

Equation (I:4), developed by Benbrahim and Gavaland (2003), shows larger residuals based on the data used in Paper I than in the cited study (up to 6 cm versus <1 cm). This difference might be due to differences in data structure. The cited authors used data obtained from trees in young stands (7-8 years) with a mean height of 13 m and mean DBH of 12 cm, while the fitting data used in Paper I were obtained from trees growing in stands with a mean age of 22 years (range 14-43 years), mean height of 23 m and mean DBH of 23 cm. Further, structure was observed in the residuals in Paper I, while Benbrahim and Gavaland (2003) observed no such structure. Generally, young poplars, such as those included in the study of Benbrahim and Gavaland, have not developed butt-swells on the stems. In contrast, the older trees sampled in Paper I had distinct and developed butt-swells from the ground to ca. 0.5 m up the stem. None of the studied equations could fully grasp this butt-swell, and most of the large residuals for the simple equations are related to this part of the stem. The occurrence of relatively large residuals related to the stump region is more pronounced for the simple equations (I:2) and (I:3) than for the other equations considered in this study (Figure 3 in Paper I).

5.2 Stem volume equations

When an upper height diameter is included as an independent variable together with DBH and H in volume equations (third entry equations) the performance of the predictions increases notably (Brandel, 1990; Kozak; 2004). Analysis of the volume equations in Paper II, based on the evaluation statistics for both the fitting and validation data presented in Tables 4 and 5 in Paper II, confirms these findings.

Including crown height as an additional independent variable did not improve the stem volume predictions, as corroborated by the relationships between crown heights and: stem volumes, diameters at breast height and total heights. Plots of crown height versus these variables show at best weak correlations with small R^2 values. Volume equation (II:7) had lower values of RMSE, absolute bias (AB) and relative absolute bias (AB%) values and thereby provided notably better predictions of the total volume compare to the other volume equations (II:1) to (II:6) in Paper II. Moreover, equation (II:6) has notably higher AB values than the other equations for both the fitting and validation data sets. Equation (II:3) has a simple bias > 2 for the fitting data, which is indicative of under-estimations while the B values of the other volume equations are < 1.

The newly-constructed equation (II:7) yielded the lowest RMSE values also in the validation exercise and had the lowest AB for both the fitting and the validation data (Tables 3 and 4 in Paper II). According to Parresol et al. (1987), AB values provide a clear distinction between examined equations and are important statistics for drawing conclusions and making recommendations regarding the suitability of equations for use in practical surveys. Results from the T-test on the validation data show that the mean error of standard deviation (SE) were lower for equation (II:7) than for the other equations. This equation are thus suitable when there is a need for high precision and accuracy and trees are cut, or when time and technical equipment are available to measure diameters high above ground. Moreover, when this equation are applied to trees of varying sizes their performance varies less between modeling small and large trees than the two entry equations (Table 3). This equation is also suitable for precise and accurate measurements and evaluations of individual trees in research trials.

The findings presented in Tables 4 and 5 in Paper II of higher bias (called mean in t-tests) in the validation data set than in the fitting data can be partly explained by the differences in structure between the two sets. This is partly because the fitting data include measurements of trees with smaller diameters at breast height than the validation set, for which the bias/residuals tend to be larger in terms of absolute m^3 values. These findings can also be partly explained by the differences in size and distribution of the sets. The validation data set is smaller (n =17) than the fitting set (n=51), and the limited numbers of trees in the validation data were found to have a slight larger distribution around the mean compared to the fitting data, especially for trees with a diameter at breast height >30 cm, Figure 2.

6 Conclusion

6.1 Taper equations

The poor performance for all equations at 90 % of stem height (Table 5 in Paper I) are not important from a practical point of view (Figueiredo-Filho, et al., 1996), since the top part of poplar stems (some meters below the top) is not used for any practical purpose except for bio-fuel. The variable exponent equation (I:6) yielded the lowest values of the absolute bias (AB),relative absolute bias (AB%), SSRR, and RMSE in evaluation statistics of all equations considered in Paper I (Tables 4 and 5 in Paper I) and performed well on the validation data. The statistical complexity with difficulties in rearrange equation (I:5 & 6) to predict height for a given diameter could be a practical reason for choosing other simpler equations. Other equations that could be recommended partly depend on the importance assigned to the criterion of the predicted diameter at the top of the tree (h=H). Equations (I:1) and (I:4) did not meet the zero top diameter prediction criterion. If a strict zero diameter prediction criteria at the top is not required, which is mostly the case from a practical point of view, then the second ranked constructed polynomial equation (I:1) is recommended.

6.2 Stem volume equations

The three entry equation (II:7) perform well and is recommended when precise and accurate volume predictions are needed. However, due to the need of measuring an upper diameter, it is less useful and not appropriate for routine surveys and inventories, for which equations (II:1), (II:3), (II:4) and (II:5) are more suitable. There are small differences in RMSE, B, AB and AB% values between these equations for the fitting data (Table 4 in Paper II), but in a combined evaluation, also considering their performance on the validation data, equations (II:1) and (II:4) are recommended.

The studied and recommended volume equations can be used for robust calculations of volume at stand level. By volume calculations of individual stems and the sum of these volumes can then be recalculated to obtain estimates of volume per unit area (ha). This enables the development of volume tables and matrices of volume per hectare (m^3 ha^{-1}), which are some of the most important and frequently used tools in forest planning and management operations. The volume equations can also be used to calculate mean annual increment (MAI).

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